Topological defects in nematics: fundamentals and applications

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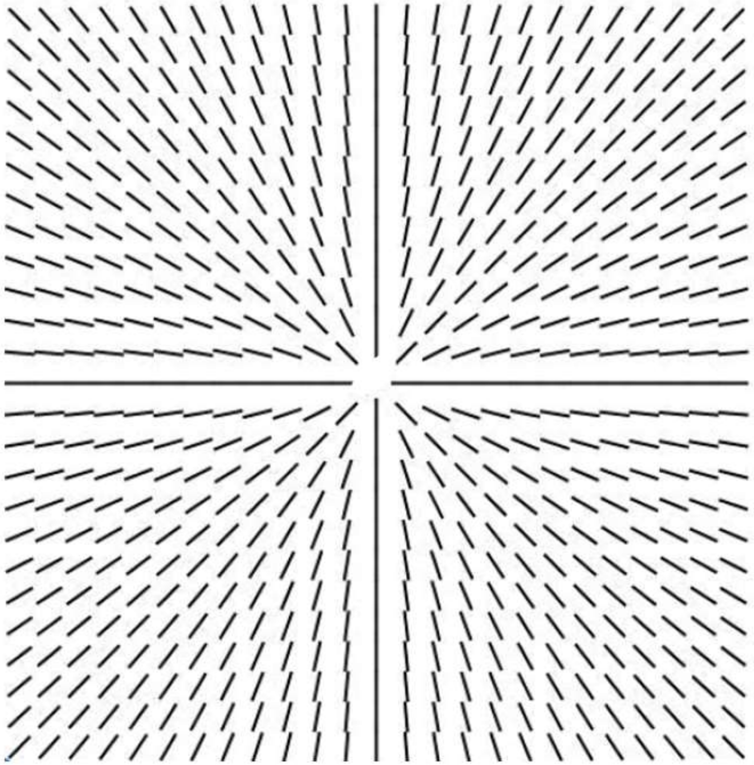
Abstract:

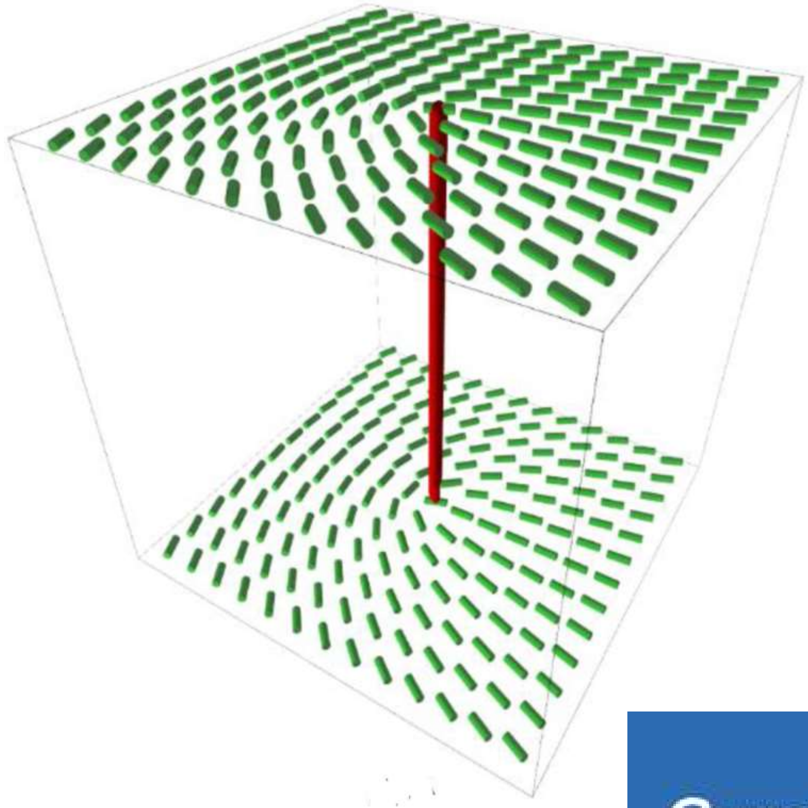
Topological defects (TDs) constitute topologically protected frustrated regions in a host field of an ordered manifold. They are ubiquitous in nature and appear at all scales, including the realms of particle physics, condensed matter, cosmology… We demonstrate that a simple plane parallel cell that confines a nematic liquid crystal (LC) could host diverse complex and multistable configurations of TDs, which we stabilized using the AFM scribing method. These competitive states could be reversibly and robustly reconfigured by appropriate external electric fields. Furthermore, we show that complex lattices of line defects, which are otherwise unstable or stable in a narrow interval of temperatures, could be stabilized efficiently by doping LCs with appropriate nanoparticles. We demonstrate that such TD configurations have potential for diverse applications, particularly in nano- and biotechnology: *e.g.*, for nanotechnology-based devices based on reconfigurable conducting nanowires, tunable photonic devices, sensitive sensors… Furthermore, our study of TDs might provide some insight into still unresolved problems of fundamental physics. Namely, LCs could exhibit so-called “chargeless” twist disclinations, which commonly decay into a defectless state. Twist TDs could simultaneously act as *defects* and *antidefects* [3], and such neighboring pairs could be mutually annihilated. These configurations bear some resemblance to intriguing Majorana particles.

Keywords: topological defects; liquid crystals; nanoparticles

Crystals 2020

I. INTRODUCTION





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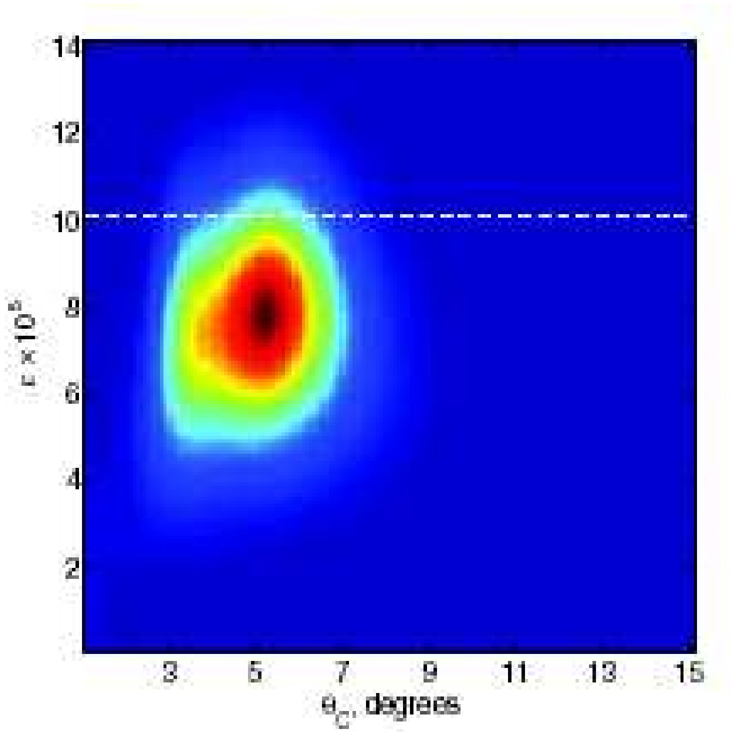

Science **318** , 1612 (2007)

A Cosmic Microwave Background Feature Consistent with a Cosmic Texture

M. Cruz,[1,2*] N. Turok,[3] P. Vielva,[1] E. Martínez-González,[1] M. Hobson[4]

The Cosmic Microwave Background provides our most ancient image of the universe and our best tool for studying its early evolution. Theories of high-energy physics predict the formation of various types of topological defects in the very early universe, including cosmic texture, which would generate hot and cold spots in the Cosmic Microwave Background. We show through a Bayesian statistical analysis that the most prominent 5°-radius cold spot observed in all-sky images, which is otherwise hard to explain, is compatible with having being caused by a texture. From this model, we constrain the fundamental symmetry-breaking energy scale to be $\phi_0 \approx 8.7 \times 10^{15}$ gigaelectron volts. If confirmed, this detection of a cosmic defect will probe physics at energies exceeding any conceivable terrestrial experiment.

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There are no particles, there are only fields

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ahobson@uark.edu

Quantum foundations are still unsettled, with mixed effects on science and society. By now it should be possible to obtain consensus on at least one issue: Are the fundamental constituents fields or particles? As this paper shows, experiment and theory imply unbounded fields, not bounded particles, are fundamental. This is especially clear for relativistic systems, implying it's also true of non-relativistic systems. Particles are epiphenomena arising from fields. Thus the Schroedinger field is a space-filling physical field whose value at any spatial point is the probability amplitude for an interaction to occur at that point.

Am.J.Phys. **81** (3), 211 (2013)

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Skyrmions :

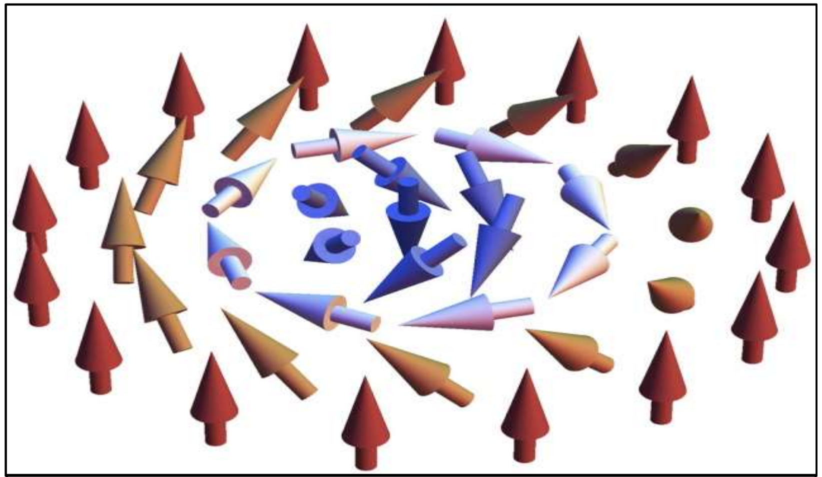
as p, n in a pion-field

T. Skyrme,

A unified field theory of mesons and bayrons,

Nucl. Phys. **31**, 556 (1962).

Typical Skyrmion



Localized topological distortion in a continuous field

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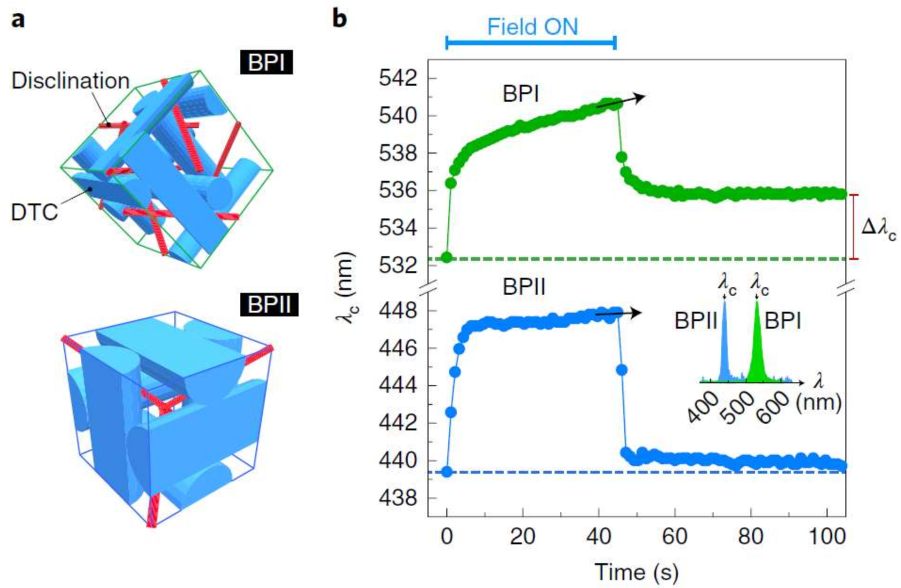
Applications

ARTICLES
https://doi.org/10.1038/s41563-019-0512-3
nature materials

Reconfiguration of three-dimensional liquid-crystalline photonic crystals by electrostriction

Duan-Yi Guo[1,4], Chun-Wei Chen[2,4], Cheng-Chang Li[1], Hung-Chang Jau[1], Keng-Hsien Lin[1], Ting-Mao Feng[1], Chun-Ta Wang[1], Timothy J. Bunning[3], Iam Choon Khoo[2*] and Tsung-Hsien Lin[1*]



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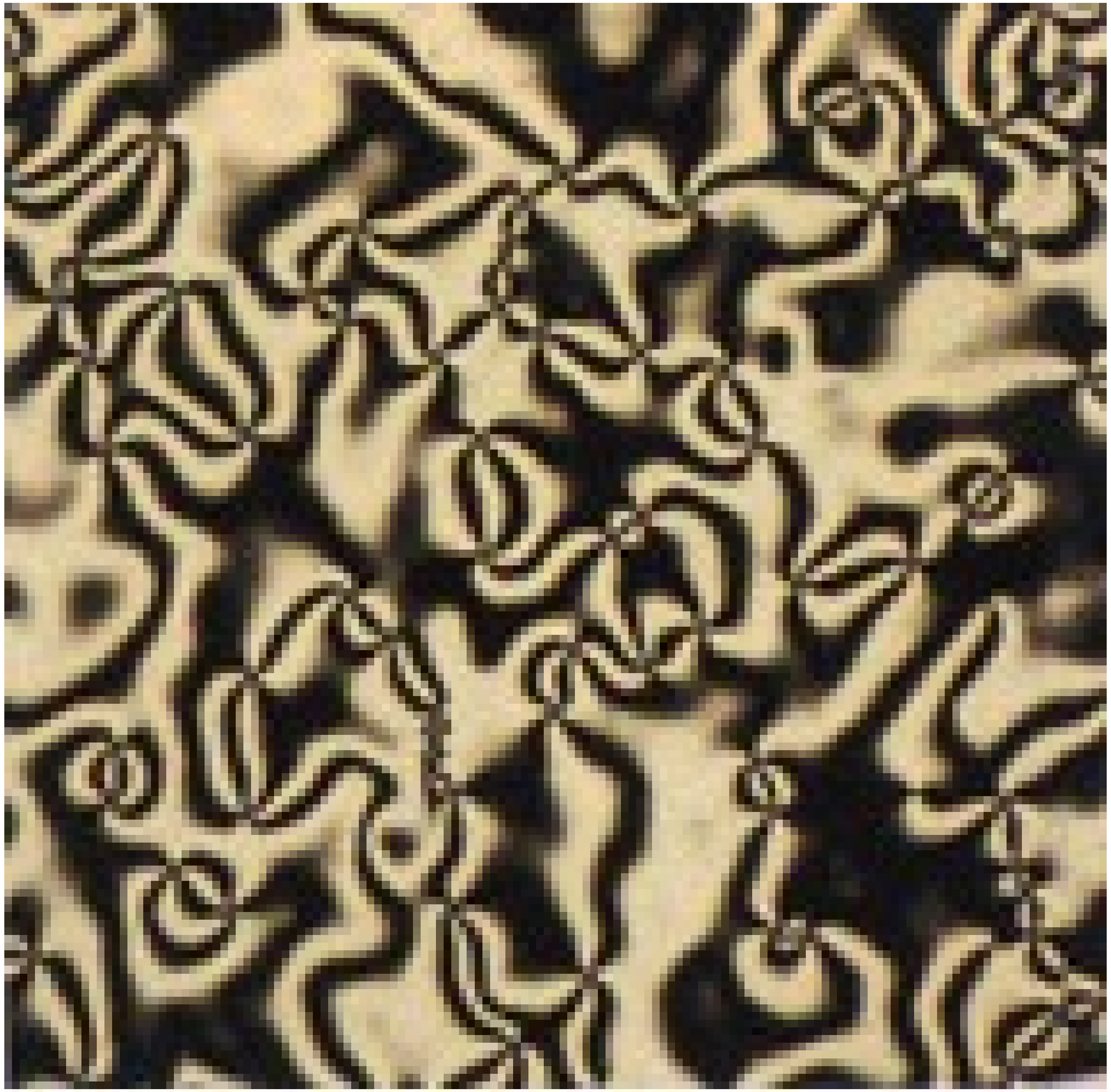
Adequate testbed – Liquid Crstals



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CONTENT:

Topological Defects in Liquid Crystals

- **Order parameter** : amplitude&phase
- **Topological charge**
- **Origin** of topological defects
  - Stabilisation : confinement
  - Stabilisation : curvature
    - Intrinsic and extrinsic curvature
  - Stabilisation of "chargeless" topological defects
  - Stabilisation with nanoparticles

- **Conclusions**

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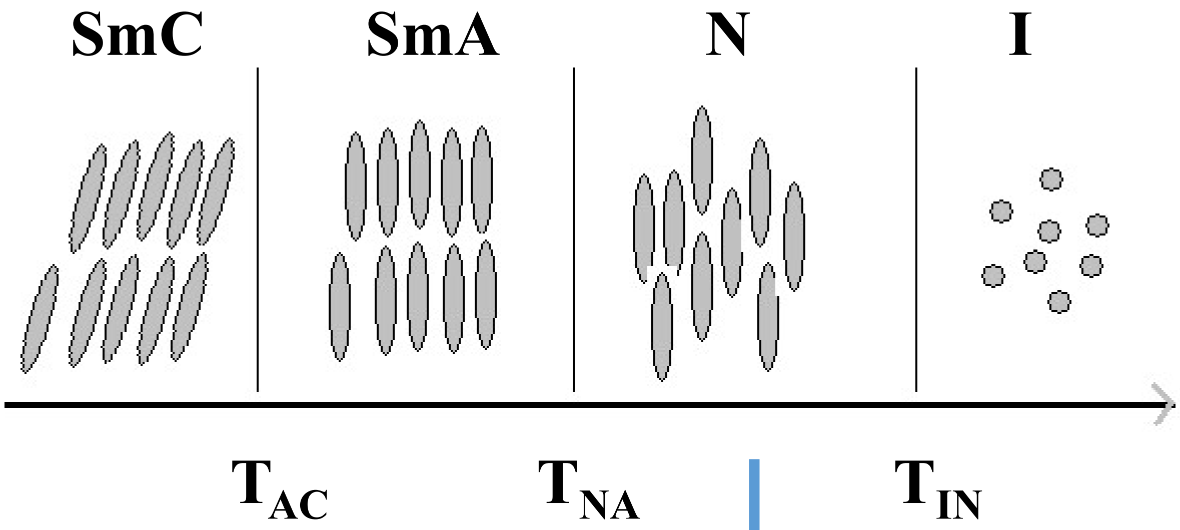
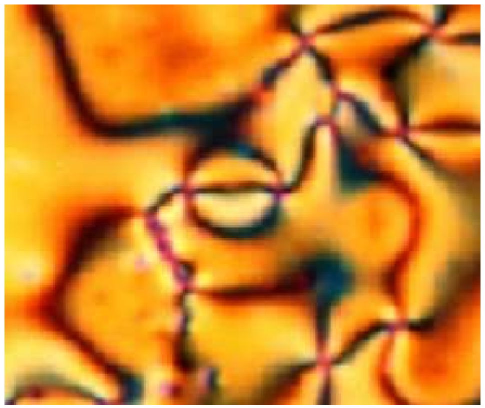
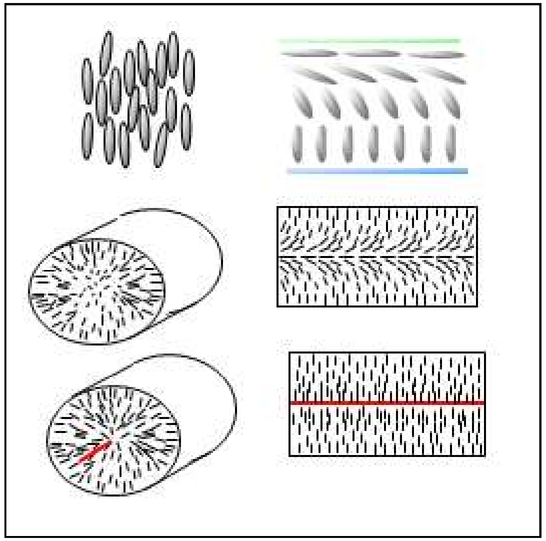
II. Topological Defects in Liquid Crystals







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Order parameter : amplitude&phase

Continuous symmetry breaking phase transition:

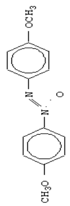
Order parameter

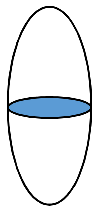
Amplitude
(hard component,
non-hydrodynamic
field)

Phase
(soft component,
hydrodynamic or
gauge field)

Palffy, Phys.Today **60**, 54 (2007).

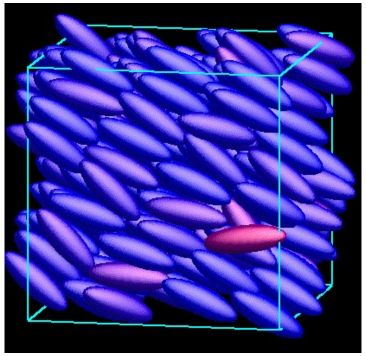
Orientational order :





$\vec{n}$

S

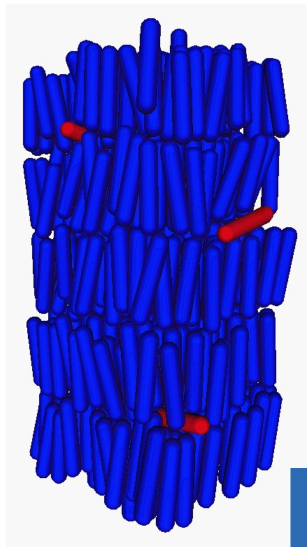


$$\underline{Q} = \frac{S}{2}\left(3\vec{n}\otimes\vec{n} - \underline{I}\right)$$

Translational order :

$$\psi = \eta e^{i\phi}$$

$$e.g.: \quad \phi = q_0 z$$



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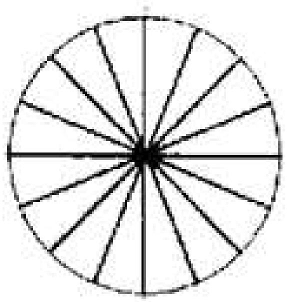
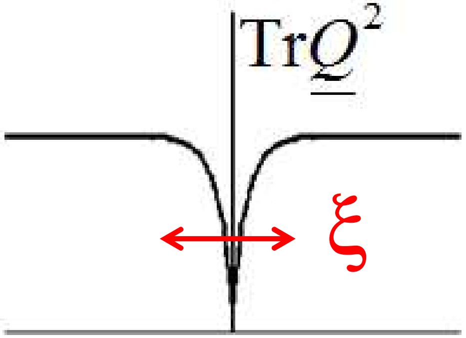
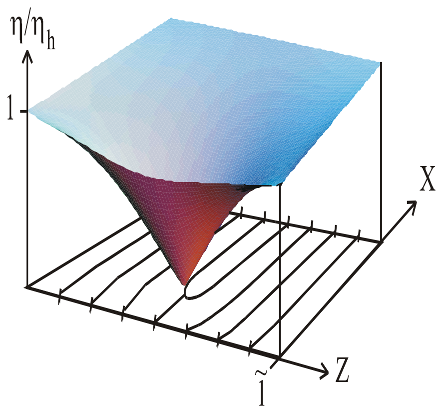
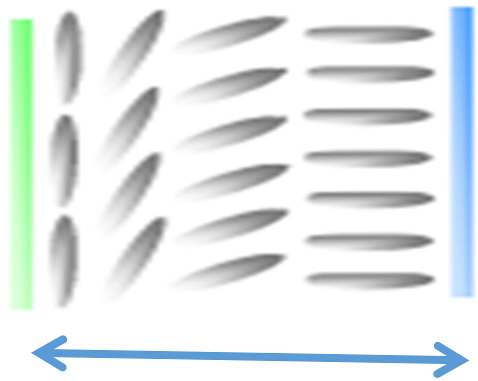
amplitude

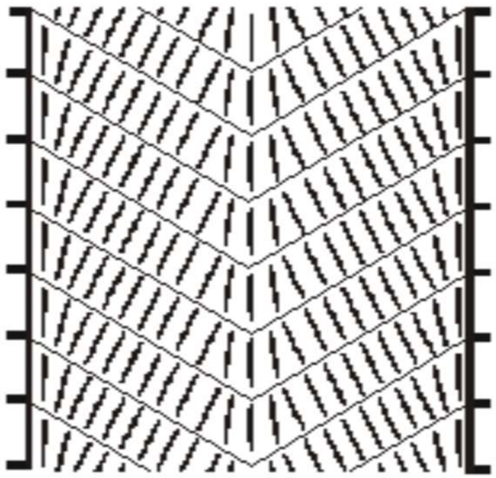






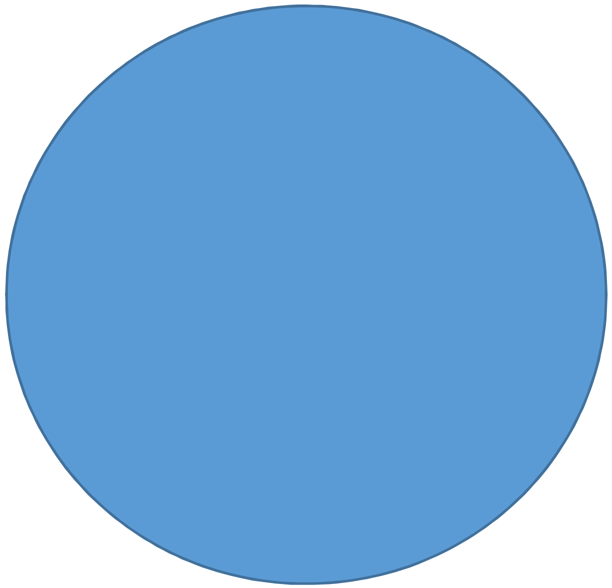
phase





Topological charge

Order parameter space



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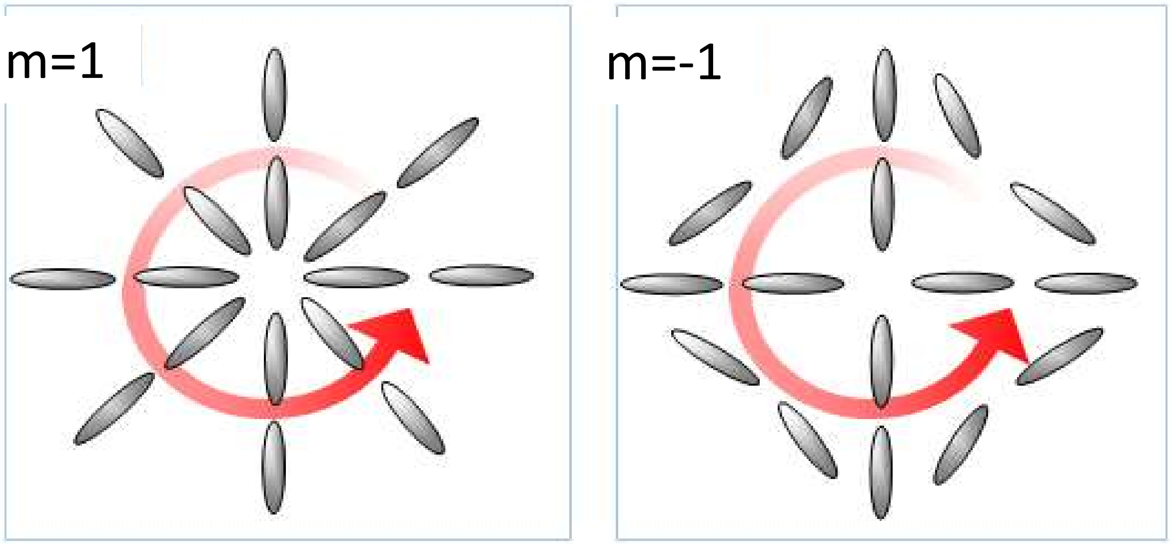

defect ***anti-defect***

$$m = \frac{\sum_{encircle} \theta_{rotation}}{2\pi}$$

$$m \in \{\pm 1/2, \pm 1, \pm 3/2 ....\}$$

Conserved quantity

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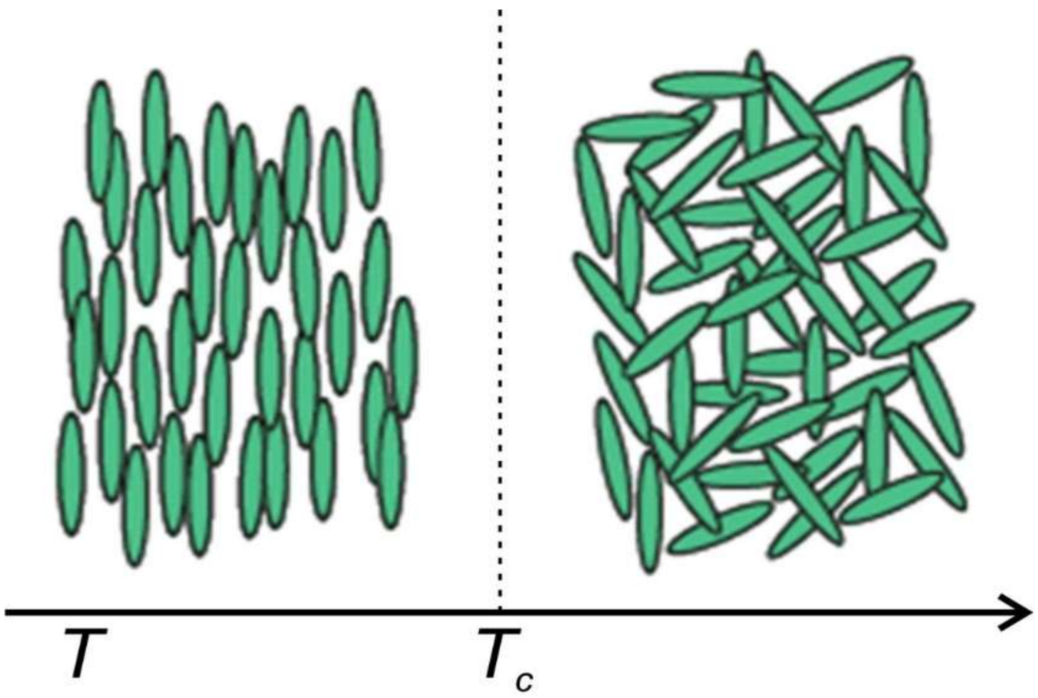
Origin of TDs

Continuous symmetry breaking



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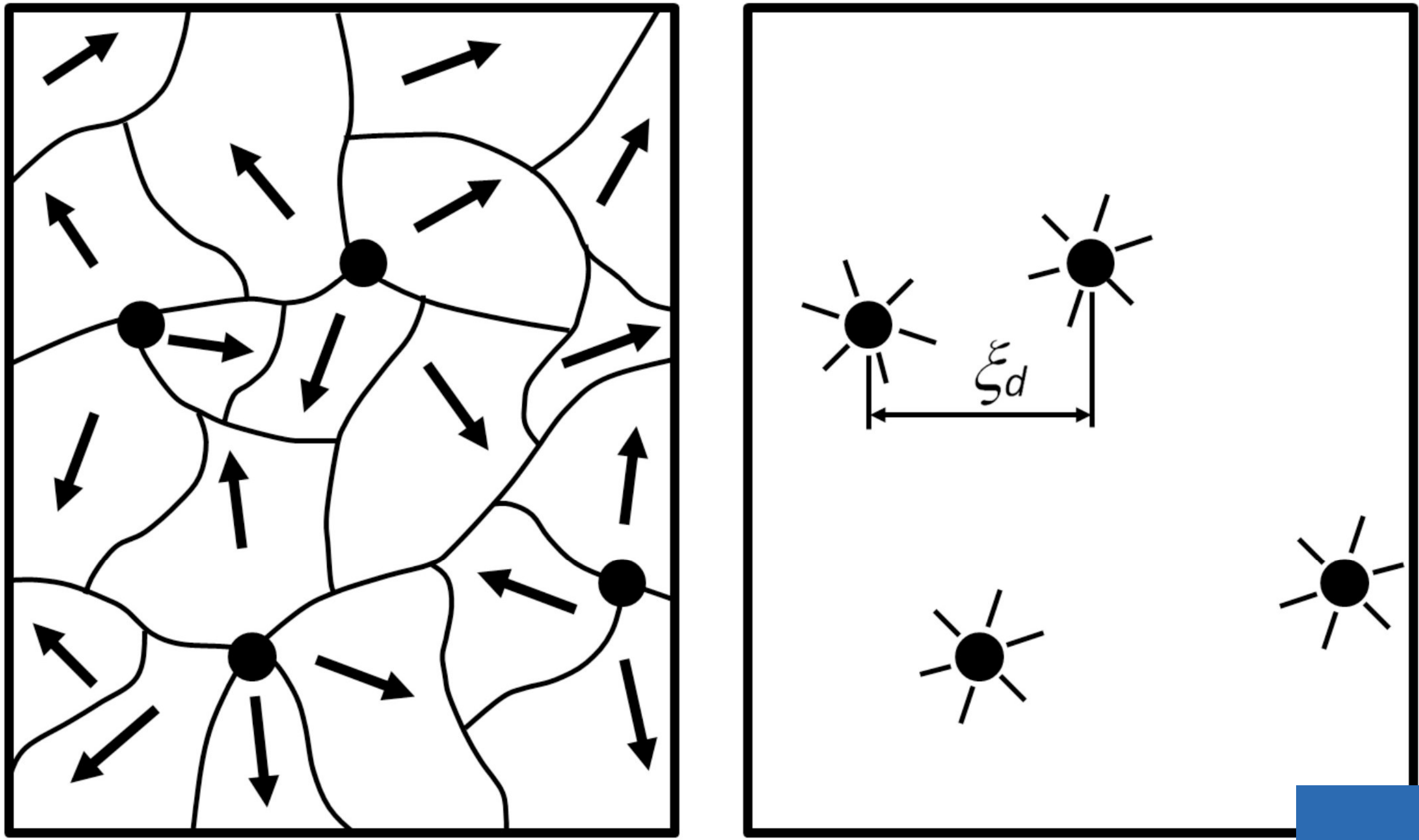
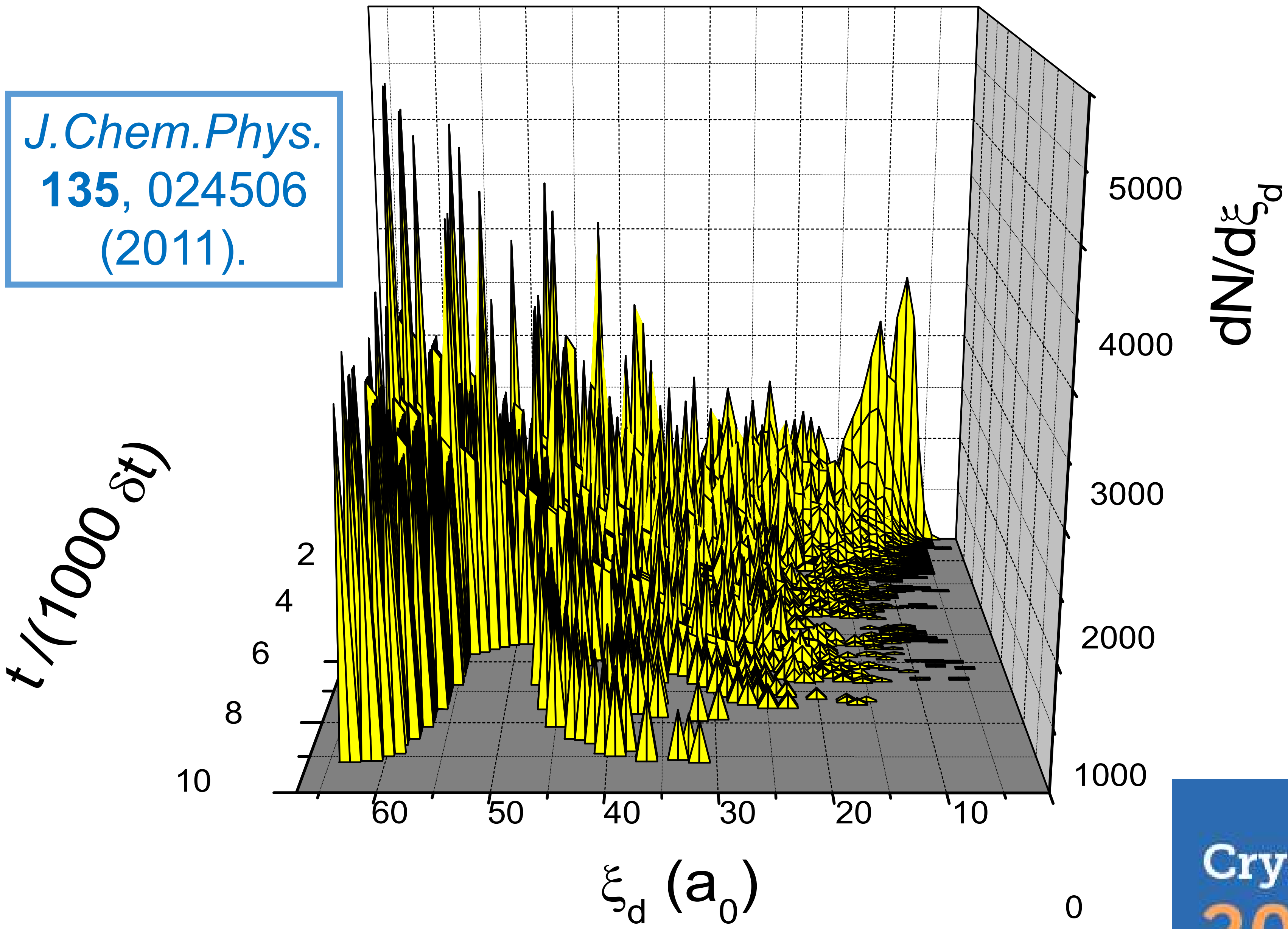

ξ_d

J.Chem.Phys. **135**, 024506 (2011).



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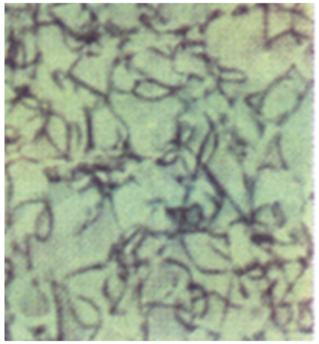
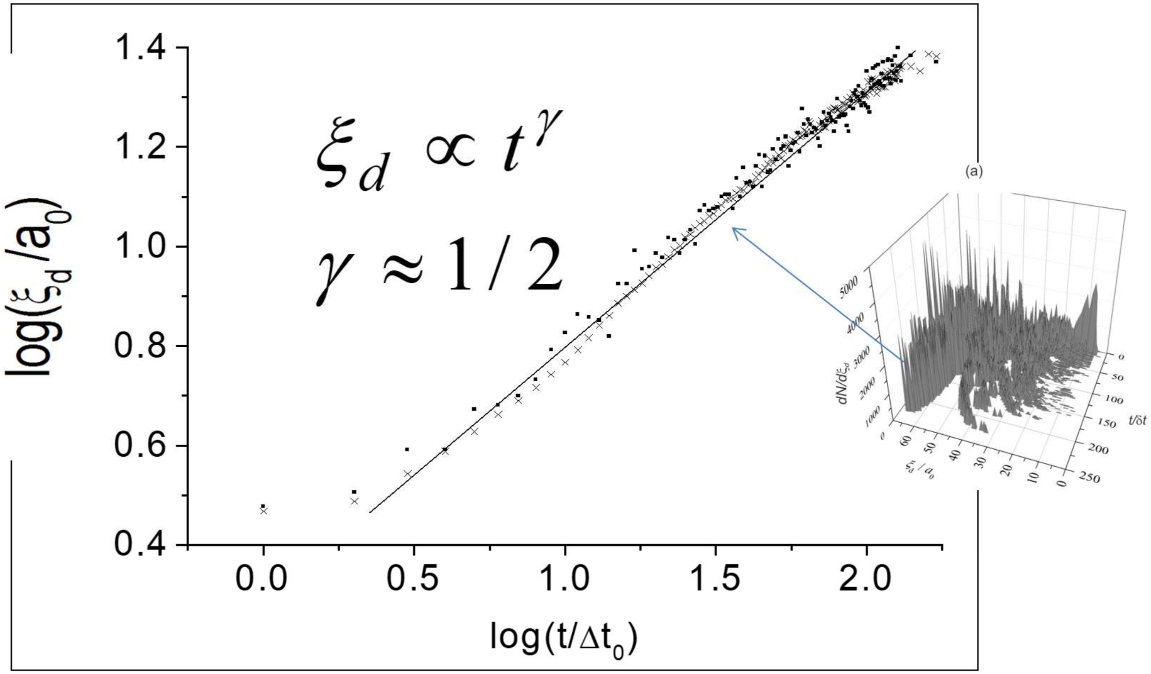
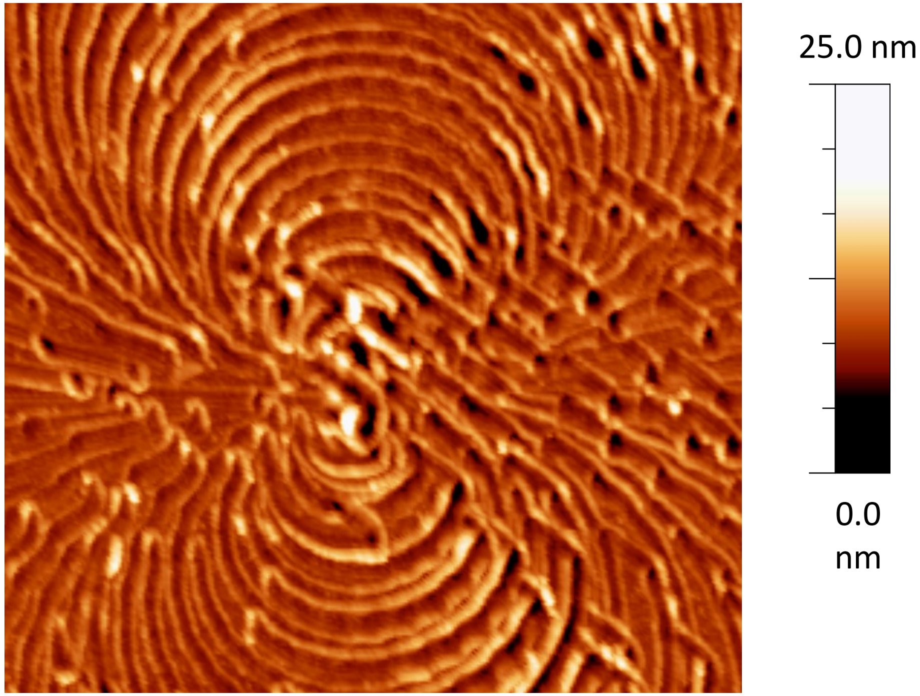




Phys.Rev.E **65**, 021705 (2002).
J.Chem.Phys. **135**, 024506 (2011).

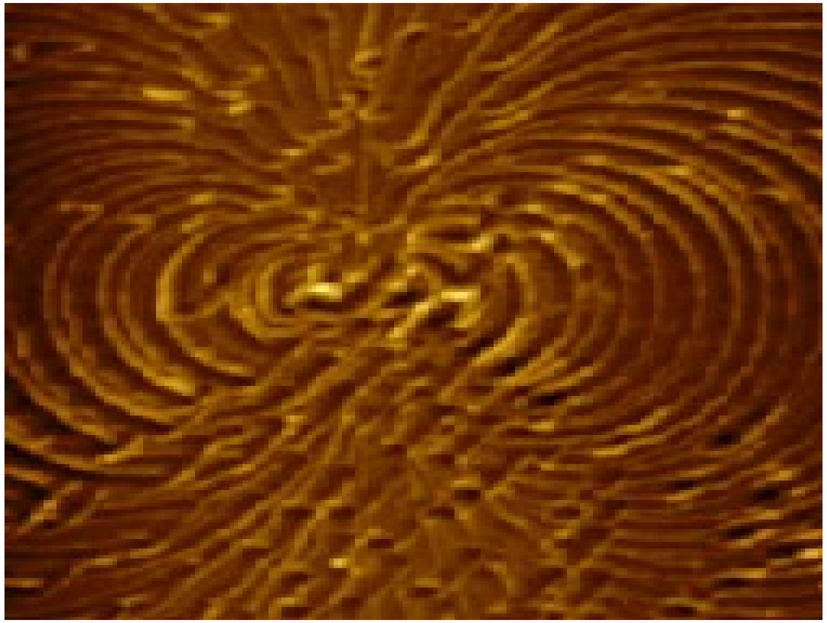
Stabilisation : confinement

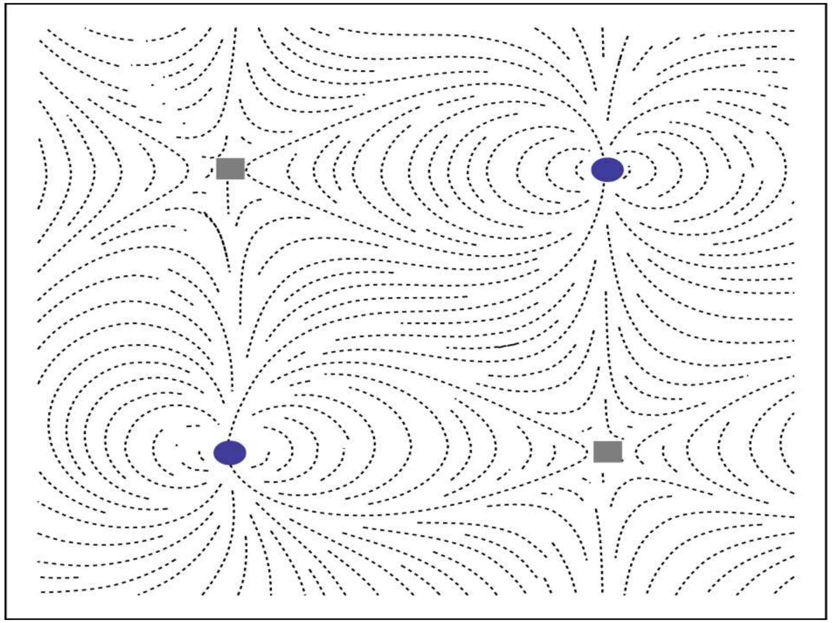


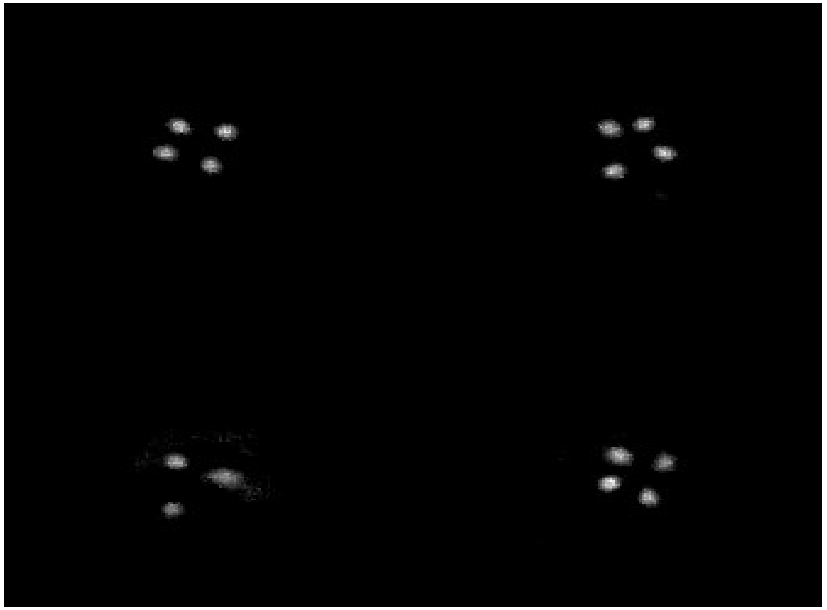
$$\theta = \sum_{i=1}^{N} \left(m_i \; ArcTan\left(\frac{y - y_i}{x - x_i} \right) + \theta_0^{(i)} \right)$$

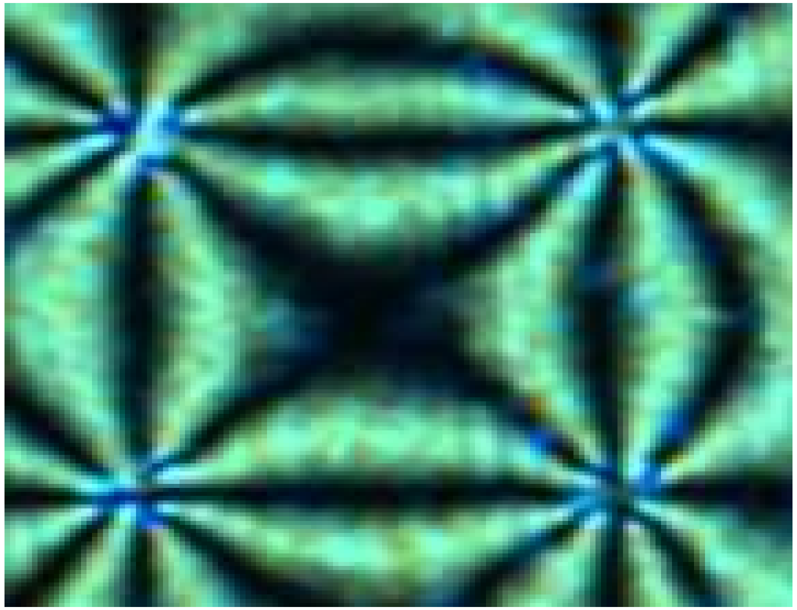
Phys. Rev. E **95**, 042702 (2017)

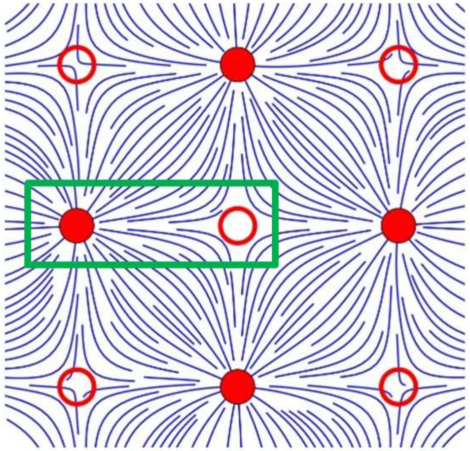
Crystals
2020

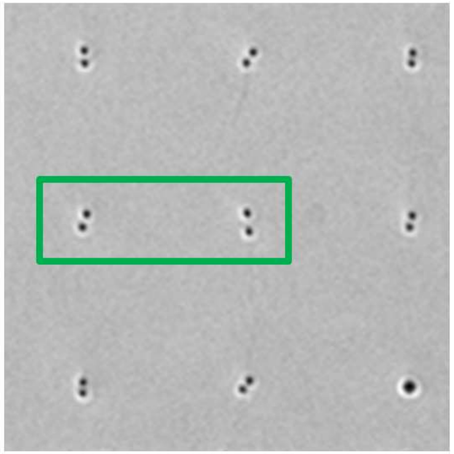






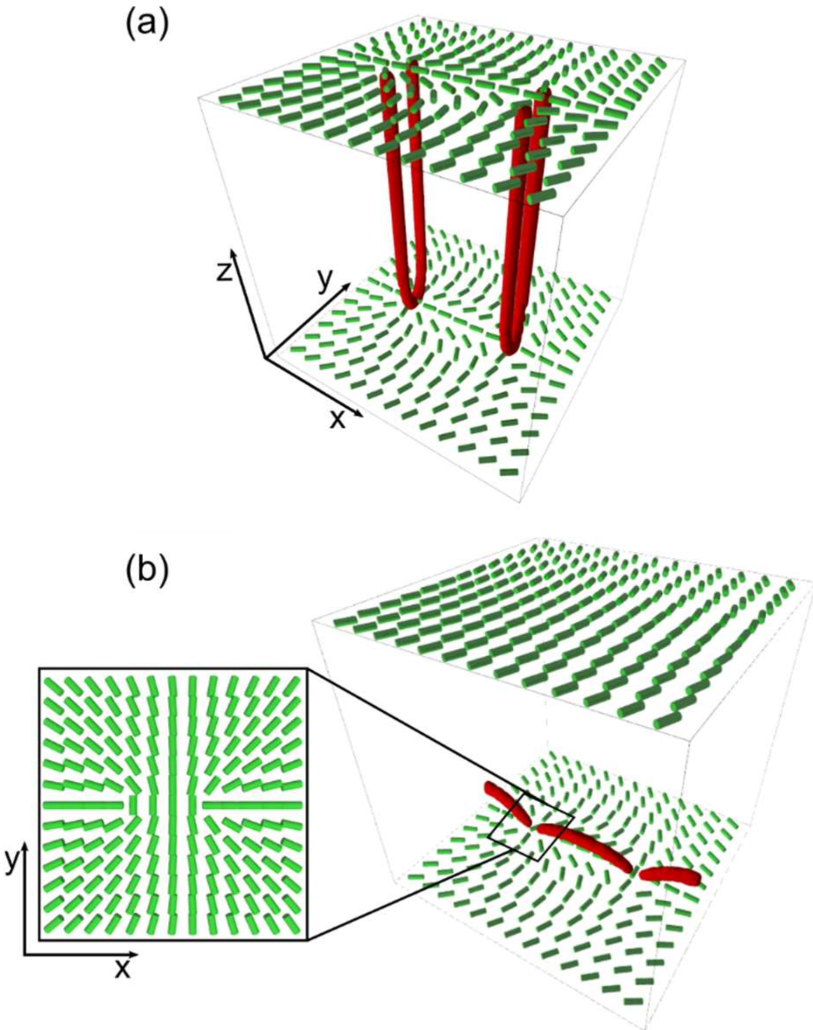






m=+1

m=-1



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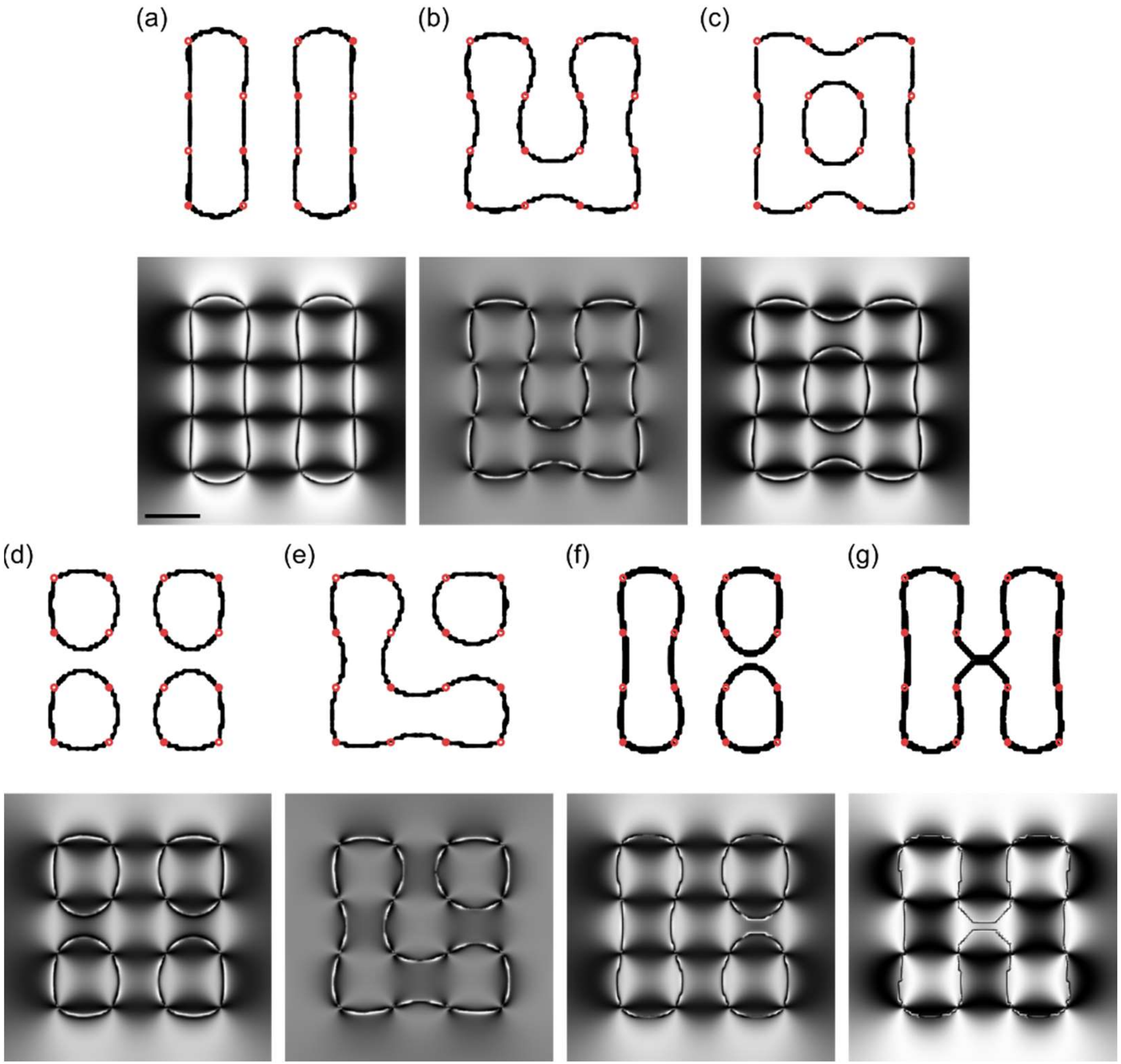
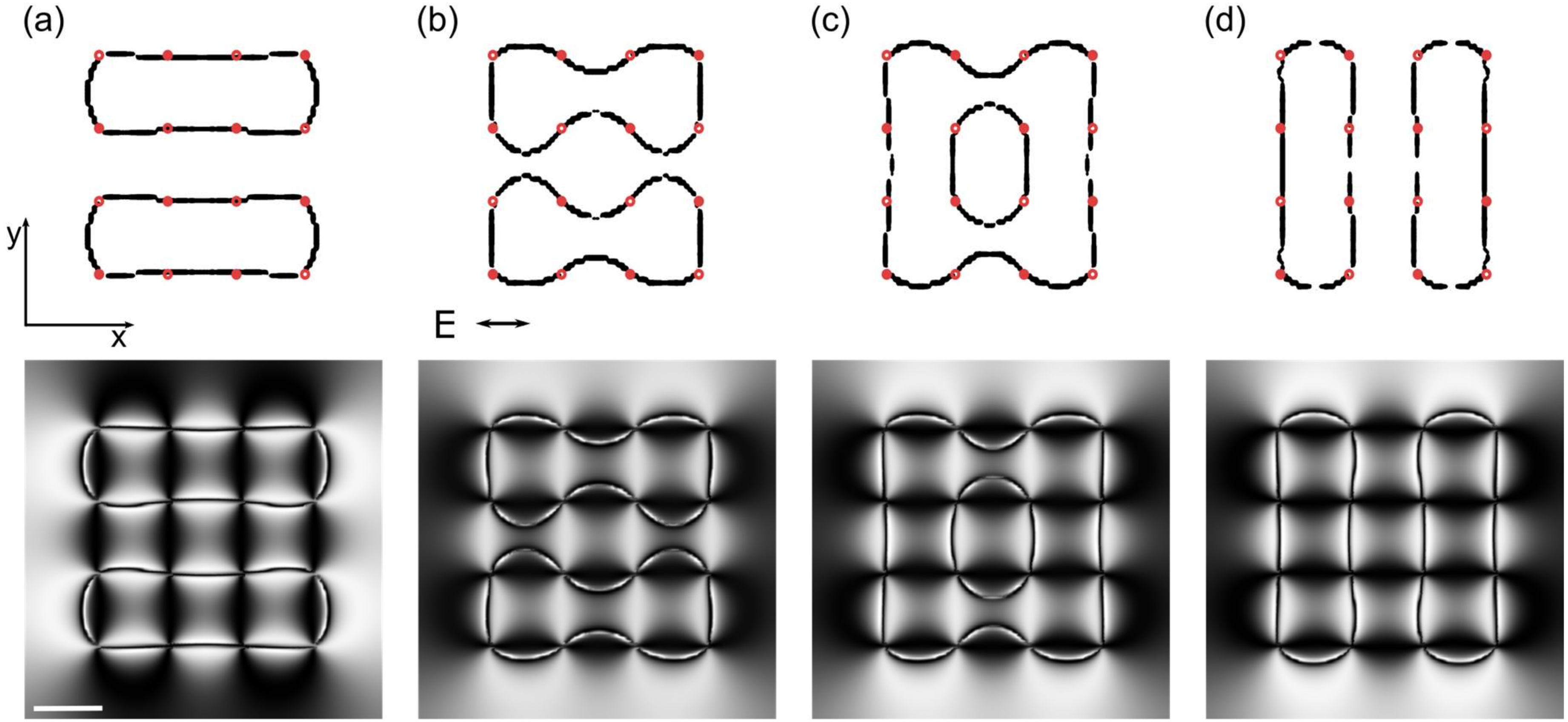

(a)
(b)
(c)
(d)
(e)
(f)
(g)



Phys. Rev. Res. 2, 0131761 (2020).

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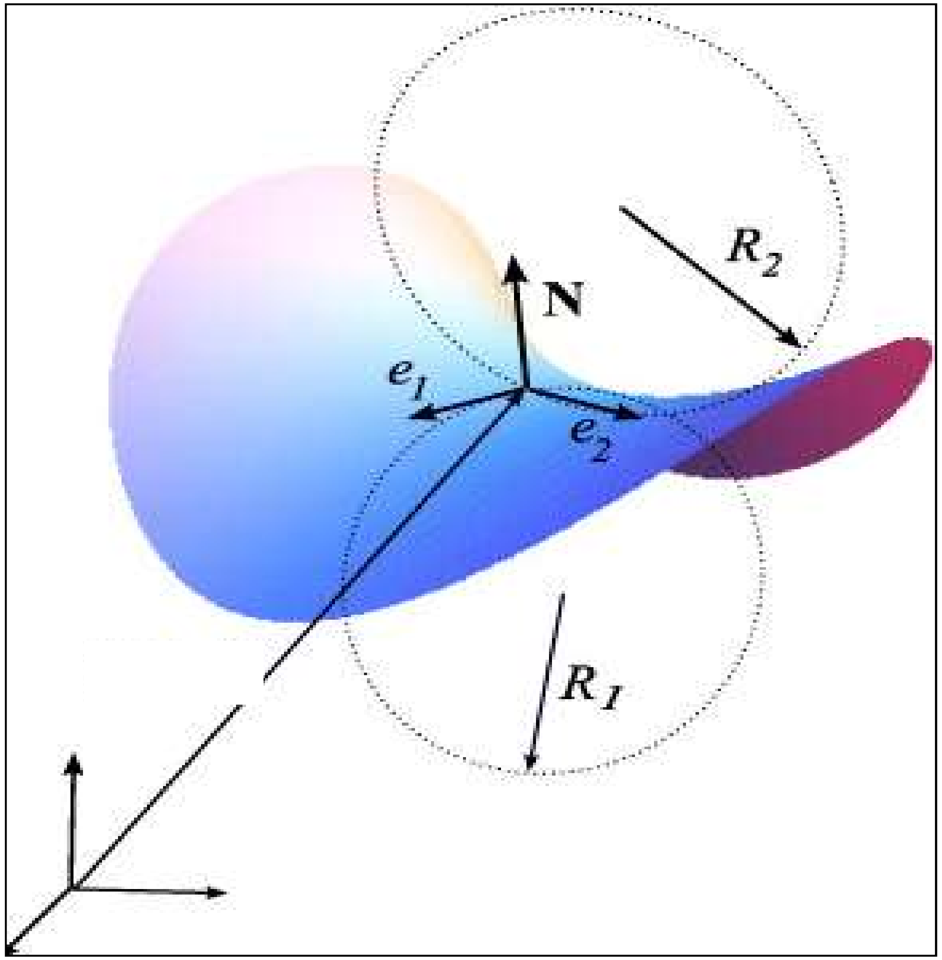
Stabilisation : curvature



Gaussian curvature

$$K = C_1 C_2 = \frac{1}{R_1 R_2}$$

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Gauss – Bonnet & Poincaré-Hopff theorem

$$m_{tot} = \frac{1}{2\pi} \oiint K d^2\vec{r} = \chi$$

Poincare, J.Math.Pures **2** (IV), 151 (1886).

Geometry		χ
Sphere		2
Torus		0
Double torus		-1

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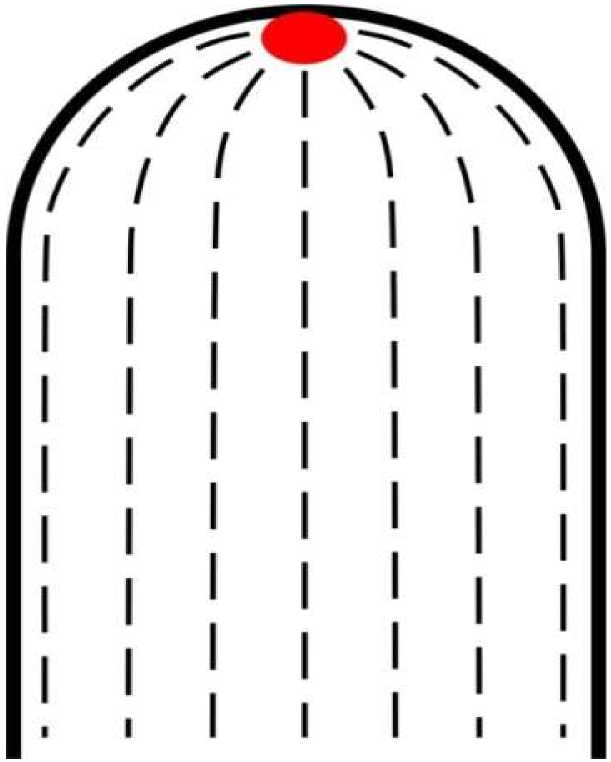
Stabilisation : curvature

Effective Topological Charge Cancelation (ETCC) mechanism

$$\Delta m_{eff}(\Delta\zeta) = \frac{1}{2\pi}\iint_{\Delta\zeta} K d^2\vec{r}$$

$$\boldsymbol{\Delta m_{eff}(\Delta\zeta) \rightarrow 0}$$

Sci. Rep. 6, 27117 (2016).



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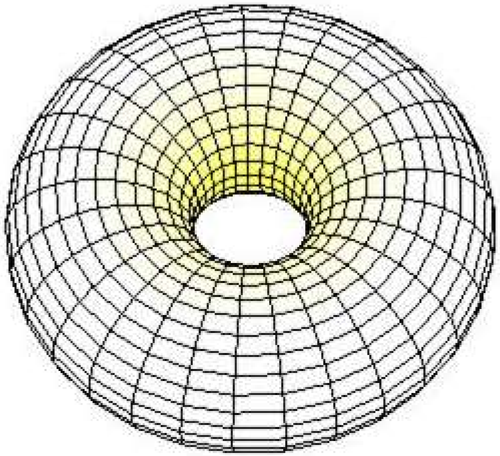
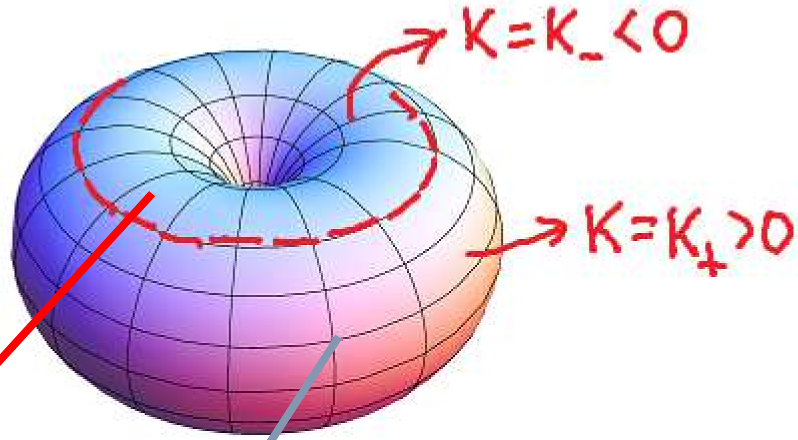
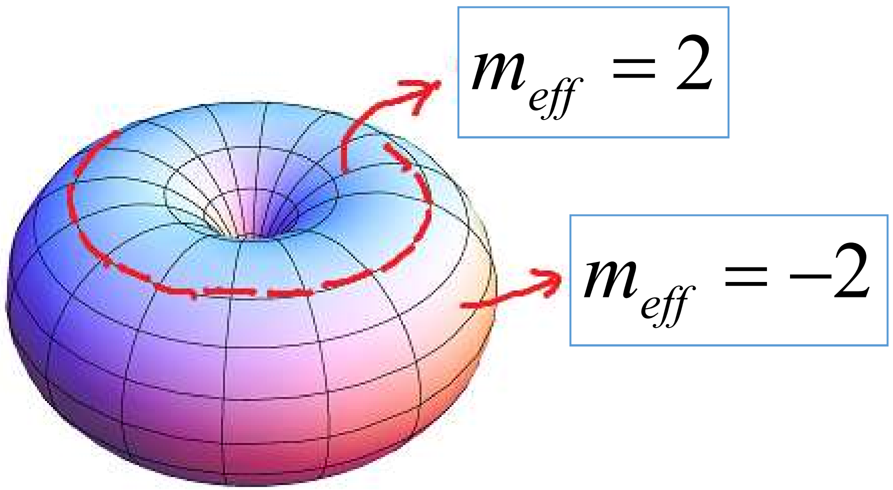
TORUS

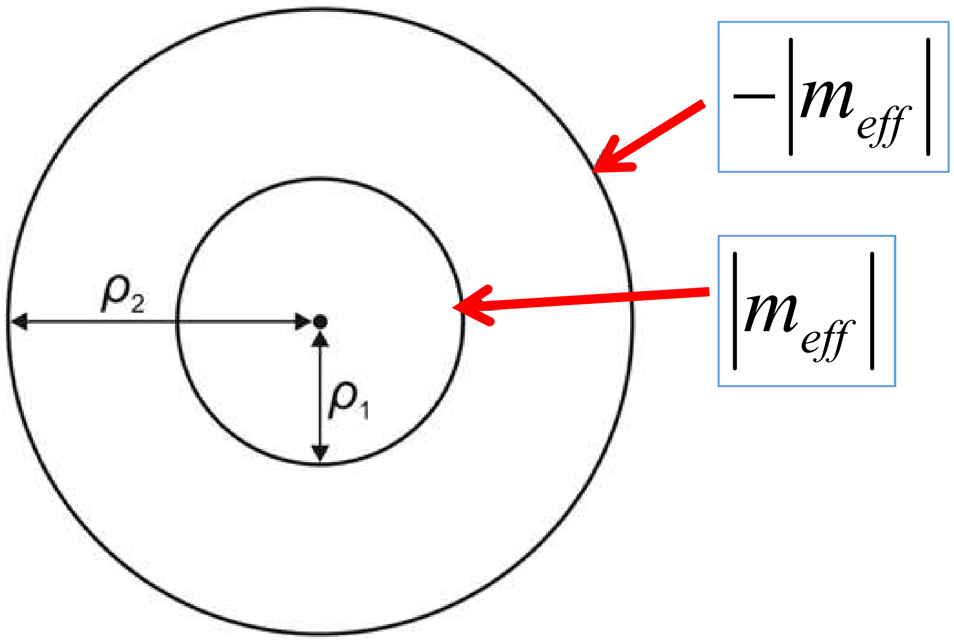
m_{tot}=0

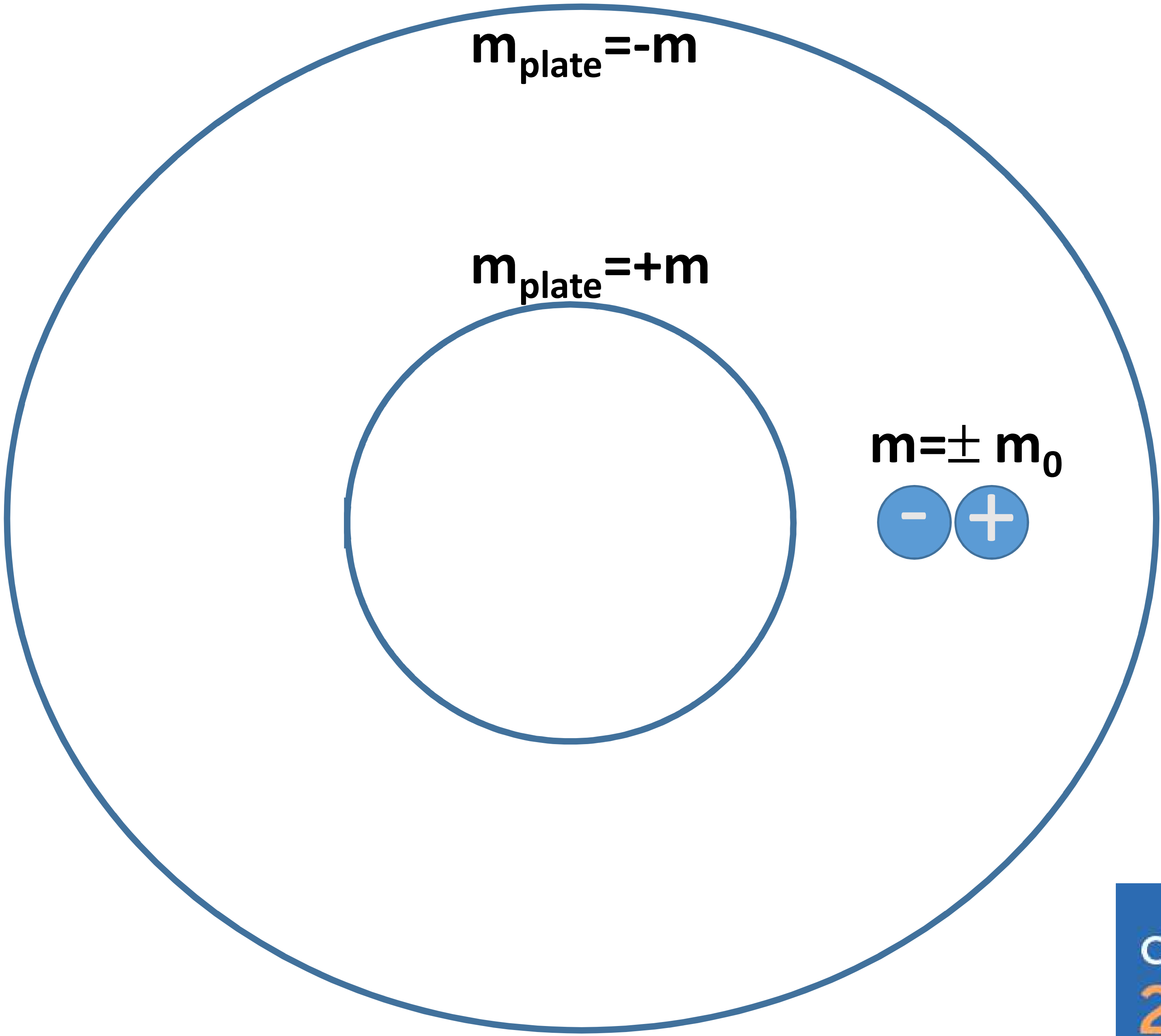




$$\frac{1}{2\pi}\int K d^2\vec{r} = \frac{1}{2\pi}\int K_- d^2\vec{r} + \frac{1}{2\pi}\int K_+ d^2\vec{r} = -2 + 2 = 0$$


$m_{eff} = 2$
$m_{eff} = -2$


$-|m_{eff}|$
$|m_{eff}|$
ρ_2
ρ_1



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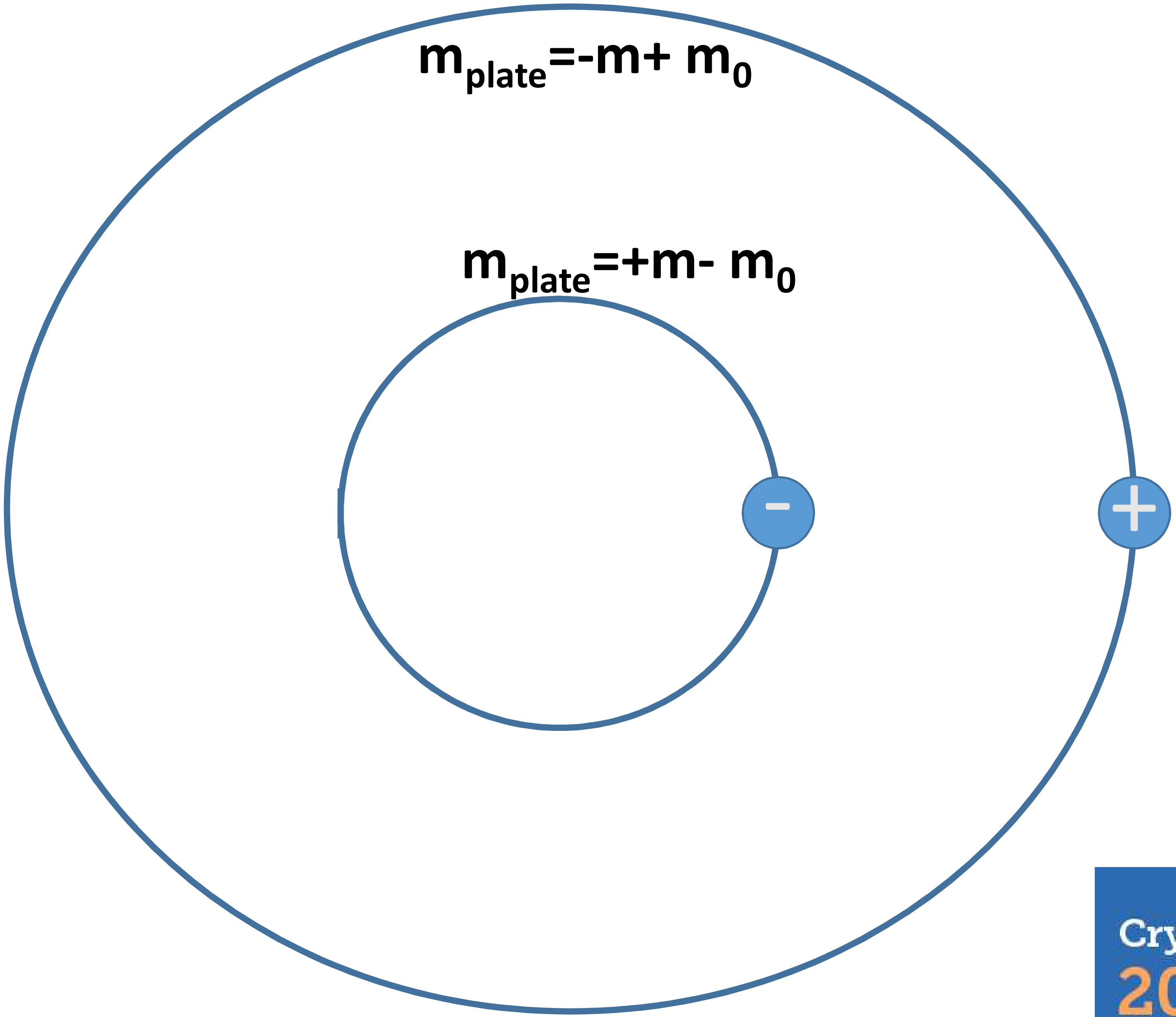

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2D electrostatics :	2D nematic :
$$W_p = \frac{e_1 e_2}{2\pi\varepsilon_0 h} ln\rho + const.$$	$$W_{\text{int}} \sim m^2 4\pi K_f \; ln\left(\frac{\rho}{a_0}\right)$$
$$F_{12} = E_1 e_2 = \frac{1}{2\pi\varepsilon_0 h}\frac{e_1 e_2}{\rho}$$	$$F_{12} = Em \sim 4\pi K_f \frac{m^2}{\rho}$$
$$E = \frac{1}{2\pi\varepsilon_0 h}\frac{e}{\rho}$$	$$E \sim 4\pi K_f \frac{m}{\rho}$$

$$1 + \tfrac{1}{2}\left(\ln\left(\frac{\rho_2}{\rho_1} - 1\right)\frac{\rho_1}{2\xi}\right) \sim \Delta m_{eff} \ln\left(\frac{\rho_2}{\rho_1}\right)$$

penalty = gain

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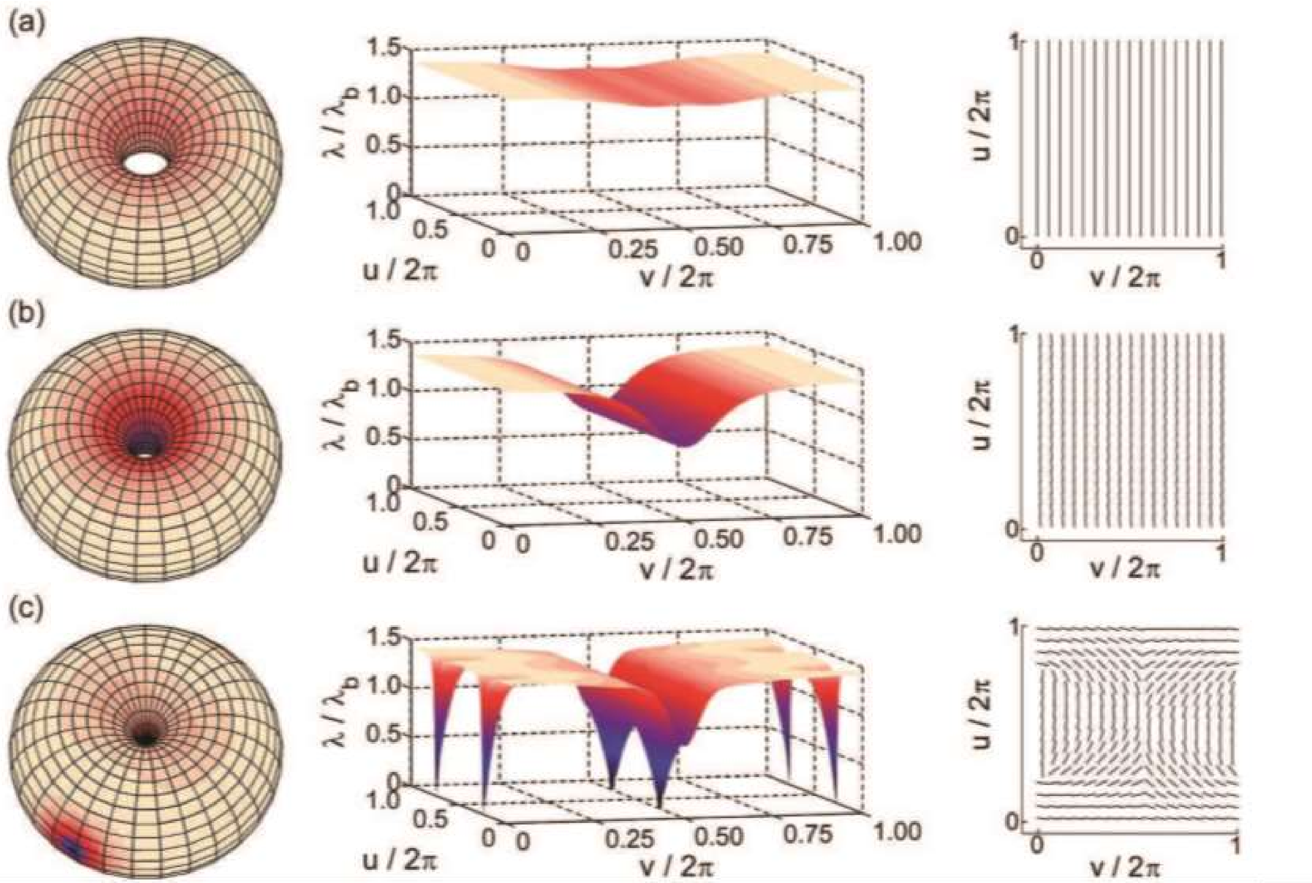

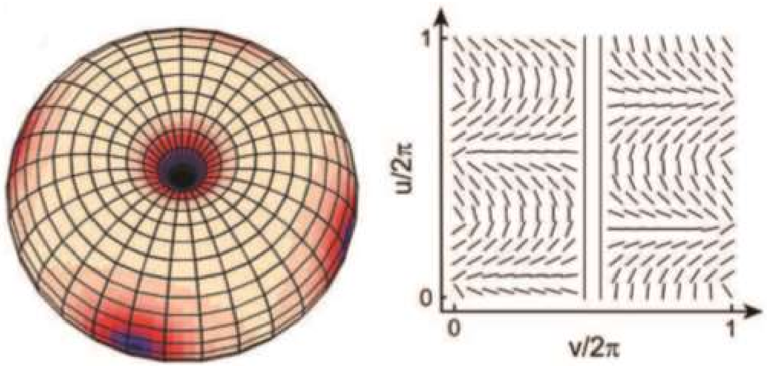

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Intrinsic and extrinsic curvature

Key curvature terms

Selinger et al,
J. Phys. Chem. B **115**,
13989 (2011).

$$W_{\text{int}} = -J \sum_{i,j(nn)} \vec{S}_i . \vec{S}_j$$

$$W_{int} = \iint \frac{K}{2} \left| \nabla \vec{S} \right|^2 dA$$

$$\vec{S} = \vec{e}_1 \cos\theta + \vec{e}_2 \sin\theta$$

$$\left| \nabla \vec{S} \right|^2 = \left| \nabla\theta - \vec{A} \right|^2 + \vec{e}_1 . \underline{C}^2 \vec{e}_1$$

Intrinsic — Extrinsic

$$curl\vec{A} \propto K = \frac{1}{R_1 R_2}$$

$$\underline{C} = \begin{vmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{vmatrix}$$

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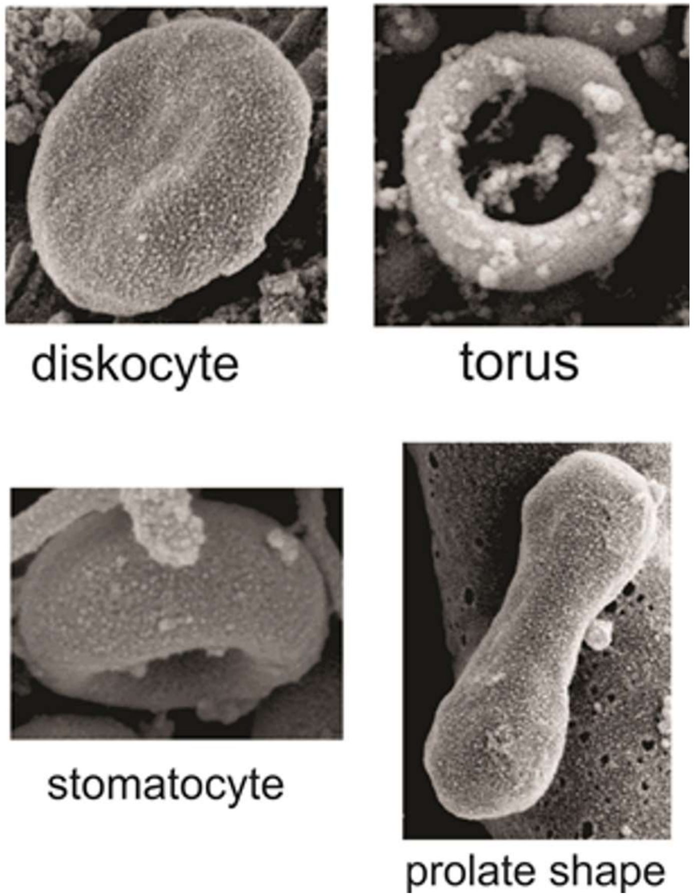
Variational parameters: $\underline{Q}, \quad \underline{C}$

Vary relative volume :

$$v = V / V_0$$

$$S = 4\pi R_0^2$$

$$V_0 = 4\pi R_0^3 / 3$$



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Intrinsic curvature term

$$K\left|\nabla\theta - \vec{A}\right|^2 \rightarrow 0$$

$$\nabla\theta = \vec{A}$$

$$curl\nabla\theta \equiv 0 \neq curl\vec{A} \propto \boldsymbol{K}$$

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$$f^{(intrinsic)} = K_e \left|\nabla\theta - \vec{A}\right|^2$$

$$\nabla \times \nabla\theta = 0 \neq \nabla \times \vec{\boldsymbol{A}} \propto \boldsymbol{K_g} = \frac{1}{\boldsymbol{R_1 R_2}}$$

Smectic A LC phase

$$f^{(compress)} = C_{\parallel}|(i\vec{n}q - \nabla)\psi|^2 \sim C_{\parallel}\eta^2|(\vec{n}q - \nabla\theta)|^2$$

$$\nabla \times \nabla\theta = 0 \neq \boldsymbol{\nabla} \times \vec{\boldsymbol{n}}$$

$\psi = \eta e^{i\theta}$ $chirality \rightarrow \nabla \times \vec{n} \neq 0$

Superconductor

$$f = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\hbar^2}{4m}\left|(\nabla - \frac{2ie}{\hbar c}\vec{A})\psi\right|^2 + \frac{(\nabla \times \vec{A})^2}{4\mu_0\mu}$$

$$f^{(coupl)} \sim \frac{\hbar^2}{4m}\eta^2\left|\nabla\theta - \frac{2e}{\hbar c}\vec{A}\right|^2$$

$$\nabla \times \nabla\theta = 0 \neq \nabla \times \vec{\boldsymbol{A}} = \vec{\boldsymbol{B}}$$

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Minimal model

$$f = \kappa Tr\underline{C}^2 - \alpha Tr\underline{Q}^2 + \beta/2\left(Tr\underline{Q}^2\right)^2 + k_i\left|\nabla\underline{Q}\right|^2 + k_e\underline{Q}.\underline{C}^2$$

$$\underline{Q} \to \underline{Q}/\lambda_0 \qquad \lambda_0 = \sqrt{\alpha/\beta} \qquad \underline{C} \to \underline{C}/R_0$$

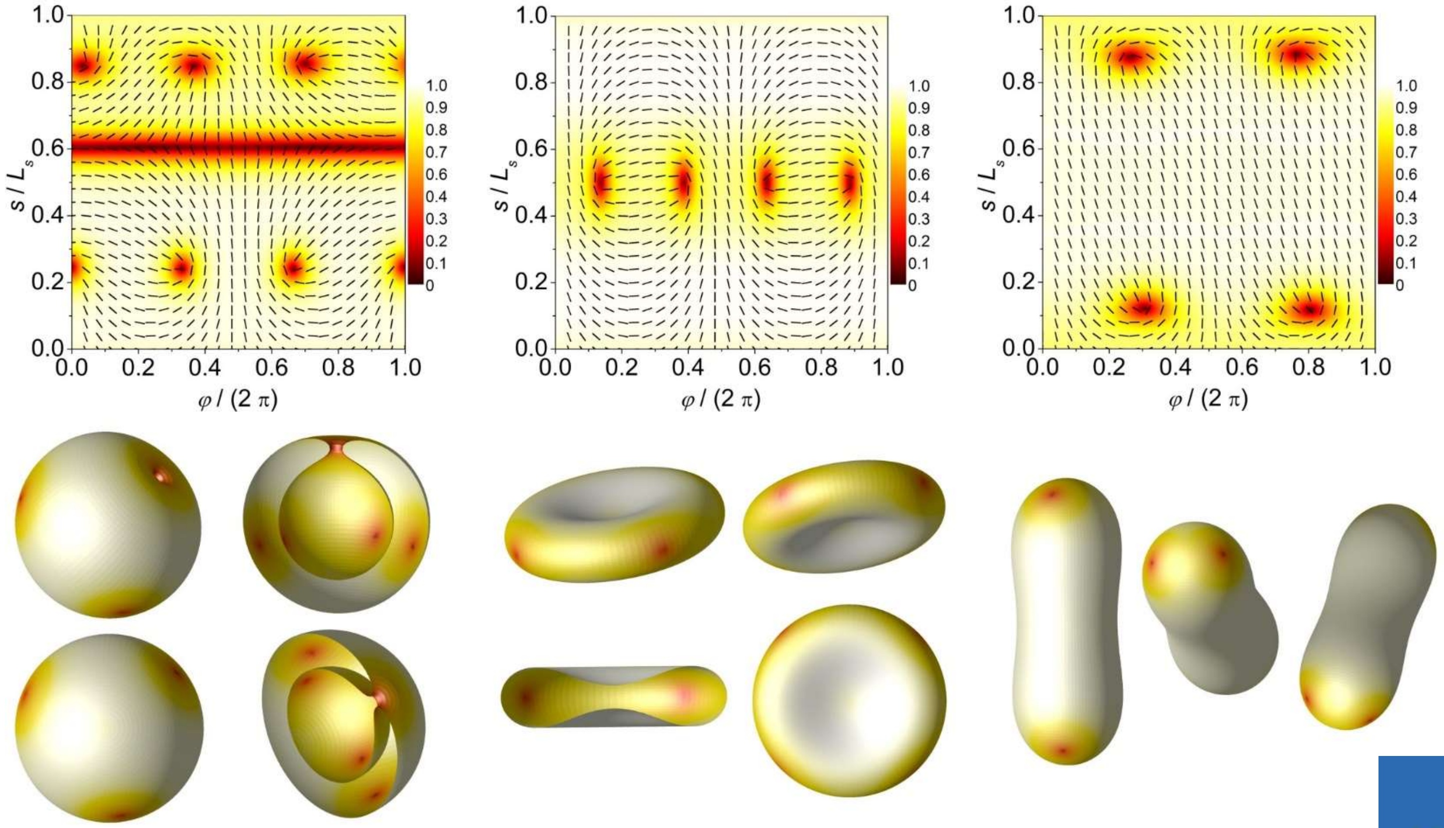
$$f= \frac{\kappa}{R_0^2}Tr\underline{C}^2 + \alpha\lambda_0^2\left(-Tr\underline{Q}^2 + \frac{1}{2}\left(Tr\underline{Q}^2\right)^2 + \left(\frac{\xi}{R_0}\right)^2\left(\left|\nabla\underline{Q}\right|^2 + \frac{1}{\lambda_0}\frac{k_e}{k_i}\underline{Q}.\underline{C}^2\right)\right)$$

Extrinsic term dominates close to a continuous phase transition!

Sci. Rep. 9, 19742 (2019).

Crystals 2020

Without extrinsic term :



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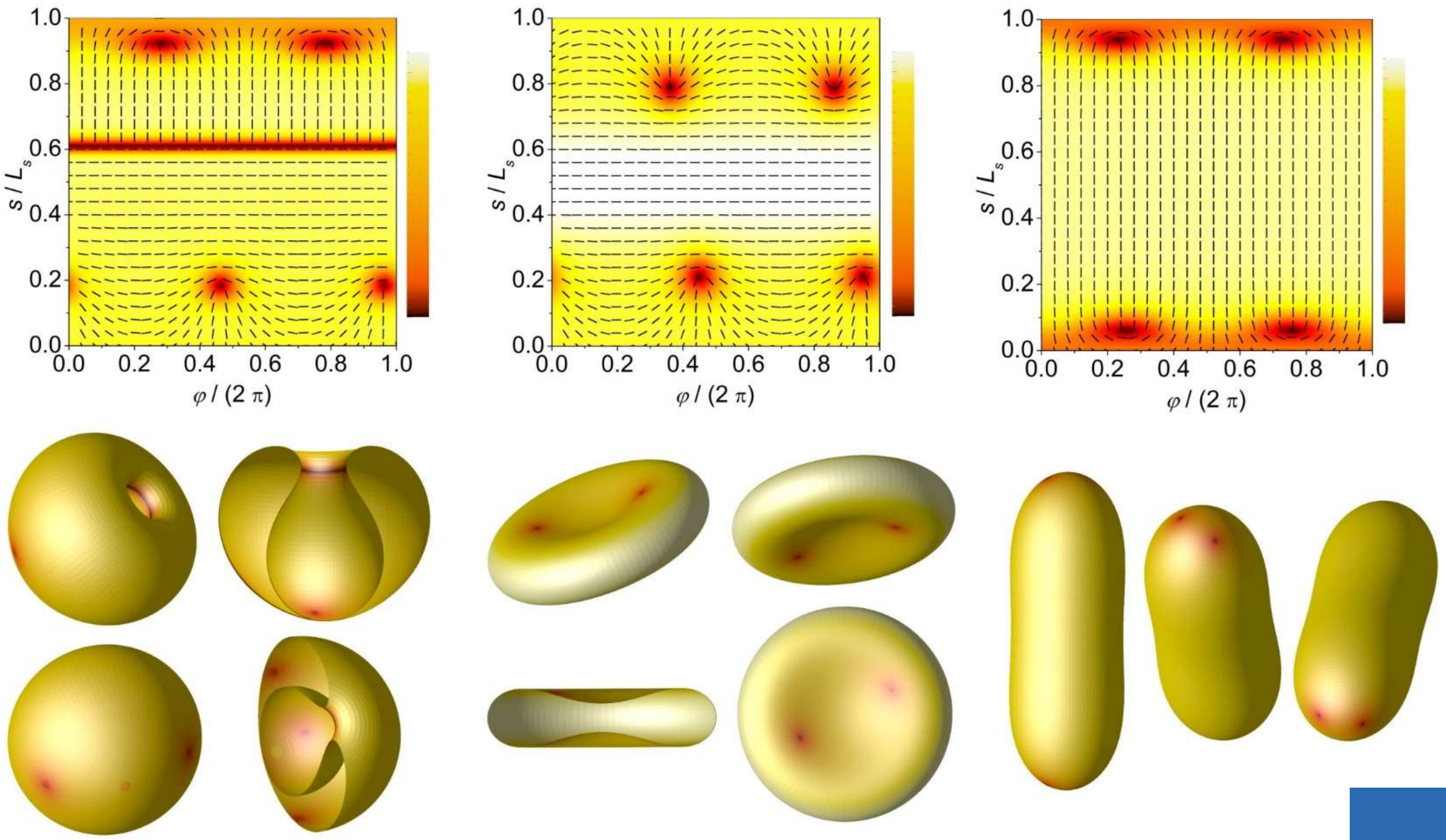
With extrinsic term :



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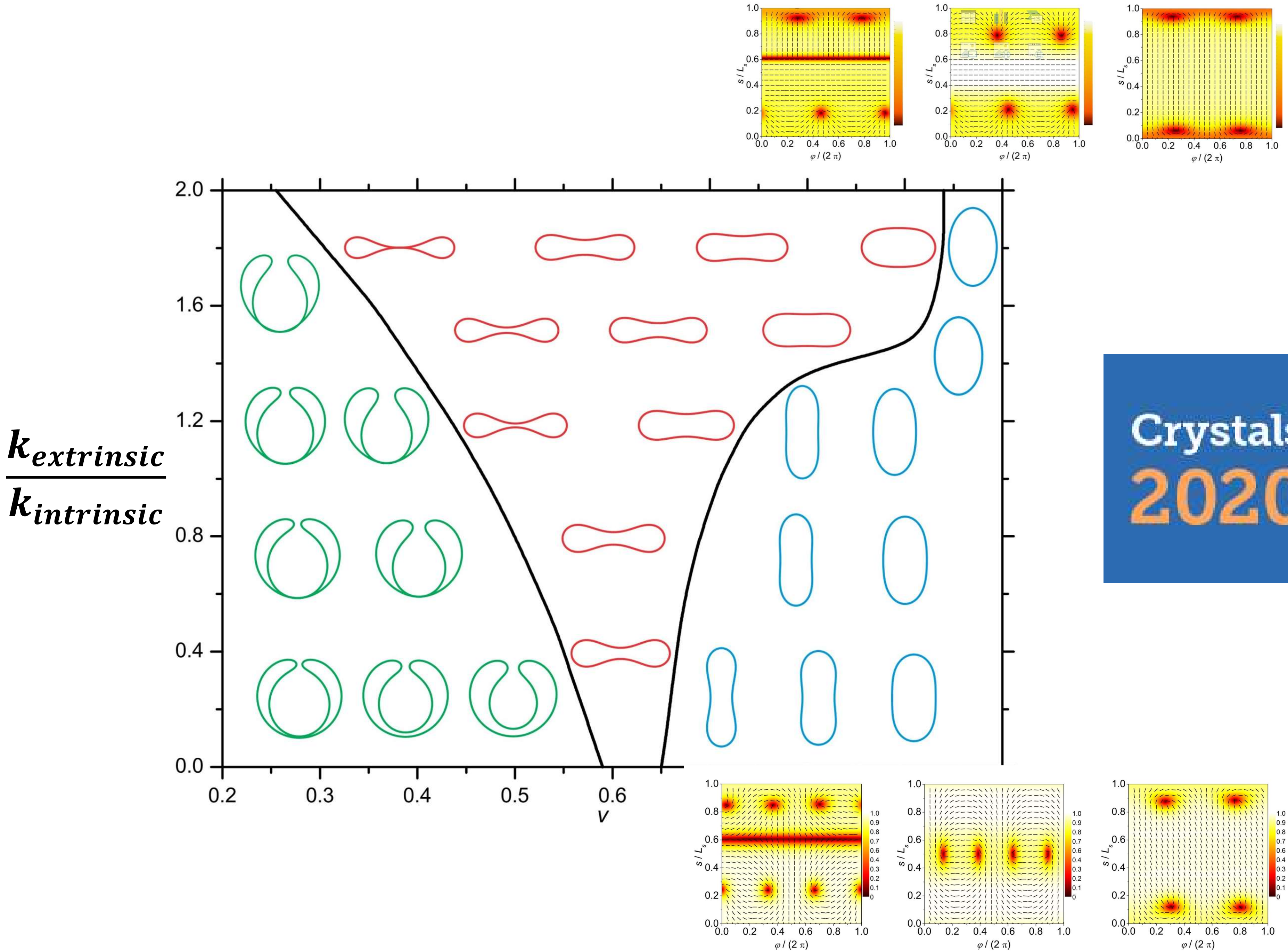

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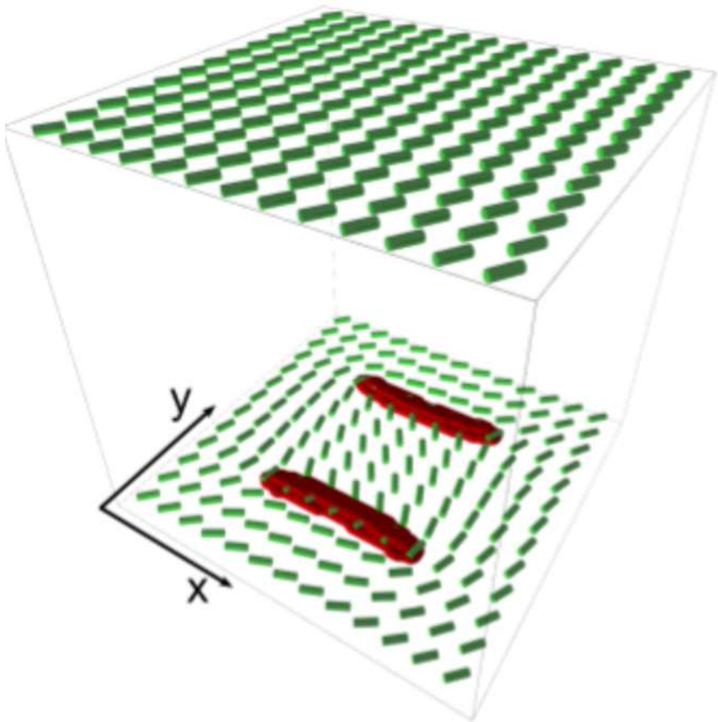
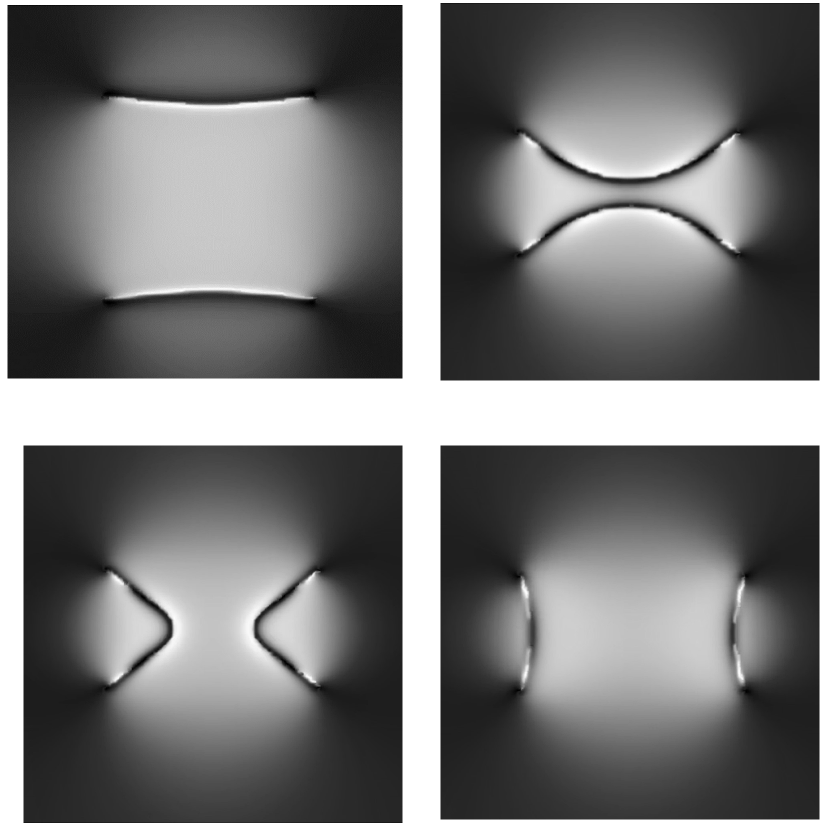
Stabilisation of "chargeless" TDs





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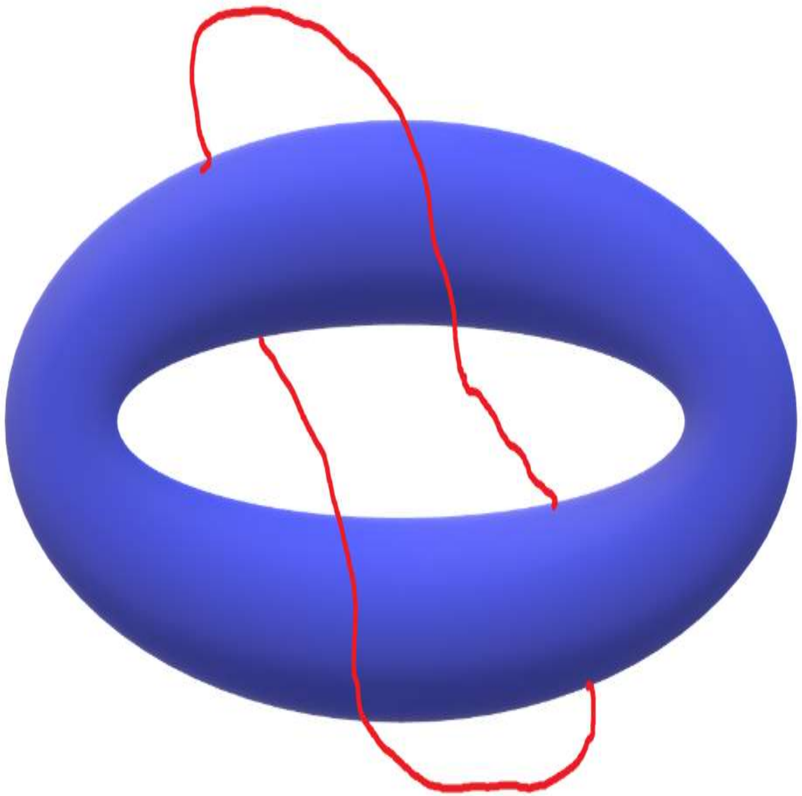

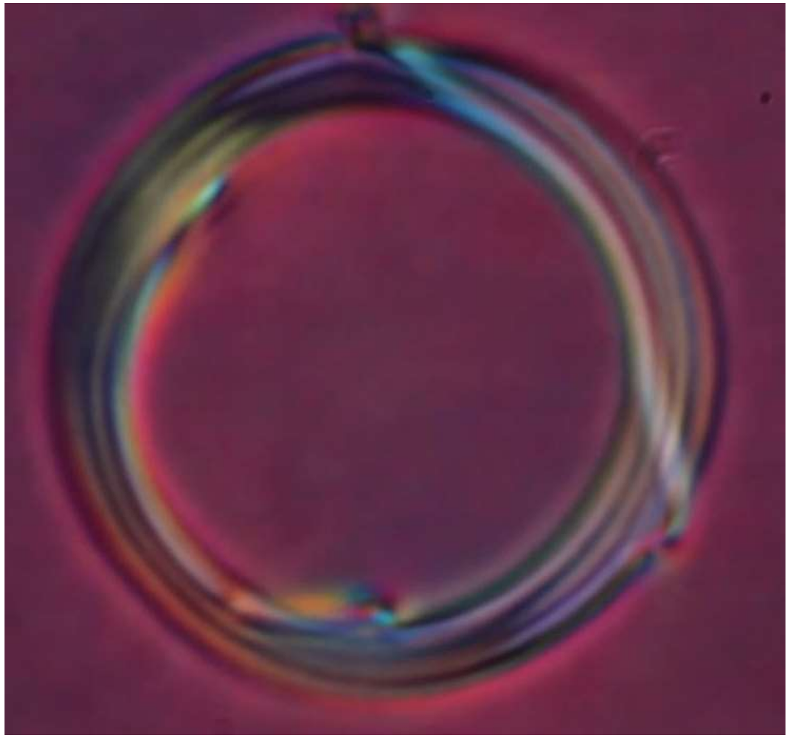

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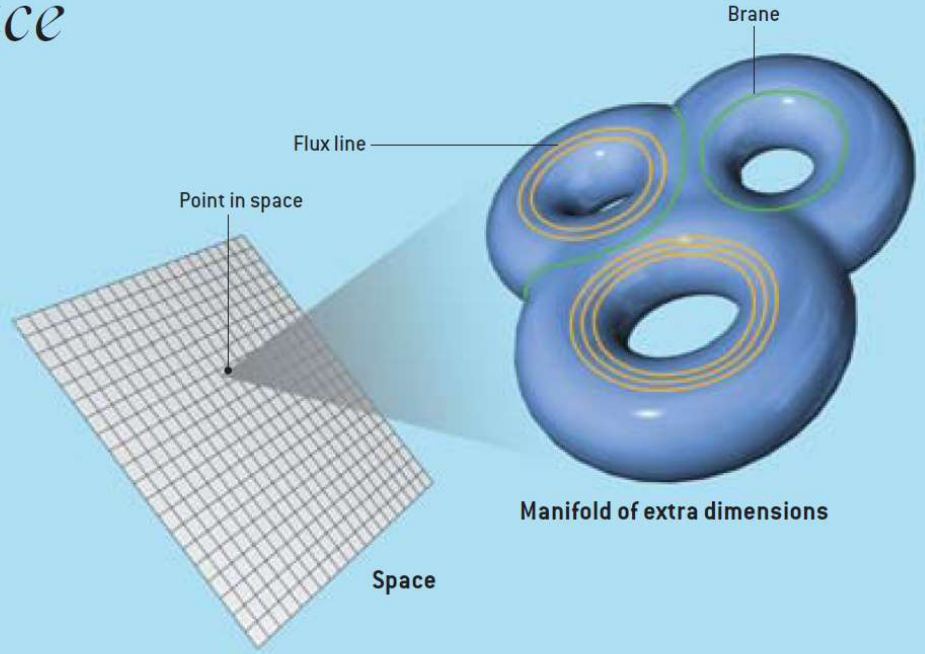
The Hidden Space

Any given solution to the equations of string theory represents a specific configuration of space and time. In particular, it specifies the arrangement of the small dimensions, along with their associated branes (*green*) and lines of force known as flux lines (*orange*). Our world has six extra dimensions, so every point of our familiar three-dimensional space hides an associated tiny six-dimensional space, or manifold—a six-dimensional analogue of the circle in the top illustration on page 81. The physics that is observed in the three large dimensions depends on the size and the structure of the manifold: how many doughnutlike "handles" it has, the length and circumference of each handle, the number and locations of its branes, and the number of flux lines wrapped around each doughnut.



THE STRING THEORY LANDSCAPE

By Raphael Bousso and Joseph Polchinski

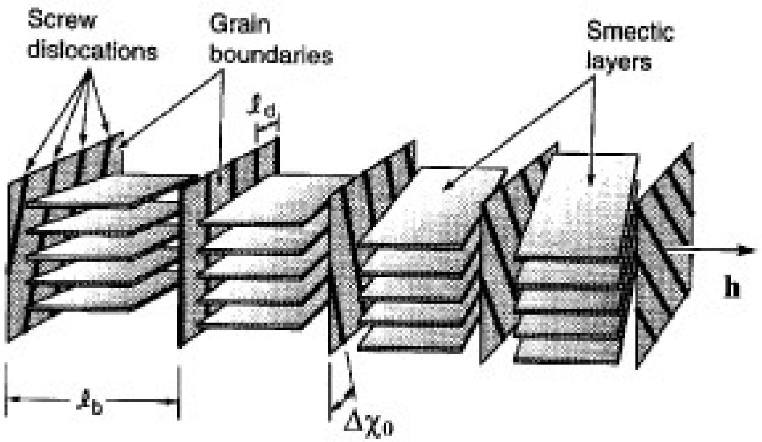
2004 SCIENTIFIC AMERICAN,

Stabilisation with nanoparticles

Strong enough chirality

SmA   TGBA   N*   BPs   I

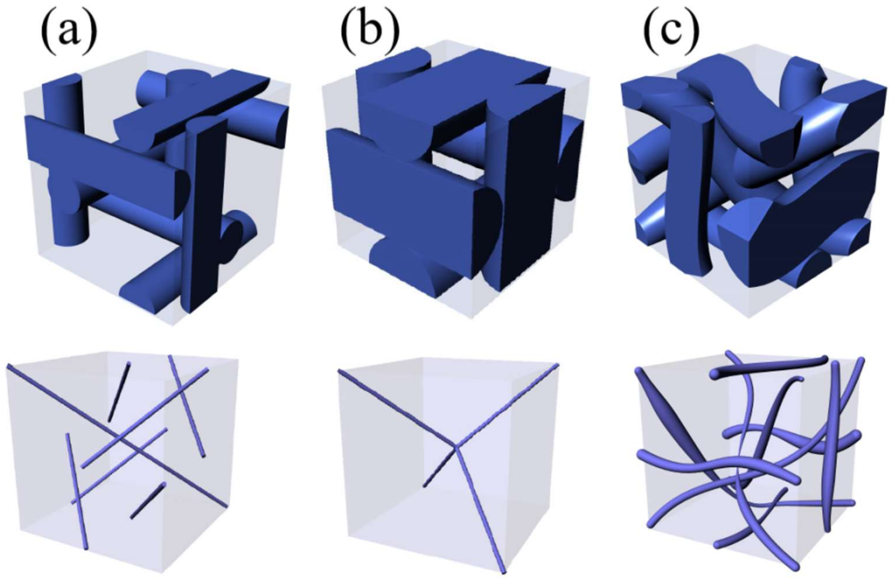
Temperature

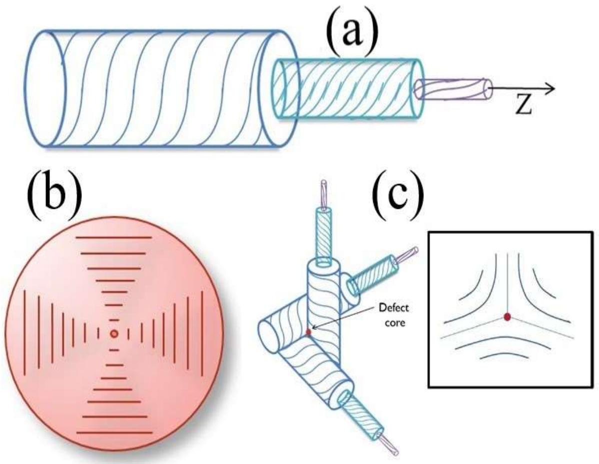


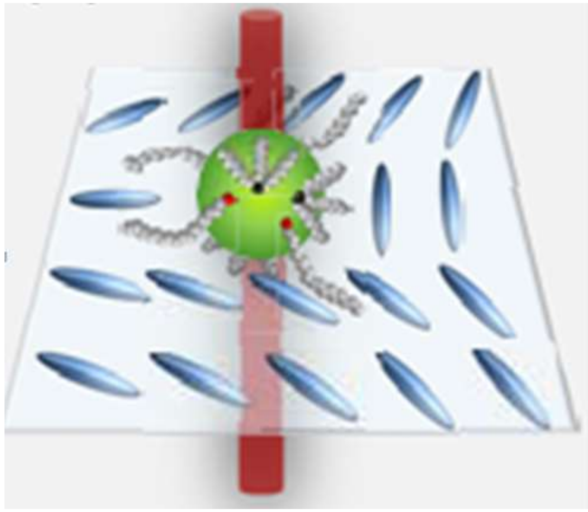
Renn&Lubensky, *Phys.Rev. A* **38**, 2132 (1988).

Wight&Mermin, *Rev.Mod.Phys.* **61**, 385 (1989).

Chiral LCs : Blue Phases







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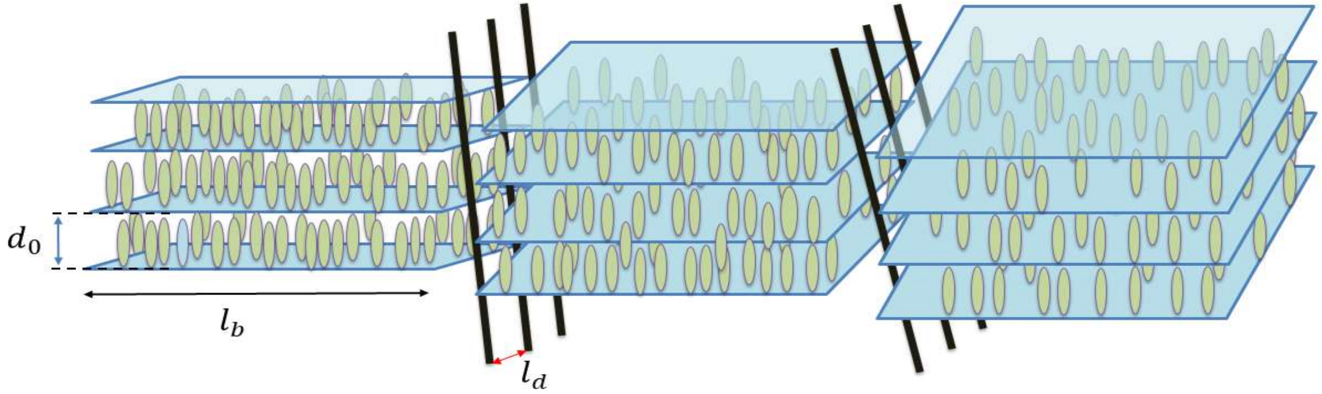
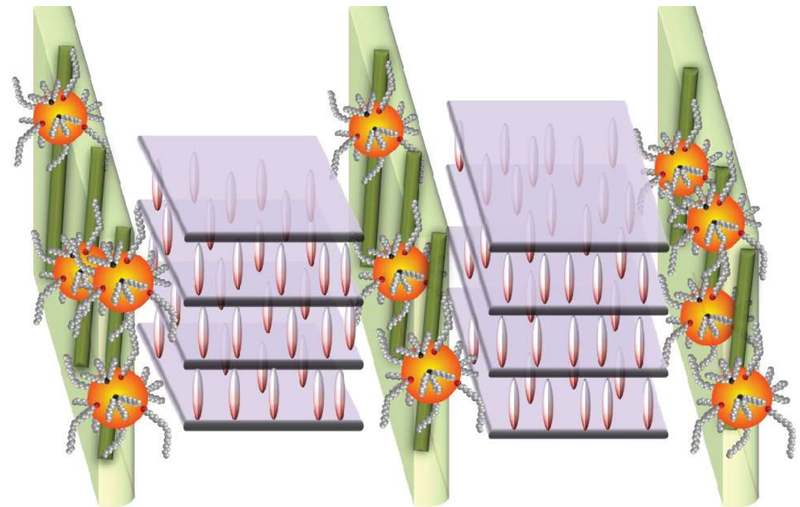
Twist grain boundary phases





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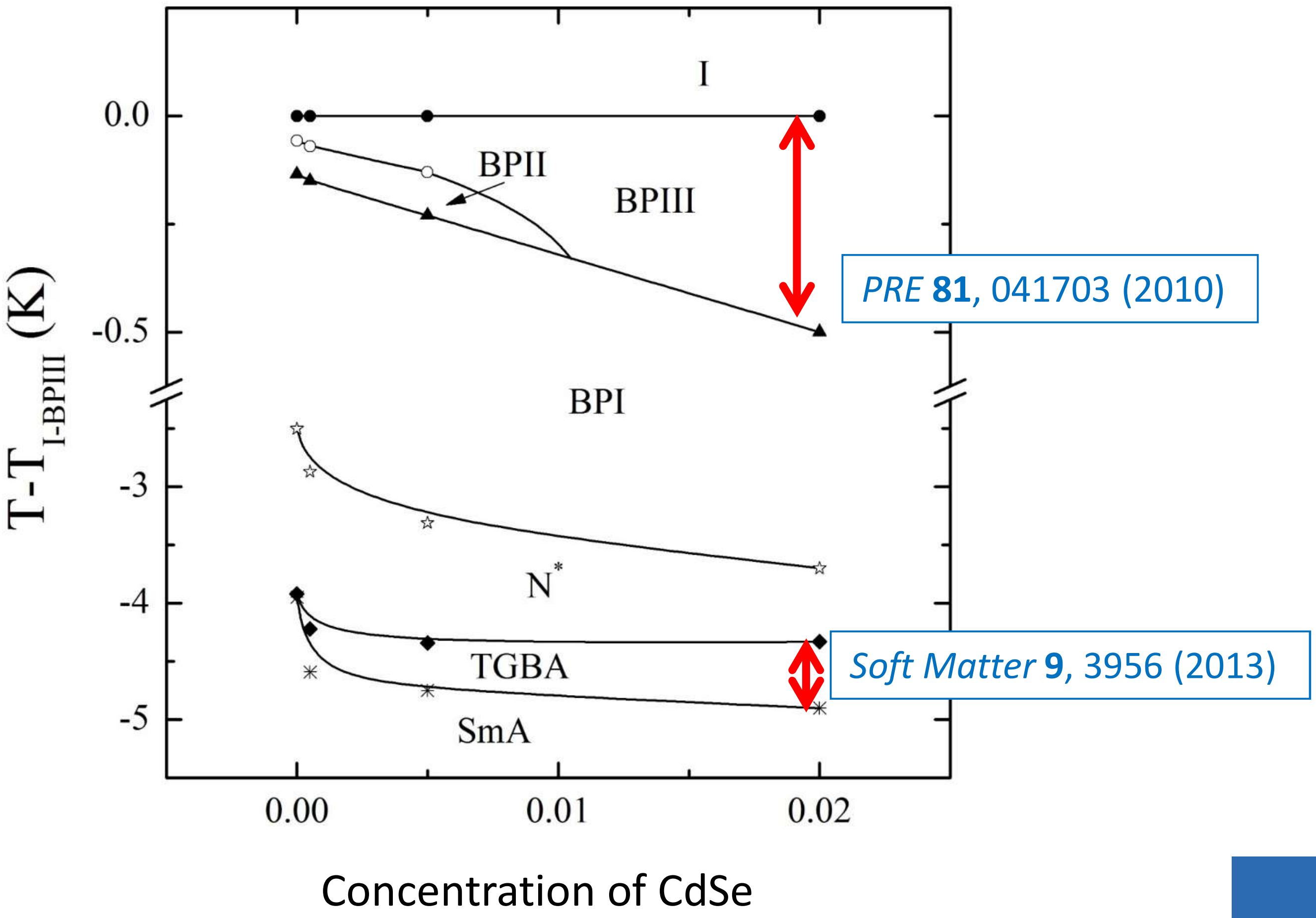

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III. Conclusions

- **Stabilisation of defects**: robust manipulations among different stable configurations of defects

- **Potential applications**: rewirable conductive wires, information storage, photonics, sensors…

- **Fundamental science**: fields as fundamental entities

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