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Multifractal detrended fluctuation analysis of relative humidity over Greece

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Abstract: Water, in its various forms, is considered a key parameter in climate change studies. Water vapor is recognized as the most important natural greenhouse gas playing a vital role in the hydrological cycle. Thus, studying air humidity fluctuations may contribute towards a deeper understanding of the radiative and thermodynamic processes that take part in the Earth's atmosphere. Traditional statistical analysis is not always efficient to describe complex physical processes with high temporal variability. In addition, a more thorough study of the variations of climatic parameters requires examination of their time series fluctuations over multiple time scales. Fractal theory offers robust solutions that satisfy the above requirements. In this work, the Multifractal Detrended Fluctuation Analysis (MF-DFA) is used in order to investigate the intrinsic dynamics of daily relative humidity time series over the Greek region from a nonlinear perspective. The scaling properties and the multifractal structure of the time series are studied by examining the fluctuation function, the multifractal spectrum and the Hurst exponent.

Keywords: relative humidity; Multifractal Detrended Fluctuation Analysis; nonlinear dynamics; climate change

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1. Introduction

Meteorological time series are generally characterized by a nonlinear behavior. Therefore, conventional statistical methods, which include the autocorrelation function or spectral analysis, are not always capable of revealing the complex behavior of natural processes and parameters where non-stationarities may exist. In addition, traditional statistical methods, such as trend analysis, usually examine time series taking into account a single time scale and neglecting time series features that occur over a wide range of temporal scales. The development of fractal theory has offered robust solutions in order to overcome these issues. Fractal approaches, in general, are based on the division of a time series into self-similar parts and the detection of the power-law behavior that reflects the scaling characteristics of the system. Kantelhardt et al. [1] introduced the Multifractal Detrended Fluctuation Analysis (MF-DFA) in order to determine the scaling behavior of time series with statistical properties that vary temporally. It is widely considered a valuable tool for time series analysis and has been used in a significant number of environmental studies [2-4]. Concerning the Greek region, in particular, Kalamaras et al. [5-6] studied the multifractal characteristics of daily temperature time series from meteorological stations of the Hellenic National Meteorological Service network as well as their geographical distribution. Philippopoulos et al. [7] also investigated the multifractal properties of daily temperature time series using the ERA-Interim reanalysis dataset.

Tzanis et al. [8] have also applied the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) [9] in order to investigate the multifractal structure of the cross-correlation between global methane and temperature. In this work, our scope is to study the multifractal characteristics and the scaling behavior of daily relative humidity time series [10] from meteorological stations at different locations within the Greek region.

2. Experiments

2.1 Data

Surface relative humidity (RH) observations during the synoptic hours (6, 12 and 18 UTC) were used in this work. The RH data cover a complete 30-year period from 1975 to 2004 and were obtained from three meteorological stations of the Hellenic National Meteorological Service (HNMS) network, namely Thessaloniki, Athinai (Hellinikon), located in the city of Athens, and Herakleion (Figure 1). The geographical characteristics of the three meteorological stations are summarized in Table 1. At this point, it should be noted that significant data gaps exist after the selected time period and since this work focuses only on the scaling properties of the time series the use of this data was avoided. Prior to the application the MF-DFA method, the daily means (RH_{daily}) of the 6-hour relative humidity data were calculated.

Table 1. List of meteorological stations

Station	WMO ID	Lat (N)	Lon (E)	Elev. a.s.l. (m)
Thessaloniki	16622	40° 31′ 29″	22° 58′ 18″	1.68
Athinai (Hellinikon)	16716	37° 53′ 23″	23° 44′ 31″	43.13
Herakleion	16754	35° 20′ 07″	25° 10′ 55″	39.00

Figure 1. Locations of meteorological stations under study

2.2 Methodology

The annual and semi-annual seasonal components that were identified in the daily relative humidity time series were subsequently eliminated using the Wiener filter [11]. The MF-DFA methodology was then applied to the time series of the deseasonalized surface RH data. A brief description of the MF-DFA method is presented below:

1. The profile $X(i)$ is firstly constructed:

$$X(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle] \tag{1}$$

where by x_k and $< x >$ the time series and its mean value are designated, respectively. The upper bound of summation i takes values from 1 to N which corresponds to the length of the time series.

2. $X(i)$ is partitioned into an integer number of $N_S = \mathrm{int}(N/s)$ non-intersecting segments all of which have the same length s, i.e., time scale. The segmentation procedure is also repeated for the retrograde time series of the profile. Thus, we get $2N_S$ segments in total.

3. Within each segment, a third-order ($m = 3$) polynomial $\tilde{X}_v$ is fitted to the profile, representing the local trend, where $v = 1,\dots,2N_S$ is the number of each segment. The local trend is then subtracted from the profile.

4. The detrended variance $F^2(s,v)$ is then calculated:

$$F^2(s,v) = \begin{cases} \dfrac{1}{s}\displaystyle\sum_{i=1}^{s} \{X[(v-1)s+i] - \tilde{X}_v(i)\}^2, & for\ v = 1,\dots,N_s \\ \dfrac{1}{s}\displaystyle\sum_{i=1}^{s} \{X[(N-(v-N_s)s+i] - \tilde{X}_v(i)\}^2, & for\ v = N_s+1,\dots,2N_s \end{cases} \tag{2}$$

5. Considering the average of all segments, we get the q^{th} order fluctuation function:

$$F_q(s) = \left\{ \frac{1}{2N_S} \sum_{v=1}^{2N_S} [F^2(s,v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \tag{3}$$

For $q = 0$ we have,

$$F_0(s) = exp\left(\frac{1}{4N_S} \sum_{v=1}^{2N_S} ln[F^2(s,v)] \right) \tag{4}$$

$F_q(s)$ is determined only for $s \geq m+2$. For $q = 2$, the MF-DFA results are identical to the DFA procedure [12-16].

6. $F_q(s)$ is computed for all values of s. The scaling behavior of $F_q(s)$ is examined through the plot of $\log(F_q(s))$ against $\log(s)$ for each moment q. For time series which are long-range correlated, $F_q(s)$ follows a power law:

$$F_q(s) \sim s^{h(q)} \tag{5}$$

For monofractal time series the scaling exponent $h(q)$ remains constant and it is equal to the Hurst exponent H. For multifractal time series $h(q)$ depends strongly on q, i.e. the scaling behavior is different for fluctuations of different magnitude. In these cases, $h(q)$ is the generalized form of the Hurst exponent. For values of $h(q)$ between 0 and 0.5 the time series is characterized by long-range negative correlation, denoting an anti-persistent character; for $h(q) > 0.5$, it is characterized by long-range positive correlation (persistent behavior); for $h(q) = 0.5$ it is considered uncorrelated, i.e. white noise.

Using the relationship $\tau(q)=qh(q)$-1 and applying a Legendre transformation we get

$$\tau'(q) = \alpha \tag{6}$$

and

$$f(a) = q\alpha - \tau(q) = q[\alpha - h(q)] + 1 \tag{7}$$

The plot of $f(\alpha)$ against α is the multifractal spectrum and gives information about the multifractal structure of the time series. The value of α at which $f(a)$=max is called the dominant Hurst exponent α_0 and corresponds to the prevailing scaling behavior [17]. Along with α_0, the spectral width w is also a key feature. It can be estimated by fitting a second-order polynomial around α_0 as proposed by [18] and measuring the distance between α_{max} and α_{min}, the two points where the fitting curve intersects the horizontal axis:

$$P(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C \tag{8}$$

A multifractal spectrum with a broad width indicates rich multifractality in the time series while smaller widths are associated with a more monofractal character.

3. Results

After applying MF-DFA on the deseasonalized RH times series of the three meteorological stations, the basic plots of the method are derived, namely a) the log-log plot of the fluctuation function $F_q(s)$ against s, b) the plot of the generalized Hurst exponent $h(q)$ against the moments q and c) the multifractal spectrum $f(\alpha)$ against α. In Figure 2 the plots concerning the meteorological station of Thessaloniki are shown, however similar plots were obtained for the rest of the meteorological stations as well. The time scales used in the MF-DFA process range between 30 months and N/5 where by N the length of the time series is denoted. The values of q also range from -6 to +6. From examination of Figure 2 it can be observed that $\log(Fq(s))$ increases linearly with $\log(s)$ and the slopes $h(q)$ are different for each q. This indicates that the relative humidity time series display multifractal characteristics. In addition, $h(q)$ is greater than 0.5 for all moments q. From this, it can be deduced that the time series of daily relative humidity are characterized by a persistent behavior (i.e. they are long-range positively correlated). This indicates that past events exert an influence on the succeeding values, i.e. an increase in the values of relative humidity is likely to be followed by an increase as well.

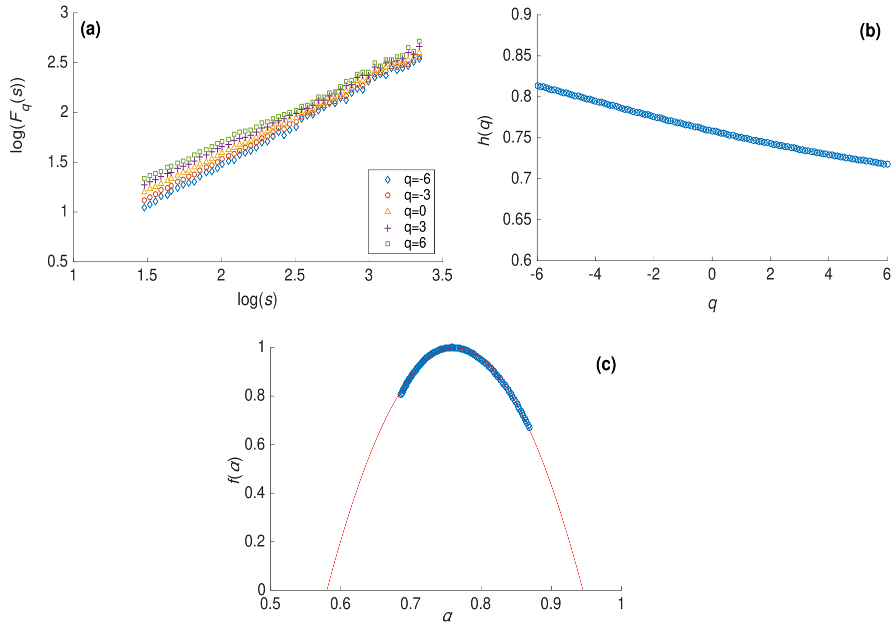

Figure 2. Multifractal Detrended Fluctuation Analysis (MF-DFA) results for RH from Thessaloniki station; **(a)** Plot of log(Fq(s)) against log(s); **(b)** Plot of h(q) against q; and, **(c)** Multifractal spectrum f(a) against a.

4. Discussion

Among the three stations examined, Thessaloniki demonstrates the highest value of α_0, i.e. it exhibits the greatest persistence, while the lowest value was observed at Herakleion (Figure 3). This implies that the distribution of α_0 varies geographically with its values increasing with latitude. This could be attributed to the fact that the northern locations are more frequently influenced by atmospheric disturbances and the descent of dry cold air masses. This can cause significant temperature changes which may affect the persistence in the behavior of daily temperature time series. A decrease of daily temperature values leads to a decrease of the local atmosphere's water-holding capacity and thus an increase in daily relative humidity affecting also its persistence. Regarding the

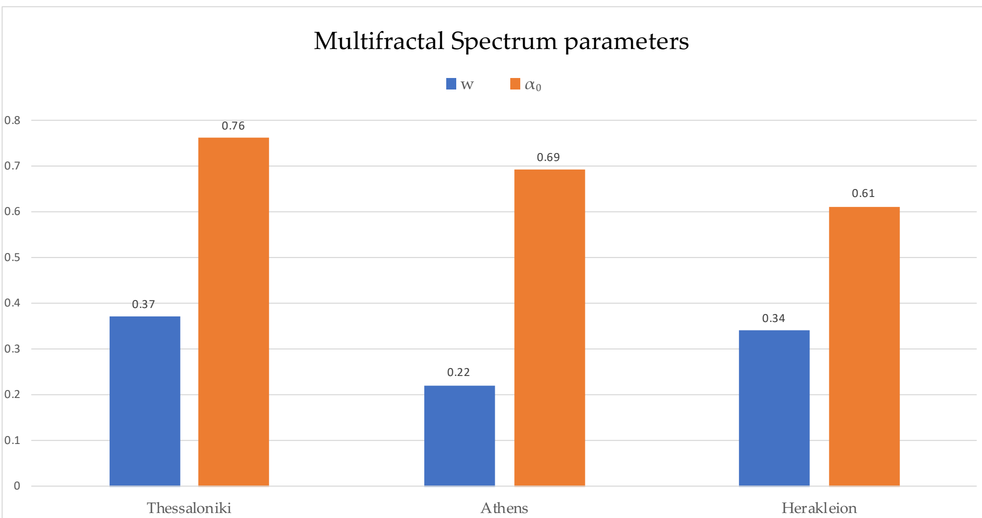

Figure 3. Multifractal spectrum parameters (w, α_0) for the meteorological stations under study.

spectral width w, the stations of Thessaloniki and Herakleion exhibit similar values and thus a similar degree of multifractality. On the other hand, the Athinai (Hellinikon) station presents a lower value of w. This finding suggests that the time series of the meteorological station in Athens possesses weaker multifractal features compared to the stations of Thessaloniki and Herakleion and therefore they are characterized by a smaller degree of complexity. This could be attributed to the different climatic conditions that prevail in the greater area of Athens.

5. Conclusions

In this work, daily relative humidity time series were examined for three meteorological stations at different geographical regions of Greece using the MF-DFA method. The most interesting results can be summarized as follows:

- Daily relative humidity time series are long-range positively correlated, which means that an increase in the values of relative humidity is likely to be followed by an increase as well.
- The values of the prevailing scaling exponent α_0 increase with increasing latitude. This could be explained by temperature and thus relative humidity changes.
- Smaller values of spectral width w, and therefore weaker multifractality were found for the meteorological station of Athinai (Hellinikon). This could be attributed to the different climatic conditions that prevail in Athens.

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