Fermion masses and mixing within a $SU(3)$ family symmetry model with five sterile neutrinos

Albino Hernández-Galeana

ESFM - Instituto Politécnico Nacional, México

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Open questions:

There are several open questions, motivations, to look for physics ”Beyond the Standard Model” (BSM):

- Neutrino masses and mixing, oscillation
- Hierarchy of masses
- Dart Matter (DM), Dart Energy (DE), Matter-Antimatter Asymmetry.

Main goal of this BSM: To account for the hierarchy of fermion masses:

$$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e,$$

and quark and lepton mixing matrices: V_{CKM} and U_{PMNS}.

Standard Model "SM"

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2} Y$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$q^o_{jL} = \begin{pmatrix} u^o_{jL} \\ d^o_{jL} \end{pmatrix}$	3	2	$\frac{1}{3}$
u^o_{jR}	3	1	$\frac{4}{3}$
d^o_{jR}	3	1	$-\frac{2}{3}$
$l^o_{jL} = \begin{pmatrix} \nu^o_{jL} \\ e^o_{jL} \end{pmatrix}$	1	2	-1
e^o_{jR}	1	1	-2
ν^o_{jR}	1	1	0
$\phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$	1	2	+1
$\tilde{\phi}_{SM} = i\,\sigma_2\,\phi^*_{SM}$	1	2	-1

Table: SM fermion content and charges, $j = 1, 2, 3$ family index

Comments:

- $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group
- Neutrinos are massless
- Anomalies cancel for each family, $j = 1, 2, 3$.
- $\overline{q^o_{iL}}\, \phi_{SM}\, u^o_{jR}$, $i, j = 1, 2, 3$ is gauge invariant. So, the Yukawa couplings $Y_{i,j}\ \overline{q^o_{iL}}\, \phi_{SM}\, u^o_{jR}$, $i, j = 1, 2, 3$ are gauge invariant, with $Y_{i,j}$ completely arbitrary.
- SM gauge bosons: Z, A(photon), and H(Higgs) couplings to fermions do not change flavor.
- $\frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\ V_{CKM}\, \gamma_\mu \Psi_{dL}\ W^{+\mu}$

MODEL WITH LOCAL $SU(3)$ FAMILY SYMMETRY

	$SU(3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table: Fermion content and charges

	$SU(3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
η_1, η_2	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1

Table: Scalars fields and charges

- η_1, η_2 are introduced to break spontaneously $SU(3)$.
- Φ^u, Φ^d are introduced to break spontaneously the $SU(2)_L \times U(1)_Y$.
- u-quarks and neutrinos coupled only to Φ^u
- d-quarks and charged leptons couple only to Φ^d

$\overline{\psi_q^o}\, \Phi^u\, \psi_{uR}^o$ **and** $\overline{\psi_q^o}\, \Phi^d\, \psi_{dR}^o$ **are forbidden by the** $SU(3)$ **family symmetry.**

$\bar{3} \times 3 = 1 + 8\ ,\ 3 \times 3 = \bar{3} + 6\ ,\ \bar{3} \times 3 \times 3 = (1+8) \times 3 = 3 + \bar{6} + 3 + 15$

$$i\mathcal{L}_{int,SU(3)} =$$

$$\frac{g_H}{2}\left(\bar{f}^o_1\gamma_\mu f^o_1 - \bar{f}^o_3\gamma_\mu f^o_3\right) Z^\mu_1 + \frac{g_H}{2\sqrt{3}}\left(\bar{f}^o_1\gamma_\mu f^o_1 + \bar{f}^o_3\gamma_\mu f^o_3 - 2\bar{f}^o_2\gamma_\mu f^o_2\right) Z^\mu_2$$

$$+\frac{g_H}{\sqrt{2}}\left(\bar{f}^o_1\gamma_\mu f^o_2 Y^+_1 + \bar{f}^o_1\gamma_\mu f^o_3 Y^+_2 + \bar{f}^o_2\gamma_\mu f^o_3 Y^+_3 + h.c.\right)$$

g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and $Y^\pm_i$, $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons, which have **flavor changing couplings to both left and right handed SM fermions**

$$g_H \begin{pmatrix} \bar{f}^o_1 & \bar{f}^o_2 & \bar{f}^o_3 \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y^+_1}{\sqrt{2}} & \frac{Y^+_2}{\sqrt{2}} \\ \frac{Y^-_1}{\sqrt{2}} & -\frac{Z_2}{\sqrt{3}} & \frac{Y^+_3}{\sqrt{2}} \\ \frac{Y^-_2}{\sqrt{2}} & \frac{Y^-_3}{\sqrt{2}} & -\frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} \end{pmatrix}^\mu \begin{pmatrix} f^o_1 \\ f^o_2 \\ f^o_3 \end{pmatrix}$$

Yukawa couplings:

Dirac Yukawa couplings:

$$
\begin{aligned}
&h_u\,\overline{\psi^o_q}\,\Phi^u\,U^o_R \;+\; h_{1u}\,\overline{\psi^o_{uR}}\,\eta_1\,U^o_L \;+\; h_{2u}\,\overline{\psi^o_{uR}}\,\eta_2\,U^o_L \;+\; M_U\,\overline{U^o_L}\,U^o_R\\
&h_d\,\overline{\psi^o_q}\,\Phi^d\,D^o_R \;+\; h_{1d}\,\overline{\psi^o_{dR}}\,\eta_1\,D^o_L \;+\; h_{2d}\,\overline{\psi^o_{dR}}\,\eta_2\,D^o_L \;+\; M_D\,\overline{D^o_L}\,D^o_R\\
&h_\nu\,\overline{\psi^o_l}\,\Phi^u\,N^o_R \;+\; h_{1\nu}\,\overline{\psi^o_{\nu R}}\,\eta_1\,N^o_L \;+\; h_{2\nu}\,\overline{\psi^o_{\nu R}}\,\eta_2\,N^o_L \;+\; m_D\,\overline{N^o_L}\,N^o_R\\
&h_e\,\overline{\psi^o_l}\,\Phi^d\,E^o_R \;+\; h_{1e}\,\overline{\psi^o_{eR}}\,\eta_1\,E^o_L \;+\; h_{2e}\,\overline{\psi^o_{eR}}\,\eta_2\,E^o_L \;+\; M_E\,\overline{E^o_L}\,E^o_R\\
&\hfill +h.c
\end{aligned}
$$

Majorana Yukawa couplings:

$$
h_{1R}\,\overline{\psi^o_{\nu R}}\,\eta_1\,(N^o_R)^c \;+\; h_{2R}\,\overline{\psi^o_{\nu R}}\,\eta_2\,(N^o_R)^c \;+\; m_R\,\overline{N^o_R}\,(N^o_R)^c
$$

$$
h_L\,\overline{\psi^o_l}\,\Phi^u\,(N^o_L)^c \;+\; m_L\,\overline{N^o_L}\,(N^o_L)^c \;+h.c
$$

Spontaneous Symmetry breaking (SSB)

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$$SU(3) \times G_{SM} \xrightarrow{\langle\eta_2\rangle,\langle\eta_1\rangle} G_{SM} \xrightarrow{\langle\Phi^u\rangle,\langle\Phi^d\rangle} SU(3)_C \times U(1)_Q$$

$SU(3) \times G_{SM}$	G_{SM}	$SU(3)_C \times U(1)_Q$
SM fermions are massless	SM fermions still massless	SM fermions become massive (PDG known values)

$SU(3)$ family symmetry breaking

Two SM singlet scalars are introduced in the fundamental representation of $SU(3)$:

$$\langle \eta_1 \rangle = \begin{pmatrix} \Lambda_1 \\ 0 \\ 0 \end{pmatrix} \quad , \quad \langle \eta_2 \rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix}$$

$$SU(3) \times G_{SM} \quad \xrightarrow{\langle \eta_2 \rangle} \quad SU(2)_F \; ? \times G_{SM} \quad \xrightarrow{\langle \eta_1 \rangle} \quad G_{SM}$$

FCNC ?

Λ_2: 5 very heavy boson masses of order M_2 ($M_2^2 = \frac{g_H^2 \Lambda_2^2}{2}$)

Λ_1: 3 boson masses of order M_1 ($M_1^2 = \frac{g_H^2 \Lambda_1^2}{2}$)

To suppress properly FCNC like, for instance: $\mu \to e\gamma$ **(**$Br < 5.7 \times 10^{-13}$**)** , $\mu \to e\,e\,e$ **(**$Br < 1 \times 10^{-12}$**)** , $K^o - \bar{K}^o$, $D^o - \bar{D}^o$, $M_1 \gtrsim 10^3$ **TeV's**

The above scalar fields and VEV's break completely the $SU(3)$ family symmetry, generating the mass terms

- $\langle \eta_1 \rangle : \quad \frac{g_H^2 \Lambda_1^2}{2} (Y_1^+ Y_1^- + Y_2^+ Y_2^-) + \frac{g_H^2 \Lambda_1^2}{4} (Z_1^2 + \frac{Z_2^2}{3} + 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_2 \rangle : \quad \frac{g_H^2 \Lambda_2^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + g_H^2 \Lambda_2^2 \frac{Z_2^2}{3}$

Electroweak Symmetry Breaking

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets;
$\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1)$:

$$
{}^{u} = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \& \langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix} , \ \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} \& \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix}
$$

$$
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad \xrightarrow{\langle \Phi^u \rangle \, , \, \langle \Phi^d \rangle} \quad SU(3)_C \times U(1)_Q
$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2 + v_d^2)\,W^+W^- + \frac{(g^2 + g'^2)}{8}\,(v_u^2 + v_d^2)\,Z_o^2$$

$$\sqrt{g^2 + g'^2}\,g_H\,Z_o\left[(v_{1u}^2 - v_{2u}^2 - v_{1d}^2 + v_{2d}^2)\,Z_1 + (v_{1u}^2 + v_{2u}^2 - 2v_{3u}^2 - v_{1d}^2 - v_{2d}^2 + 2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}\right.$$

$$\left. v_{1u}v_{2u} - v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u} - v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}} + 2\,(v_{2u}v_{3u} - v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\right]$$

$+$ **tiny contributions to the** $SU(3)$ **gauge boson masses**

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Tree level Fermion Masses

$\bar{\psi}^o_{SM,L}\,\Phi^{u,d}\,\psi^o_{SM,R}$ **are not** $SU(3)$ **invariant**
Allowed Tree Level Yukawa couplings:

$$h_e\,\bar{\psi}^o_l\,\Phi^d\,E^o_R\;+\;h_{1e}\,\bar{\psi}^o_e\,\eta_1\,E^o_L\;+\;h_{2e}\,\bar{\psi}^o_e\,\eta_2\,E^o_L\;+\;M_E\,\bar{E}^o_L\,E^o_R\;+h.c$$

Dirac See-Saw Mechanisms
In the gauge basis ${\psi^o_{L,R}}^T=(e^o,\mu^o,\tau^o,E^o)_{L,R}$, the mass terms read as $\bar{\psi}^o_L\mathcal{M}^o\psi^o_R+h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h_e\,v_{d1}$
$\overline{\mu^o_L}$	0	0	0	$h_e\,v_{d2}$
$\overline{\tau^o_L}$	0	0	0	$h_e\,v_{d3}$
$\overline{E^o_L}$	$h_{1e}\,\Lambda_1$	$h_{2e}\,\Lambda_2$	0	M_E

$\equiv$

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	a_1
$\overline{\mu^o_L}$	0	0	0	a_2
$\overline{\tau^o_L}$	0	0	0	a_3
$\overline{E^o_L}$	b_1	b_2	0	M

Table: Generic tree level Dirac mass matrix $\mathcal{M}^o$ for u and d quarks, charged leptons and Dirac neutrinos

$$V_L^{o\,T}\mathcal{M}^o\mathcal{M}^{o\,T}\ V_L^o = V_R^{o\,T}\mathcal{M}^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,\lambda_3^2,\lambda_4^2)$$

$$V_L^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,-\lambda_3,\lambda_4)$$

The non-zero eigenvalues may be obtained from diagonalization of the 2x2 matrix

$$m_o = \begin{pmatrix} 0 & a \\ b & M \end{pmatrix}\ ,\ a = \sqrt{a_1^2 + a_2^2 + a_3^2}\ ,\ b = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

$$v_L = \begin{pmatrix} c_\alpha & s_\alpha \\ & \\ -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad v_R = \begin{pmatrix} c_\beta & s_\beta \\ & \\ -s_\beta & c_\beta \end{pmatrix}$$

$$\lambda_3\ c_\alpha\ c_\beta = \lambda_4\ s_\alpha\ s_\beta$$

$$v_L{}^T\, m_o\, v_R = Diag(0,0,-\lambda_3,\lambda_4)$$

$$v_L{}^T\, m_o\, m_o^T\, v_L = v_R{}^T\, m_o^T\, m_o\, v_R = Diag(\lambda_3^2,\lambda_4^2)$$

One loop contribution to fermion masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\lambda_3$, $m_4^o = \lambda_4$, and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} \, F(M_Y) \quad i,j = 1,2,3,$$

$$F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$$

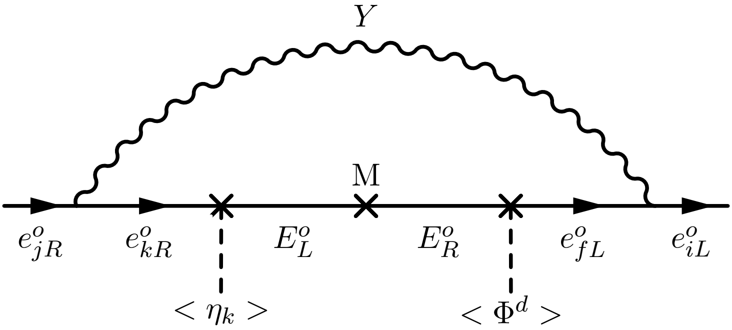

Figure: Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	0	0
$\overline{\mu_L^o}$	D_{21}	D_{22}	0	0
$\overline{\tau_L^o}$	D_{31}	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: One loop mass matrix $\mathcal{M}_1^o$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R$$

Generic mass matrix up to one loop for quarks and charged leptons.

$$\mathcal{M} = Diag(0, 0, -\lambda_3, \lambda_4) + {V_L^o}^T \mathcal{M}_1^o \, V_R^o$$

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & -\lambda_3 + c_\alpha c_\beta \, m_{33} & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & \lambda_4 + s_\alpha s_\beta \, m_{33} \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\,\Psi_{L,R}$; $\bar{\chi}_L\,\mathcal{M}\,\chi_R = \bar{\Psi}_L\,{V^{(1)}_L}^T \mathcal{M}\,V^{(1)}_R\,\Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$
{V^{(1)}_L}^T \mathcal{M}\,\mathcal{M}^T\,V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\,\mathcal{M}\,V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,,
$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons.

Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$
\psi_L^o = V_L^o\,V_L^{(1)}\,\Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\,V_R^{(1)}\,\Psi_R
$$

and for neutrinos $\Psi_\nu^o = U_\nu^o\,U_\nu\,\Psi_\nu$.

Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \bar{f^o}_{uL}\gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}} \bar{\Psi}_{uL} \, [(V^o_{uL} \, V^{(1)}_{uL})_{3\times4}]^T \, (V^o_{dL} \, V^{(1)}_{dL})_{3\times4} \, \gamma_\mu \Psi_{dL} \, W^{+\mu},$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V^o_{uL} \, V^{(1)}_{uL})_{3\times4}]^T \, (V^o_{dL} \, V^{(1)}_{dL})_{3\times4}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V^o_{eL} \, V^{(1)}_{eL})_{3\times4}]^T \, (U^o_\nu \, U_\nu)_{3\times8}$$

Neutrino masses: $3+5$ Scenario

Tree level Dirac Neutrino masses

$$h_D\, \overline{\Psi^o_l}\, \Phi^u\, N^o_R \,+\, h_{\nu 1}\, \overline{\Psi^o_{\nu_R}}\, \eta_1\, N^o_L \,+\, h_{\nu 2}\, \overline{\Psi^o_{\nu_R}}\, \eta_2\, N^o_L \,+\, m_D\, \overline{N^o_L}\, N^o_R \,+ h.c$$

h_D, h_1, h_2, and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis ${\Psi^o_{\nu}}^T_{L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\, \left[v_{u1}\, \bar{\nu}^o_{eL} + v_{u2}\, \bar{\nu}^o_{\mu L} + v_{u3}\, \bar{\nu}^o_{\tau L} \right]\, N^o_R + \left[h_{\nu 1}\, \Lambda_1\, \bar{\nu}^o_{eR} + h_{\nu 2}\, \Lambda_2\, \bar{\nu}^o_{\mu R} \right]\, N^o_L$$
$$+ m_D\, \bar{N}^o_L\, N^o_R + h.c.$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	$h_{\nu 1}\, \Lambda_1$	$h_{\nu 2}\, \Lambda_2$	0	m_D

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L\,\bar{\Psi}^o_l\,\Phi^u(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c + h.c$$

and

$$h_{1R}\,\bar{\Psi}^o_{\nu R}\,\eta_1\,(N^o_R)^c \;+\; h_{2R}\,\bar{\Psi}^o_{\nu R}\,\eta_2\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c$$

$$+\left[h_{1R}\,\Lambda_1\,\bar{\nu}^o_{eR} + h_{2R}\,\Lambda_2\,\bar{\nu}^o_{\mu R}\right]\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iL}\ (\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	$h_{1R}\Lambda_1$
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	0
N^o_R	$h_{1R}\Lambda_1$	$h_{2R}\Lambda_2$	0	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iR}\ (\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	$h_1\Lambda_1$	$h_2\Lambda_2$	0	M_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	$h_1\Lambda_1$	0	0	0	$h_{1R}\Lambda_1$
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2\Lambda_2$	0	0	0	$h_{2R}\Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	$h_{1R}\Lambda_1$	$h_{2R}\Lambda_2$	0	m_R

Table: Tree Level Majorana mass matrix

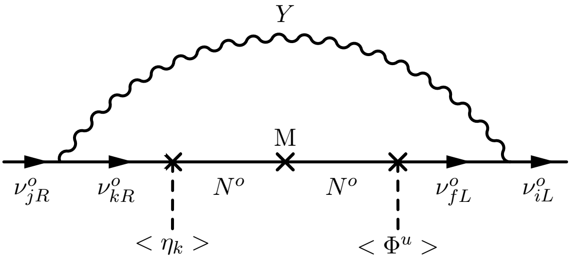

Figure: Generic one loop diagram contribution to the Dirac mass term $m_{ij}\,\bar{\nu}^o_{iL}\nu^o_{jR}$. M $= M_D, m_L, m_R$

One loop Majorana neutrino masses

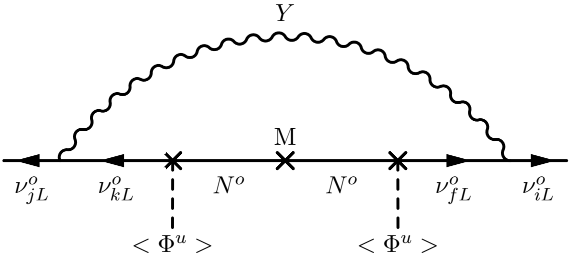

Figure: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iL} (\nu^o_{jL})^T$. M $= M_D, m_L, m_R$

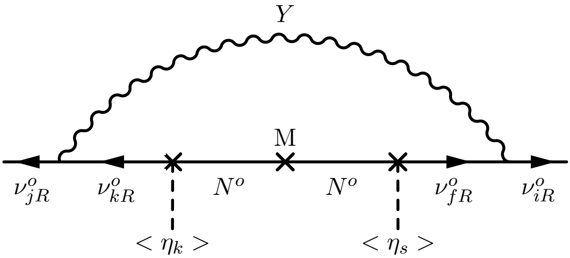

Figure: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iR}(\nu^o_{jR})^T$. M $= M_D, m_L, m_R$

Thus, in the Ψ^o_ν basis, we may write the one loop contribution for neutrinos as

$$\mathcal{M}^o_{1\,\nu} = \begin{pmatrix} L_{\nu\,11} & L_{\nu\,12} & L_{\nu\,13} & 0 & D_{\nu\,15} & D_{\nu\,16} & 0 & 0 \\ L_{\nu\,12} & L_{\nu\,22} & L_{\nu\,23} & 0 & D_{\nu\,25} & D_{\nu\,26} & 0 & 0 \\ L_{\nu\,13} & L_{\nu\,23} & L_{\nu\,33} & 0 & D_{\nu\,35} & D_{\nu\,36} & D_{\nu\,37} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{\nu\,15} & D_{\nu\,25} & D_{\nu\,35} & 0 & R_{\nu\,55} & R_{\nu\,56} & 0 & 0 \\ D_{\nu\,16} & D_{\nu\,26} & D_{\nu\,36} & 0 & R_{\nu\,56} & R_{\nu\,66} & 0 & 0 \\ 0 & 0 & D_{\nu\,37} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}$$

SSB and $U(1)_{PQ}$ global symmetry

$$[SU(3) \otimes G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L \otimes U(1)_{PQ}]_{\text{global}}$$

$$\downarrow \langle \eta_2 \rangle \sim \Lambda_3 \geq 10^8 \ TeV$$

$$[SU(2)_F \otimes G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L]_{\text{global}}$$

$$\downarrow \langle \eta_1 \rangle \sim \Lambda_2 \approx \text{ few } TeV$$

$$[G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L]_{\text{global}}$$

$$\downarrow \langle \Phi_u \rangle, \langle \Phi_d \rangle \approx \ 250 \ GeV$$

$$[SU(3)_C \otimes U(1)_{QED}]_{\text{gauge}} \times [U(1)_B]_{\text{global}}$$

	$U(1)_B$	$U(1)_L$	$U(1)_{PQ}$
ψ_q^o	$\frac{1}{3}$	0	1
ψ_{uR}^o	$\frac{1}{3}$	0	2
ψ_{dR}^o	$\frac{1}{3}$	0	2
ψ_l^o	0	1	0
ψ_{eR}^o	0	1	1
$\psi_{\nu R}^o$	0	1	1
$U_{L,R}^o$	$\frac{1}{3}$	0	1
$D_{L,R}^o$	$\frac{1}{3}$	0	1
$E_{L,R}^o$	0	1	0
$N_{L,R}^o$	0	1	0

Table: All the Yukawa couplings and the scalar potential are invariant under the global symmetry $U(1)_B \times U(1)_L \times U(1)_{PQ}$, where B is the baryon number, L is the lepton number, and $U(1)_{PQ}$ the candidate for the Peceei-Quinn symmetry to address the strong CP problem

	$U(1)_B$	$U(1)_L$	$U(1)_{PQ}$
η_1, η_2	0	0	1
Φ^u	0	0	0
Φ^d	0	0	0

Dirac mass:
γ^5 field transformation $\psi' = \gamma^5 \psi$
$-m\,\overline{\psi'}\,\psi' \longrightarrow m\,\overline{\psi}\,\psi$

Majorana mass: Mohapatra, Unification and Supersymmetry, p 147;
We can diagonalize the complex and symmetric mass matrix M which can be done, in general, as follows:
$U\,M\,U^T\,\Lambda = M_D$,
where M_D is diagonal with positive, real eigenvalues and Λ is a diagonal unitary matrix.
Thus, in some sense, the phase of the Majorana mass term gives the the C-phase of the neutrino.

$$M = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}, \quad m_R \gg m_D$$

$$\text{heavy neutrino} \quad m_N = \tfrac{1}{2}(m_R + \sqrt{m_R^2 + 4m_D^2}) \approx m_R$$

$$\text{light neutrino} \quad m_\nu = \tfrac{1}{2}(m_R - \sqrt{m_R^2 + 4m_D^2}) \approx -\tfrac{m_D^2}{m_R}$$

Numerical results

Particular parameter space region at the M_Z scale:

Input values for the $SU(3)$ family symmetry:

$$M_1 = 5300\,\text{TeV's} \quad , \quad M_2 = 10^3\ M_1 \quad , \quad \frac{\alpha_H}{\pi} = 0.1266$$

with M_1, M_2 the horizontal boson masses, and the coupling constant, respectively,

$$M_1^2 = \frac{g_H^2\,\Lambda_1^2}{2} \qquad , \qquad M_2^2 = \frac{g_H^2\,\Lambda_2^2}{2}$$

$$g_H = 2.23561\ ,\ \Lambda_1 = 3352.7\ TeV\ ,\ \Lambda_2 = 10^3\,\Lambda_1$$

Quark masses and $(V_{CKM})_{4\times4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 32445.2 \\ 0 & 0 & 0 & 153081 \\ 0 & 0 & 0 & 188414. \\ 7.09\times10^9 & -1.3905\times10^9 & 0 & 5.04313\times10^9 \end{pmatrix} \text{MeV}\,,$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 0 & -164.78 & -450.207 & -644.992 \\ 2.05472 & 600.381 & 1627.36 & 2331.45 \\ -2.47402 & 1593.14 & -173000. & 39878.4 \\ -0.0000393617 & 0.0253469 & 0.442859 & 8.81106\times10^9 \end{pmatrix} \text{MeV}$$

the u-quark masses

$$(\,m_u\,,\,m_c\,,\,m_t\,,\,M_U\,) = (\,1.38049\,,\,638.077\,,\,173016\,,\,8.81\times10^9\,)\ \text{MeV}$$

the mixing matrices:
$V_{uL} = V^{o}_{uL}\, V^{(1)}_{uL}$:

$$\begin{pmatrix} 0.985383 & 0.105814 & -0.133508 & 2.67779 \times 10^{-6} \\ 0.000823743 & -0.78665 & -0.617398 & 0.0000129872 \\ -0.170353 & 0.608264 & -0.775239 & 0.000015552 \\ 0 & 4.73287 \times 10^{-7} & 0.0000204323 & 1. \end{pmatrix}$$

$V_{uR} = V^{o}_{uR}\, V^{(1)}_{uR}$:

$$\begin{pmatrix} -0.000293496 & 0.187295 & -0.563405 & 0.804671. \\ -0.00149523 & 0.982274 & 0.101149 & -0.157814 \\ 0.999999 & 0.00152369 & -0.0000144046 & 0. \\ 3.50745 \times 10^{-7} & 0.00752158 & 0.819966 & 0.572364 \end{pmatrix}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 478.509. \\ 0 & 0 & 0 & 2132.96. \\ 0 & 0 & 0 & 2914.63. \\ 3.19851 \times 10^6 & -501085. & 0 & 1.97236 \times 10^6 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_d = \begin{pmatrix} 0 & -3.71878 & -29.5924 & -48.5743 \\ 34.2 & 45.5029 & -1.54773 & -2.54051 \\ -45.6 & 34.1272 & -2860. & 412.606. \\ -0.0228 & 0.0170636 & 0.125684 & 3.79101 \times 10^6 \end{pmatrix} \text{ MeV}$$

$$(\, m_d \,,\, m_s \,,\, m_b \,,\, M_D \,) = (\, 2.82069 \,,\, 57. \,,\, 2860.72 \,,\, 3.79 \times 10^6 \,) \text{ MeV}$$

Mixing matrices:
$V_{d\,L} = V^{o}_{d\,L}\, V^{(1)}_{d\,L}$:

$$\begin{pmatrix} -0.982152 & -0.123914 & -0.141504 & 0.0000673452 \\ 0.183872 & -0.790966 & -0.583578 & 0.000358725 \\ 0.0396112 & 0.599181 & -0.799633 & 0.000487473 \\ -0.0000191258 & 0 & 0.000608673 & 1. \end{pmatrix}$$

$V_{d\,R} = V^{o}_{d\,R}\, V^{(1)}_{d\,R}$:

$$\begin{pmatrix} 0.0824904 & -0.123915 & -0.515747 & 0.843709 \\ 0.593446 & -0.790966 & 0.0687286 & -0.132177 \\ 0.800455 & 0.59918 & -0.0159327 & -7.32943 \times 10^{-9} \\ 0.0169952 & -3.18402 \times 10^{-8} & 0.853831 & 0.520273 \end{pmatrix}$$

Quark mixing matrix

$$V_{CKM} = \begin{pmatrix} -0.974392 & -0.224827 & -0.003696 & -0.0000163 \\ -0.224475 & 0.973562 & -0.042288 & 0.0000214 \\ -0.013106 & 0.0403763 & 0.999098 & -0.0006083 \\ 3.74 \times 10^{-7} & -1.28 \times 10^{-6} & -0.000020 & 1.24 \times 10^{-8} \end{pmatrix}$$

Lepton masses and $(U_{PMNS})_{4\times 8}$ mixing

Charged leptons:

Tree level:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 2670.25 \\ 0 & 0 & 0 & 11902.6 \\ 0 & 0 & 0 & 16264.7 \\ 1.21882\times 10^{10} & -2.32202\times 10^{9} & 0 & 6.07835\times 10^{10} \end{pmatrix} \text{MeV}\,,$$

up to one loop corrections:

$$\mathcal{M}_e = \begin{pmatrix} 0 & -19.9797 & -83.226 & -16.9884 \\ 0.6408 & 71.9782 & 293.027 & 59.814 \\ -0.8544 & 168.853 & -1712.54 & 480.432 \\ -2.74\times 10^{-7} & 0.000054 & 0.000755 & 6.20\times 10^{10} \end{pmatrix} \text{MeV}$$

the charged lepton masses

$$(\, m_e\,,\ m_\mu\,,\ m_\tau\,,\ M_E\,) = (\,0.486031\,,\ 102.717\,,\ 1746.17\,,\ 6.20\times 10^{10}\,)\ \text{MeV}$$

Mixing matrices:

$V_{eL} = V^{o}_{eL}\, V^{(1)}_{eL}$:

$$\begin{pmatrix} 0.986458 & 0.0744614 & -0.146138 & 4.30921 \times 10^{-8} \\ 0.00276675 & -0.898433 & -0.439101 & 1.93334 \times 10^{-7} \\ -0.163991 & 0.43275 & -0.886473 & 2.62497 \times 10^{-7} \\ 0 & 5.68933 \times 10^{-8} & 3.23887 \times 10^{-7} & 1 \end{pmatrix}$$

$V_{eR} = V^{o}_{eR}\, V^{(1)}_{eR}$:

$$\begin{pmatrix} -0.00110967 & 0.258469 & -0.945829 & 0.196466 \\ -0.0055735 & 0.965887 & 0.256182 & -0.0374297 \\ 0.999984 & 0.00567042 & 0.00037637 & 0 \\ 9.59174 \times 10^{-6} & -0.0149293 & 0.199442 & 0.979796 \end{pmatrix}$$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	30.9559	0	0	0	13.2472
$\overline{\nu^o_{\mu L}}$	0	0	0	434.898	0	0	0	62.502
$\overline{\nu^o_{\tau L}}$	0	0	0	1980.48	0	0	0	76.9286
$\overline{N^o_L}$	30.9559	434.898	1980.48	40	790642.	114364	0	4000
$\overline{(\nu^o_{eR})^c}$	0	0	0	790642.	0	0	0	9.8860×10^6
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	114364.	0	0	0	1.4086×10^6
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	13.2472	62.502	76.9286	4000.	9.8860×10^6	1.4086×10^6	0	100000

Table: Tree Level Majorana masses

Tree level neutrino masses:

$\mathcal{M}_\nu^o$:

$$\begin{pmatrix} 0 & 0 & 0 & 30.9559 & 0 & 0 & 0 & 13.2472 \\ 0 & 0 & 0 & 434.898 & 0 & 0 & 0 & 62.502 \\ 0 & 0 & 0 & 1980.48 & 0 & 0 & 0 & 76.9286 \\ 30.9559 & 434.898 & 1980.48 & 40 & 790642. & 114364 & 0 & 4000 \\ 0 & 0 & 0 & 790642. & 0 & 0 & 0 & 9.8860 \times 10^6 \\ 0 & 0 & 0 & 114364. & 0 & 0 & 0 & 1.4086 \times 10^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.2472 & 62.502 & 76.9286 & 4000. & 9.8860 \times 10^6 & 1.4086 \times 10^6 & 0 & 100000 \end{pmatrix}$$

Neutrino Majorana mass matrix up to one loop:

$\mathcal{M}_\nu$:

$$\begin{pmatrix}
4.89118 \times 10^{-7} & 0.00534 & 0.02683 & -0.02718 & 0.02291 & 0.02261 & 0.16167 & 0.16087 \\
0.00534 & -0.00830 & 0.04847 & -0.03714 & -0.02119 & -0.02098 & -0.05213 & -0.05187 \\
0.02683 & 0.04847 & -0.62927 & 0.62907 & -0.00373 & -0.00353 & -0.20960 & -0.20855 \\
-0.02718 & -0.03714 & 0.62907 & -0.53094 & -0.22186 & -0.22037 & 0.22869 & 0.22756 \\
0.02291 & -0.02119 & -0.00373 & -0.22186 & -2604.12 & -0.25395 & -0.94008 & -0.93540 \\
0.02261 & -0.02098 & -0.00353 & -0.22037 & -0.25395 & 2643.36 & 0.74848 & 0.74476 \\
0.16167 & -0.05213 & -0.20960 & 0.22869 & -0.94008 & 0.74848 & -9.97002 \times 10^{6} & 2109.94 \\
0.16087 & -0.05187 & -0.20855 & 0.22756 & -0.93540 & 0.74476 & -2109.94 & 1.00658 \times 10^{7}
\end{pmatrix}$$

Neutrino masses: eV

$$(m_1 = 0.005847, m_2 = 0.010488, m_3 = 0.051461, m_4 = 1.21534,$$

$$m_5 = 2604.12, m_6 = 2643.36, m_7 = 9.97002\times10^6, m_8 = 1.00658\times10^7)$$

Squared neutrino mass differences:

$$m_2^2 - m_1^2 = 7.58162 \times 10^{-5}\ eV^2$$

$$m_3^2 - m_2^2 = 2.53822 \times 10^{-3}\ eV^2$$

$$m_4^2 - m_1^2 = 1.47441\ eV^2$$

Neutrino mixing:

$U_\nu = U_\nu^o \, U_\nu^{(1)}$:

$$\left(\begin{array}{cccc}
-0.817815 & -0.573736 & -0.030985 & -0.030274 \\
0.398121 & -0.525222 & -0.73405 & 0.011549 \\
0.009873 & 0.027107 & 0.142716 & -0.645085 \\
0.000011 & 5.86044 \times 10^{-6} & -0.000089 & -0.000079 \\
0.013900 & -0.015841 & -0.002901 & -0.106192 \\
-0.097564 & 0.111201 & 0.020387 & 0.74528 \\
-0.40357 & 0.617751 & -0.662886 & -0.126871 \\
-9.34653 \times 10^{-7} & -4.8869 \times 10^{-7} & 7.12916 \times 10^{-6} & 6.29816 \times 10^{-6}
\end{array} \right.$$

$$\left. \begin{array}{cccc}
0.008056 & -0.008006 & 1.12646 \times 10^{-6} & -1.08644 \times 10^{-6} \\
0.115983 & -0.115003 & 6.87784 \times 10^{-6} & -6.81358 \times 10^{-6} \\
0.532465 & -0.528342 & 0.000016 & -0.000016 \\
-0.702146 & -0.707553 & -0.056523 & -0.056254 \\
-0.064448 & 0.063411 & 0.701853 & -0.698211 \\
0.450162 & -0.446888 & 0.100018 & -0.099499 \\
0.000048 & 0.000047 & 0 & 0 \\
0.056171 & 0.056604 & -0.702997 & -0.706708
\end{array} \right)$$

U_{PMNS} lepton mixing matrix :

$$
\begin{pmatrix}
-0.807257 & -0.571865 & -0.0560008 & 0.0759557 & -0.0790509 & 0.0784275 & -1.6032\times10^{-6} & 1.60766\times10^{-6} \\
-0.414308 & 0.440887 & 0.718949 & -0.291791 & 0.126822 & -0.125914 & 1.11777\times10^{-6} & -1.07963\times10^{-6} \\
-0.0640545 & 0.29044 & 0.200336 & 0.571204 & -0.524122 & 0.520029 & -0.0000179606 & 0.0000177362 \\
4.43209\times10^{-8} & -1.19151\times10^{-7} & -1.05789\times10^{-7} & -1.68405\times10^{-7} & 1.62541\times10^{-7} & -1.61267\times10^{-7} & 0 & 0
\end{pmatrix}
$$

FCNC in Neutral Mesons

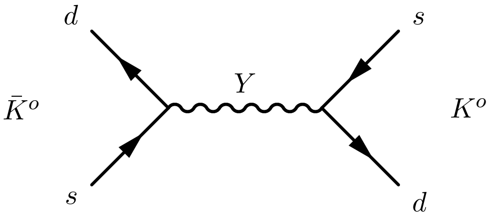

Figure: Generic tree level contribution to $K^o - \bar{K}^o$ from the $SU(3)$ horizontal gauge bosons and scalar fields

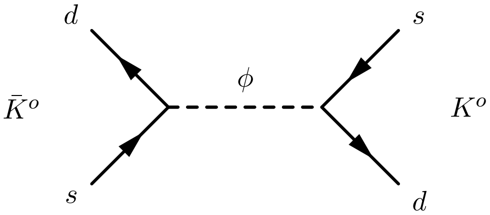

Figure: Generic tree level contribution to $K^o - \bar{K}^o$ from the

FCNC in Neutral Mesons

PDG2018: CKM quark-mixing matrix
To illustrate the level of suppression required for BSM contributions, consider a class of models in which the unitarity of the CKM is maintained, and the dominant effect of the new physics is to modify the neutral meson amplitudes by

$$\frac{z_{ij}}{\Lambda^2}\,(\bar{q}_i \gamma^\mu P_L q_j)^2$$

The existent data imply that

$$\frac{\Lambda}{\sqrt{|z_{ij}|}} \gtrsim \begin{cases} 10^4 \text{ TeV for } K^o - \bar{K}^o \\ 10^3 \text{ TeV for } D^o - \bar{D}^o \\ 500 \text{ TeV for } B^o - \bar{B}^o \\ 100 \text{ TeV for } B^o_s - \bar{B}^o_s \end{cases}$$

$K^o - \bar{K}^o$ and $D^o - \bar{D}^o$ meson mixing

$K^o - \bar{K}^o$:

$$\delta_L = 0.145437 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 32601.3 \text{ TeV's}$$

$$\delta_R = 0.469396 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 10101.1 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.369507 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 12831.8 \text{ TeV's}$$

$D^o - \bar{D}^o$:

$$\delta_L = 0.000647997 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 7.31705 \times 10^6 \text{ TeV's}$$

$$\delta_R = 0.00146872 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 3.22828 \times 10^6 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.669613 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 7080.85 \text{ TeV's}$$

Conclusions:

- Particle model with local $SU(3)$ family symmetry can account for the hierarchical spectrum of quark masses and mixing, and charged lepton masses
- FCNC in $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ Neutral Mesons may be properly suppressed
- The mass of the $SU(2)_L$ weak singlet vector-like D quark, M_D, may lie within a few TeV's region
- Simultaneously this scenario can fit the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.58 \times 10^{-5}\ eV^2$, $m_3^2 - m_2^2 \approx 2.53 \times 10^{-3}\ eV^2$, $m_4^2 - m_1^2 \approx 1.47\ eV^2$, as well as the majority of the corresponding entries of the U_{PMNS} lepton mixing matrix.