



Article

Testing General Relativity vs. Alternative Theories of Gravitation with the SaToR-G Experiment

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Abstract: A new experiment in the field of gravitation, SaToR-G, is presented. The experiment aims to compare the predictions of different theories of gravitation in the limit of weak-field and slow-motion. The ultimate goal of the experiment is to look for possible “new physics” beyond the current *standard model* of gravitation based on the predictions of General Relativity. A key role in the above perspective is the theoretical and experimental framework within which to confine our work. To this end, we will try to exploit as much as possible the framework suggested by Dicke over fifty years ago.

Keywords: General Relativity; Alternative theories of gravitation; Weak Fields; LAGEOS and LARES satellites

1. The goals of SaToR-G

SaToR-G (Satellites Tests of Relativistic Gravity) is a new fundamental physics project that aims to test gravitation beyond the predictions of Einstein’s General Relativity (GR) theory [1], in search for effects foreseen by alternative theories of gravitation (ATG) [2] and possibly connected with “new physics”. In particular, SaToR-G is dedicated to measurements of the gravitational interaction in the weak-field and slow-motion (WFSM) limit of GR by means of laser tracking to geodetic passive satellites orbiting the Earth. Indeed, this new experiment exploits — as *quasi-ideal* proof masses — the geodynamic laser-ranged satellites LAGEOS [3,4], LAGEOS II [5] and LARES [6] tracked by the powerful Satellite Laser Ranging (SLR) technique [7,8].

The activities of SaToR-G mainly focus (but not only) on metric theories of gravitation, GR being the first of this category. In the context of theories of gravitation alternative to GR, scalar-tensor and vector-tensor theories are of considerable importance. In particular, scalar-tensor theories are metric theories of gravitation quite interesting to be further investigated as ATG [9–11]. The main focus of SaToR-G will be twofold: i) measurement of possible deviations of gravity from the inverse square law for the distance between the Earth and the satellites considered, with possible constraints on a Yukawa-like long-range interaction [12–14] with a typical range correlated to the semi-major axis of satellites [15–17]; ii) precise and accurate measurements of some post-Newtonian parameters according to the PPN (Parameterized Post-Newtonian) formalism [18–20]. These are in fact in this context the most powerful tools for testing the predictions of different theories beyond GR itself.

2. The theoretical framework of SaToR-G

Precisely measuring the orbits of artificial satellites allows to test GR vs. other metric theories in their most profound aspects related to the curvature of spacetime, to geodesic



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motion and to the field equations. Metric theories of gravitation share Einstein's Equivalence Principle (EEP) [21], the Lorentzian structure of spacetime and the equations of motion. In other words, in all metric theories of gravitation the structure of spacetime is the same, as is the way in which the geometry of spacetime determines the way mass-energy moves in it. What instead profoundly distinguishes GR from the other metric theories of gravitation are the equations of the gravitational field, that is, how the mass-energy of the field *orders* the geometry of spacetime to curve [2].

As mentioned above, testing for the values of the PPN parameters represents a powerful tool to discriminate among different theories of gravitation. Anyway, within the SaToR-G strategy to test a theory of gravity, we are also interested in recovering the more general approach from which the PPN formalism itself, in its current version, was basically born. In fact, we will try as much as possible to test the different theories in the theoretical/experimental framework conceived by Robert Dicke around the mid-60s [22]. The main idea at the basis of this framework is to build up a set of experiments to be as unbiased as possible both from classical Newtonian physics and from Einstein's GR. The continuing experimental successes of GR predictions in recent decades made this quest less pressing.

On the other hand, during the 1960s and 1970s (and also, partly, for the 1980s), when experimental evidence for the validity of general relativity was still very weak [23,24], several alternative theories were proposed with a certain degree of continuity [25]. Indeed, in the early 1970s, Kip Thorne and Clifford Will proposed [26] — as a strategy for testing GR — a scheme based on both a Dicke-like approach, as well as an approach based on the nascent, at that time, PPN formalism.

However, from a practical point of view, it appears that Dicke's framework has not been fully exploited in the past, and the main tests and measurements of GR have actually been based on measurements of the PPN parameters. This aspect is not only largely true in the case of gravitational measurements within the solar system, that is, in the case of weak fields (that Thorne and Will were primarily concerned with, in 1971), but also in the context of almost strong fields such as those tested more recently in relativistic astrophysics [27,28]. Indeed, even in the cases of non-weak fields, the post-Newtonian formalism provides an excellent description of gravitational measurements [29].

For the above reasons, we believe that an effort to reconsider the Dicke framework is appropriate and of interest even in these days, to test the foundation of gravitation, especially in those aspects that are not fully covered by the PPN framework.

3. The legacy from LARASE

SaToR-G builds on the improved dynamical model of the two LAGEOS and LARES satellites achieved within the previous project LARASE (LAsER RANGed Satellites Experiment) [30]. The improvements mainly concern the modeling of the non-conservative forces (NCF) acting on the surface of the three satellites [31–34] and that of the Earth's gravitational field and tides in their precise orbit determination (POD) [34–36]. Regarding the NCF, the main improvements were the development of a model for the spin of the satellites (LASSOS: LARASE Satellites Spin mOdel Solutions) and a model for thermal thrust forces (LATOS: LARASE Thermal mOdel Solutions). For the gravitational field, the monthly solutions of the GRACE mission [37–39] have been implemented in the code used for the POD.

The main results of LARASE in the field of gravitational effects measurements were a precise and accurate measurement of the precession of the argument of pericenter of LAGEOS II [17] and of the Lense-Thirring precession from the analysis of the orbits of the two LAGEOS and LARES satellites [34,40,41]. In particular, in the case of the first measurement, constraints on non-symmetric [42,43] and torsional [44,45] theories of gravitation were set with improvements with respect to previous results in the literature. In this regard, a significant result was the constraint on a Yukawa-like interaction with a characteristic range λ close to the radius of the Earth and a strength $|\alpha| = |0.5 \pm 8 \pm$

$101| \times 10^{-12}$. In the case of the Lense-Thirring precession, the parameter μ which is used to parameterize the relativistic precession (with $\mu = 1$ in GR and $\mu = 0$ in Newtonian physics) was measured very accurately with an error budget of about 1.6%: $\mu_{\text{meas}} - 1 = (1.5 \pm 7.4 \pm 16) \times 10^{-3}$, where the first uncertainty represents the statistical formal error, at a 95% confidence level, while the second uncertainty represents the estimate of the systematic sources of error.

4. Conclusions

A number of activities have been initiated with the aim of setting new kinds of measurements in the field of gravitation with Earth-bound laser-ranged satellites. This activity will be based on a theoretical/experimental framework not “simply” described by PPN parameters, but also as close as possible to the original framework proposed by Professor R.H. Dicke.

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Sample Availability: Samples of the compounds are available from the authors.

