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Gravitational Collapse of Dust Cloud in the Presence of Dark Energy in 4D-Einstein Gauss-Bonnet Gravity

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Abstract: We investigate the gravitational collapse of a gravitational bounded object constituted of dust cloud and dark energy. We considered the the effects of homogenous and isotropic fluid on newly suggested 4D limit for Einstein-Gauss-Bonnet gravity(EGB). For this purpose, we consider the gravitational collapse of gravitational object constituted of dust cloud ρ_{DM} in the background of dark energy, $p = w\rho$ with $(w < -1/3)$. We illustrate that the procedure is qualitatively equivalent to the scenario of theory of Einstein for the collapse of the gravitational object composed of homogeneous dust. Further, we consider the collapse for dark energy by considering the equation of state $p = w\rho$ to find that black hole also may form in EGB case, which predict that end state of gravitational collapse in EGB case is consistent with results carried out in pure Einstein's gravity theory.

1. Introduction

During the previous few decades, there has been growing interest to review the fate of the universe in various theories of gravity. These theories offer awareness of DE(Dark energy), which could be the keystone for universe expansion. From the astronomical observations, it has been demonstrated that our universe is flat and right now it comprises of around $(2/3)$ DE(Dark energy) and DM(dark matter) is of $(1/3)$. Moreover, the nature of both DE and DM is still unknown [1].

In recent years, there have been some changes in the general theory of relativity(GTR), one of the most interesting theory and well known through which higher theory of gravity is tested is known by the name EGB gravity (i.e. Einstein-Gauss-Bonnet gravity). While this gravity theory is the generalization of the GTR and is a exclusive scenario of Lovelock gravity. More interestingly, it has been found that the Lovelock second-order gravity is known as Einstein Gauss-bonnet gravity while the first order is Einstein's gravity [2,3]. In the theory of EGB gravity, the study of gravitational phenomena of the system in $(D > 4)$ dimension is under discussion. This idea typically takes place in the low energy limit of the hereotic superstring theory of gravity [4,5]. However, it was suggested recently that there may be a four-dimensional non-trivial of Gauss-Bonnet theory, thus ignoring Lovelock's theorem, if one evaluates a rescaling of Gauss-bonnet(GB) coupling constant λ to eliminates the vanishing term (D$-$4) that was occurring through the alteration of the Gauss-Bonnet effect. Whereas, the Lovelock theorem specifies that the term namely GB in equation of the motion just responds to D$>$ 4 dimension respectively [6].

The nature of such a 4D-EGB is currently under consideration but questions have also been lifted regarding the suggested rescaling that eliminates the factor with the one which is unknown. Though it is not certain whether the field equation for the system has a true motion of equation but with no special symmetry [7,8]. While for the spherical system it

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is possible that the 4-dimensional solutions with high symmetries could be set up with the suggested redefinition of λ from the 4-dimensional effect of 4D-Einstein Gauss-Bonnet gravity. Such solutions can also give an interpretation physically and can examine their features as well, even though in the general theory's absences. These theories have therefore gained particular attention in finding solutions for the black hole. This EGB theory is a very fascinating scenario for researching how higher-order curvature corrections to black hole physics significantly alter the qualitative characteristics.

For instance, the stability and shadow of black holes are investigated by R.A Kanoplya and A.F Zinhailo in [9], rotating black hole have been discussed by Rahul Kumar and S. G.Ghosh in [10]. Gravitational lensing and 4 dimensional rotating black hole Shadow were discussed in [11,12]. Gravitational collapse and non-static anisotropic fluid have been investigated by G.Abbas and M.Tahir in $5D$ Einstein-Gauss-Bonnet Gravity in [13]. Whereas, the collapse phenomena of inhomogeneous dust fluid have also been considered in 5-dimensional EGB gravity in [14].

However, in this study, we will examine the gravitational collapse of non-interacting particles namely the dust cloud in the context of dark energy(DE), by "dust cloud" we mean a cloud that is formed from a matter having zero pressure. In recent decades, the final fate of gravitational collapse has been gaining attention towards it and it's also been a very interesting problem in astronomy as well as in astrophysics respectively. Although, it has been observed that, for many ages, the black hole is the keystone for the gravitational collapse. This phenomenon of collapse, influenced via the force of gravity is the core of black hole and in the last past ages expanding consideration towards it.Many authors [15,16] and [17,18] have investigated the gravitational collapse in EBG theory.Also on the space-time dimension, the final stage indicates a non-trivial dependency.

The paper plane designed as follows: In Section(II), we examine the features of gravitational collapse in GB-gravity with in the back ground of dark energy. In Section(III) we will study the gravitational collapse in 4D-Einsteins-Gauss-Bonnet for dust collapse and dark energy separately, so to explore the distinct role that they will perform during the phenomena of collapse by finding the equation of motion form of the curve for singularity and the apparent horizon (AH) for the theory of $4-$dimensional limit, and in pure Einstein theory, we will compare the corresponding outcomes. For the first case, we will investigate that the collapse of a homogenous dust cloud in EGB behaves qualitatively similar to that of the dust collapse in the Einstein's theory and the impact due to the existence of the GB term can be seen during a delay within t (i.e, a co-moving time) on which singularity is being formed. For the second case, we will examine the theory via plotting the physical quantities. In final sections (IV), we discuss the conclusion regarding to the results for the EGB theory.

All over the paper, we have assumed $G = 1$ with $c = 1$ and for the definition of energy-momentum tensor $T_{\mu\nu}$ in the field equation we have used the term $\kappa = 8\pi$

2. Gravitational Collapse in Einstein Gauss Bonnet Gravity

we begin with the action for D-dimensional Einstein Gauss-Bonnet gravity with the matter

$$S = \int d^D x \sqrt{-g}(R + \tilde{\lambda} L_{GB}) + S_{matter}, \tag{1.1}$$

where, S_{matter} denotes the action associated with matter and $\tilde{\lambda}$ is a coupling constant that we assume to be non-negative and R denoted as the Ricci scalar that indicates the relative part of action i.e Einstein term, whereas the g indicates the determinant of the metric and L_{GB} is said to be the second-order Gauss-Bonnet term and is defined as

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\mu\nu\xi}R^{\alpha\mu\nu\xi}. \tag{1.2}$$

Here $R_{\alpha\mu\nu\xi}$ and $R_{\mu\nu}$, R are known as respectively the Riemann Tensor, Ricci tensor and Ricci scalar. The variation of the action with respect to the metric $g_{\mu\nu}$ provides the equations of EGB

$$\mathcal{G}_{\mu\nu} \equiv G_{\mu\nu} + \mathcal{H}_{\mu\nu} = T_{\mu\nu}. \tag{1.3}$$

where,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{1.4}$$

is known as Einstein Tensor and

$$\mathcal{H}_{\mu\nu} = 2\tilde{\lambda}[RR_{\mu\nu} - 2R_{\mu\alpha}R^{\alpha}{}_{\nu} - 2R_{\mu\alpha\nu\xi}R^{\alpha\xi} + R_{\mu}{}^{\alpha\xi\sigma}R_{\nu\alpha\xi\sigma}] - \frac{\tilde{\lambda}}{2} g_{\mu\nu} L_{GB}, \tag{1.5}$$

is the Lanczos tensor while $T_{\mu\nu}$ is the energy-momentum tensor in the context of dark energy, defined as

$$T^{-}_{\mu\nu} = (\rho_{DM} + \rho + p) u^{-}_{\mu} u^{-}_{\nu} + p g^{-}_{\mu\nu}, \tag{1.6}$$

here energy density of the dust cloud is ρ_{DM}, whereas energy density and pressure of dark energy are ρ and p. Where u^{-}_{μ} is the n-velocity vector. Here in this study, we adopt the mechanism given in Refs. [19] and concentrate on to obtain the solution in co-moving coordinates for an n$-$dimensional spherical dynamical matter cloud, so metric has the form

$$ds^2_{-} = -e^{2\phi(r,t)} dt^2 + e^{2\psi(r,t)} dr^2 + R^2(r,t) d\Omega^2{}_{n-2}, \tag{1.7}$$

where,
$d\Omega^2_{n-2} \equiv d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + ... + \sin^2\theta_1 \sin^2\theta_2 ... \sin^2\theta_{n-3} d\theta^2_{n-2}$.

By using the bianchi identities we get $\phi' = 0$,whereas $\frac{\partial}{\partial r} = {}'$. With such an appropriate re-scaling of the co-moving time t, we would then determine the value of ϕ i.e. $\phi = 0$.

First, we will determine the field equation Eq.(1.3) for off-diagonal components i.e.

$$\mathcal{G}_{tr} = (D-2)\left(\frac{\dot{\psi}R' - \dot{R}'}{R}\right) + 2(D-2)\lambda \frac{(\dot{\psi}R' - \dot{R}')(e^{2\psi} + \dot{R}^2 e^{2\psi} - R'^2)}{e^{2\psi}R^3}, \tag{1.8}$$

Here $\frac{\partial}{\partial t} = \dot{}$ and we therefore, set $\lambda = (D-3)(D-4)\tilde{\lambda}$. From the Eq.(1.8) we have two distinct forms of $e^{2\psi}$, from which we can calculate the two branches for the collapse. First one is equal to having a form

$$2\lambda(e^{2\psi} + e^{2\psi}\dot{R}^2 - R'^2) + e^{2\psi}R^2 = 0, \tag{1.9}$$

and that will bring us to

$$e^{2\psi} = \frac{2\lambda R'^2}{R^2 + 2\lambda(\dot{R}^2 + 1)}. \tag{1.10}$$

While the other one will correspond to having the form

$$\dot{\psi}R' - \dot{R}' = 0, \tag{1.11}$$

so that gives the form

$$e^{2\psi} = \frac{R'^2}{E(r)}, \tag{1.12}$$

Here $E(r)$ is a free radial coordinate function, which concerns the initial velocity profile of the cloud that would collapse. In this scenario, we will concentrate upon the collapse that would satisfy Eq.(1.10) because it resembled that of the relevant equation in the theory of Einstein. Now the rest of the non-zero field equations could be expressed as

$$\mathcal{G}_{tt} = \rho_{DM} + \rho = \frac{(D-2)F'}{2R^{D-2}R'} \ , \tag{1.13}$$

and

$$\mathcal{G}_{rr} = p = -\frac{(D-2)\dot{F}}{2R^{D-2}\dot{R}} \ , \tag{1.14}$$

Here, we have determined the mass function for structure $F(r)$, and this is equal to that of the Misner-Sharp mass within that pure general relativity[20] as

$$F(r) = \lambda R^{D-5}(\dot{R}^2 + 1 - E)^2 + R^{D-3}(\dot{R}^2 + 1 - E). \tag{1.15}$$

If the Eqs.(1.13)and (1.14) are resolved,the other field equations, i.e., $\mathcal{G}^{\theta_i}_{\theta_j}$ are valid. Eq.(1.15) can be given by the form about an equation of the Motion within the framework as

$$\frac{F(r)}{R^{D-2}} = \lambda\dot{R}^4 + [2\lambda(1-E) + R^2]\dot{R}^2 + \lambda(1-E)^2 + R^2(1-E). \tag{1.16}$$

Observe that the equation for $\lambda = 0$ diminishes to the typical motion of the equation for general relativity(GR) dust collapse. The equation referred to the one above is a PDE of the fourth power in $\dot{R}^4$ and thus would usually allow four different branches of solution. Also, the Eq.(1.16) is quadratic in the case of $\dot{R}^2$ whereby for collapse resulting through infinity at zero velocity that is $E = 1$ possesses a positive root while the second one possesses a negative root. The positive root is the only one that would be physically meaningful, and provide two solutions regarding both the collapsing as well as expanding dust cloud. Further, the positive collapsing solution perhaps compared to the vacuum geometry at the boundary, and that's the area on which we will pay attention within the ensuing section.

The value $F(r)$ behaves as a quasi-local mass for the collapsing dust cloud and perhaps known at a certain time t for expressing the quantity of matter herein the r co-moving radius. Its reality that F is not depending upon t is however a result of selecting non-interacting particles for the content of matter, in other words, an effect of Eq.(1.14) suggesting a $\dot{F} = 0$. It, therefore, ensures there will not be inflow and outflow of matter via any shell r and within particularly from the cloud boundary, also the interior collapse could be likened with the vacuum exterior. Defining a co-moving boundary of the collapsing cloud at $r = r_b$ we would then describe $F(r_b)$ as associated with total mass of the system, provided that it should be associated with the M mass parameter within the exterior vacuum solution. That's the scenario once comparison has done with the vacuum solution investigated in [21] having the metric

$$ds_+^2 = -H(S)dT^2 + \frac{ds^2}{H(S)} + S^2 d\Omega_{n-2}^2 \ , \tag{1.17}$$

with

$$H(S) = 1 + \frac{S^2}{2\lambda}\left(1 \pm \sqrt{1 + \frac{4\lambda M}{(D-2)S^{D-1}}}\right). \tag{1.18}$$

Note that here exist two types of solutions relying upon the signs ahead of the square root, one pointing towards attractive whereas the second towards repulsive mass points, thus we prefer the form having a minus sign. For small S, the two branches have the same behavior, i.e., approaching a DeSitter-like solution (one attractive whereas the other repulsive), even though asymptotically exhibiting distinct characteristics, with that of the branch with the minus sign which has Schwarzschild-like behaviour for large S [22,23].The $r = r_b$ boundary radius in the interior refers here to collapsing boundary of such $R_b(t) = R(r_b, t)$ cloud, whereas the same boundary when seen from the exterior would be presented by $S = S_b(T)$, with $T = T(t)$ at the boundary defined by the corresponding continuity condition of g_{00}. The metric continuity upon this unit (D-2)-sphere indicates that

$$R_b(t) = S_b(T(t)). \tag{1.19}$$

M the total D-dimensional ADM mass is being linked to $F(r_b)$ for the equation of the motion at the boundary by the continuity condition (see [15]). Within simple marginally bound collapse case, shown via $E = 1$ which results in

$$\dot{R_b^2} = -\frac{R_b^2}{2\lambda}\left(1 \pm \sqrt{1 + \frac{4\lambda F(r_b)}{R_b^{D-1}}}\right), \tag{1.20}$$

the interior whereas in the Exterior is

$$S_b(T(t)) = -\frac{S_b^2}{2\lambda}\left(1 \pm \sqrt{1 + \frac{8\lambda M}{(D-2)S_b^{D-1}}}\right), \tag{1.21}$$

Above mentioned equations as well as the with boundary condition in Eq.(1.19) could only be fulfilled when

$$2M = (D-2)F(r_b). \tag{1.22}$$

The above arrangement may also be easily applied to the present theory in [6] to study the end results of a homogeneous collapse.

3. Gravitational Collapse of a Dust Cloud or Dark Energy

Within this section, we determine a collapsing cloud of dust and dark energy independently, for seeing the distinct roles that they could perform during the collapse.

3.1. Gravitational collapse with dust cloud

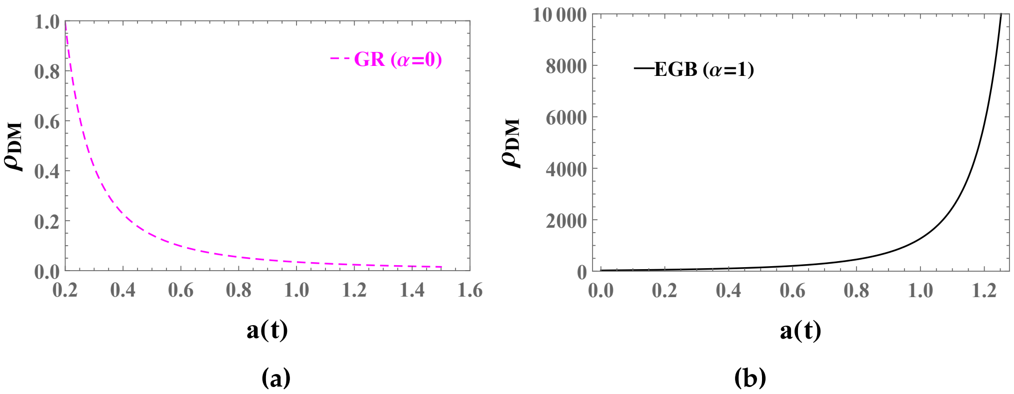

Figure 1. The left panel (a) The Evolution of the energy density ρ_{DM} of the cloud for OSD case and The right panel (b) Evolution of the energy density of the cloud ρ_{DM} for 4D-EGB has shown.

We will therefore emphasis to bound on the unique case for the collapse with $D = 4$ dimensions with the GB coupling rescaling which is presented by the form $\tilde{\lambda} \to \frac{\tilde{\lambda}}{(D-4)}$, that corresponds to $\lambda = (D-3)\tilde{\lambda} = \tilde{\lambda}$ in this scenario by defining $D = 4$. It's quite essential to know that because of the system's spherical symmetry, the reparametrization of λ presented in [6] provides the true field equations within this model unless the lower dimensional limit would be selected to remove the extra dimensions of the metric $(D-2)$-sphere component. For this scenario, the line element looks like

$$ds_-^2 = -dt^2 + \frac{R'^2}{E}dr^2 + R^2 d\Omega_2^2 \ , \tag{2.1}$$

With $d\Omega_2^2$ the ordinary line-element on the unit 2-sphere and the field equations derivation continue exactly as explained in the preceding section.

To be simple, we could now enforce some scaling dependent on the gauge freedom to define the initial radius as $R(0, r) = r$, the arbitrary nature of the $E(r)$ function,as well as the other physical necessity for the mass function (view[24] for info). Therefore,we have introduced

$$R(r,t) = ra(r,t), \tag{2.2}$$
$$E(r) = 1 - r^2 f(r), \tag{2.3}$$
$$F(r) = m(r)r^3, \tag{2.4}$$

The scenario of homogeneous dust definitely means if $a(r,t) = a(t), m(r) = m_0$ and $f(r) = j$, thus reducing the equation of the motion (1.16) to

$$\lambda(\dot{a}^2 + j)^2 + a^2(\dot{a}^2 + j) = m_0 a \ , \tag{2.5}$$

If $\lambda = 0$,the above equation agrees with the collapses of Oppenheimer-Snyder-Datt (OSD) motion of the equation [25,26].

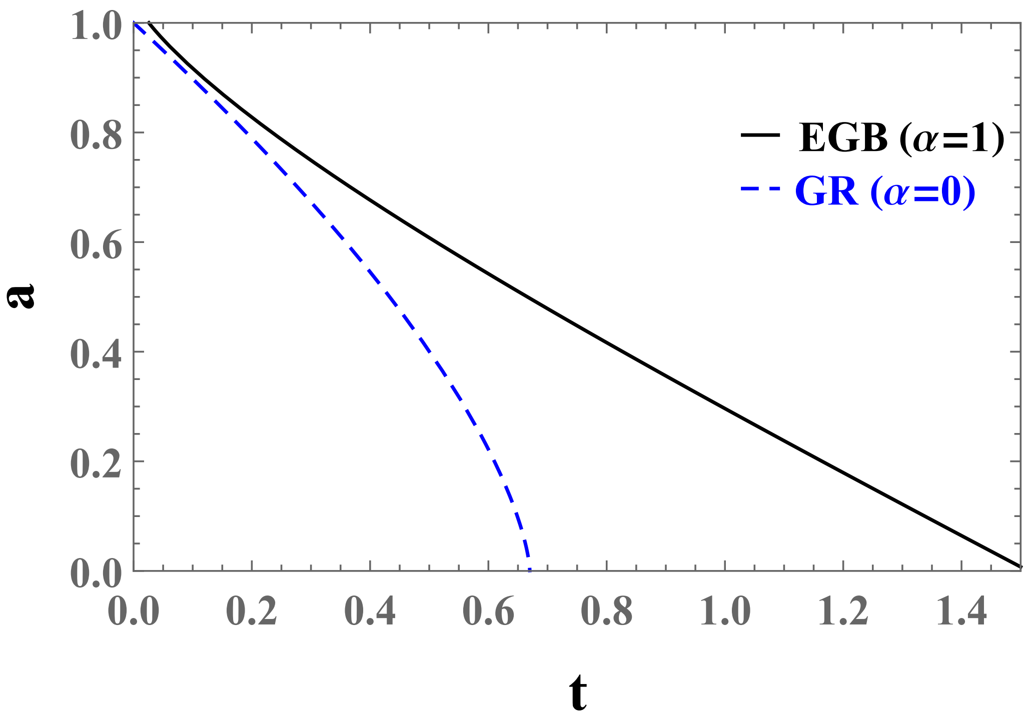

Figure 2. In Einstein's theory (blue dashed line) and in 4D-EGB theory (solid line), the scale element a(t) for marginally bound dust collapse. Observe that the Gauss-Bonnet term 's effect on collapse is to slow the scale factor velocity and slow singularity forming time. The initial time t = 0 is selected in this in such a manner that a(0) = 1 for the both respects.

Now in this case we have considered that

$$\rho_{DM} \neq 0, \ \ and \ \ \rho = p = 0. \tag{2.6}$$

So from this assumption, we have observed that with the above-mentioned rescaling of Einstein's equation for $\rho_{DM}(t)$, that is Eq. (1.13), gives

$$\rho_{DM} = \frac{3m_0}{a^3}, \tag{2.7}$$

That becomes finite at the onset of a collapse, resulting in a divergent density with $a \to 0$. Therefore it is good to assume for explanatory purposes the simplest case of marginally bound collapse that leads to $j = 0$. So Eq.(2.5) becomes

$$\lambda \dot{a}^4 + a^2 \dot{a}^2 = m_0 a \ , \tag{2.8}$$

Along with $\lambda = 0$,we regain the normal OSD solution which comes in a shape

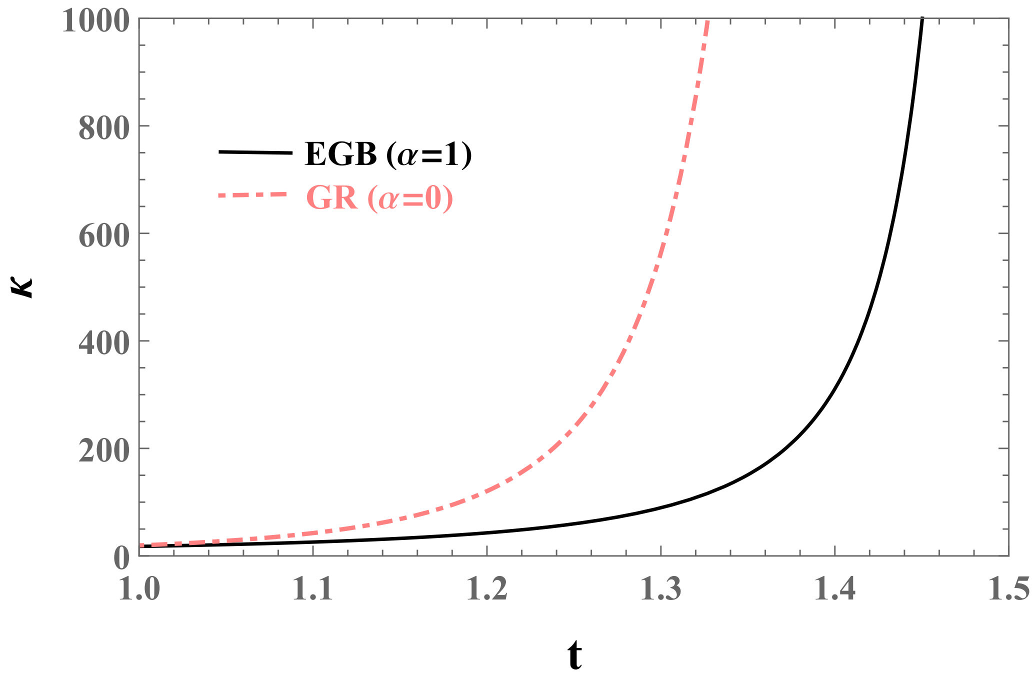

Figure 3. In Einstein 's gravity (red dotdash line) as well as in the 4D-EGB theory (solidline), compared with the Kretschmann κ scalar for collapse. We suppose here that the singularity tends to happen for both cases at the same time t_s(thereby moving the scale factor in the GR scenario). and therefore we will notice that κ diverges faster in GR with respecting to the 4D-EGB scenario as a result of the GB term.

$a(t) = (1 - 3\frac{\sqrt{m_0}t}{2})^{\frac{2}{3}}$ with a initial condition $a(0) = 1$.

Equation (2.8) is perhaps a quartic equation and has four roots, in particular, one of which defines the collapse and look-alikes to the exterior black hole (1.17). In the remaining three roots, two are complex and one is an expansion Of cloud. Therefore, the root of Eq. (2.8) explaining the collapse would be presented via

$$\dot{a} = -\frac{a}{\sqrt{2\lambda}}\sqrt{\sqrt{1 + \frac{4\lambda m_0}{a^3}} - 1}. \tag{2.9}$$

Observe that for the solution being true we should have $\lambda = 0$, perhaps we could not recover the OSD limit out of the equation above. It is obvious to get $t(a)$ by the analytical integration of equation (2.9) and that's a monotonous function of a which has shown as.

$$t = \frac{2}{3}\sqrt{\lambda}\left(\arctan(u) - \frac{1}{u}\right) + c, \tag{2.10}$$

whereas

$$u = \frac{1}{\sqrt{2}}\sqrt{\sqrt{1 + \frac{4\lambda m_0}{a^3}} - 1} \tag{2.11}$$

and c is a constant of the integration that would be found by taking the initial condition as $a(0) = 1$:

$$c = \frac{2}{3}\sqrt{\lambda}\left(\frac{1}{u_0} - \arctan(u_0)\right), \tag{2.12}$$

and

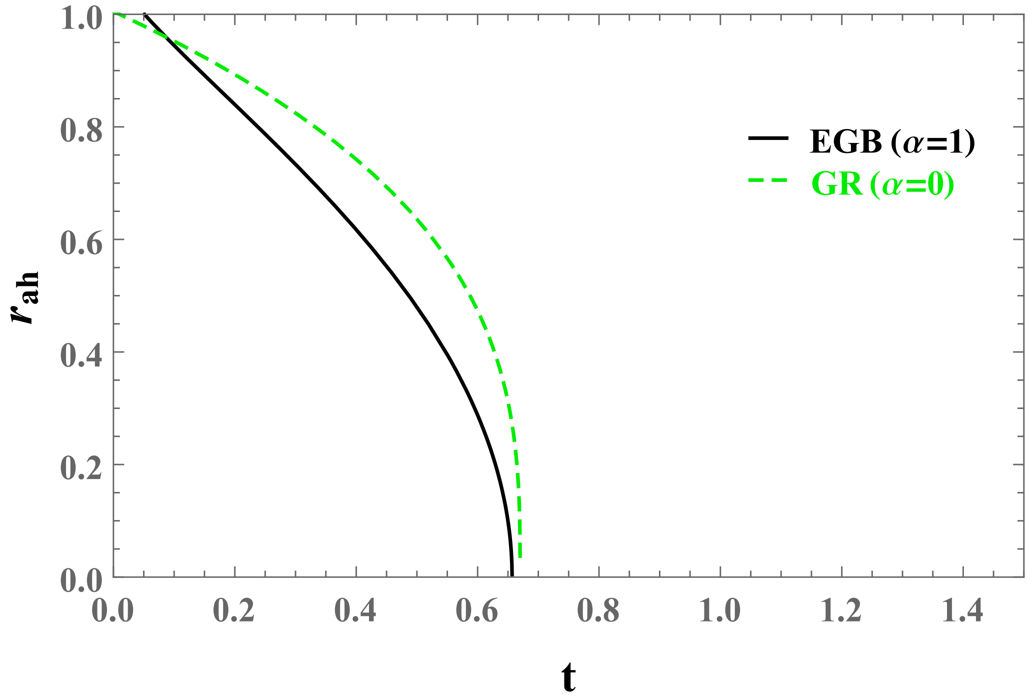

Figure 4. AH curve radius as a function of time in the gravity of purely Einstein (green dashed line) and in the recently formulated theory of 4D-EGB (black solid line). We notice that the GB term influences the formation of trapped surfaces through delayed the co-moving time where each shell has became trap, and supposing that collapsing process starts at $t = 0$ and $a(0) = 1$.

$$u_0 = \sqrt{\frac{\sqrt{1 + 4\lambda m_0} - 1}{2}}. \tag{2.13}$$

Thus the $a(t)$ scale factor is defined simply from the inverse of Eq.(2.10). Figure 2 provides a comparison of the nature of solution for Eq.(2.9) as well as the OSD case.

For the above-mentioned context, the Kretschmann scalar will be shown via

$$\kappa = 12\left(\frac{\ddot{a}^2}{a^2} + \frac{\dot{a}^4}{a^4}\right) \ , \tag{2.14}$$

This diverges for $a \to 0$, thereby demonstrating the forms of curvature singularity here as collapse end-state, similar to the pure relativistic situation. Thus, the singularity is "weaker" here as a result of the GB term than in its relativistic comparison. This could be shown by the reality that in the GR context the Kretschmann scalar diverges more quickly, as seen in figure 2.From above-mentioned pondering we see when the singularity occurred at the time t_s on which $a(t_s) = 0$. At the same time t_s, all of the shells drop into the singularity because of the homogeneous form of the collapse but also singularity is space-like. Therefore in this scenario, we do have

$$t_s = \frac{2}{3}\sqrt{\lambda}\left(\frac{1}{u_0} - \arctan(u_0)\right) - \frac{\pi}{3}\sqrt{\lambda}. \tag{2.15}$$

Now we're studying the nature of trapped surfaces with the case of marginally bound as well as homogeneous dust collapse. In the interior the AH is determined by the condition that the surface $R(t,r)$ would be null,that is $g^{\mu\nu}(\partial_\mu R)(\partial_\nu R)$.It meets the necessity that

$$1 - \frac{F(r)}{R(r,t)} = 1 - \frac{r^2 m_0}{a(t)} = 0. \tag{2.16}$$

That determines the AH curve $r_{ah}(t)$ implicitly by

$$r_{ah}(t) = \sqrt{\frac{a(t)}{m_0}}. \tag{2.17}$$

Figure 4 illustrates the formation of the AH radius within that 4D-Einstein Gauss-bonnet theory and also in the theory of Einstein. Similar with what we have studied about the scale factor and also the forming of trapped surfaces would be to delay due to the impact of the GB term.

3.2. COLLAPSE OF DARK ENERGY

In case of DE collapse,first we define the metric when $\lambda = (D-3)\tilde{\lambda}$ with $\lambda = 0$ by using the Eqs.(2.1) and (2.2) i.e,.

$$ds_-^2 = -dt^2 + \frac{a^2(t)}{E}dr^2 + a^2(t)d\Omega_2^2 \ , \tag{3.1}$$

Whereas Filed equations (1.13) and (1.14) can be written as

$$\frac{F'}{R^2 R'} = (\rho_{DM} + \rho) \tag{3.2}$$

$$\frac{\dot{F}}{R^2 \dot{R}} = p \tag{3.3}$$

Now, the interaction between both the dust cloud and dark energy is defined by the law of conservation of energy momentum tensor i.e., $T_{\mu\nu;\lambda}g^{\nu\lambda} = 0$, that would be given within the current study as

$$\dot{\rho}_{DM} + 3\left(\frac{\dot{a}}{a}\right)\rho_{DM} = Q \tag{3.4}$$

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = -Q \tag{3.5}$$

Here $Q = Q(t)$ expresses the interaction within the dust cloud as well as the dark energy. As we're primarily dealing with the gravitational collapse in this article, we suppose

$$\dot{a} < 0 \tag{3.6}$$

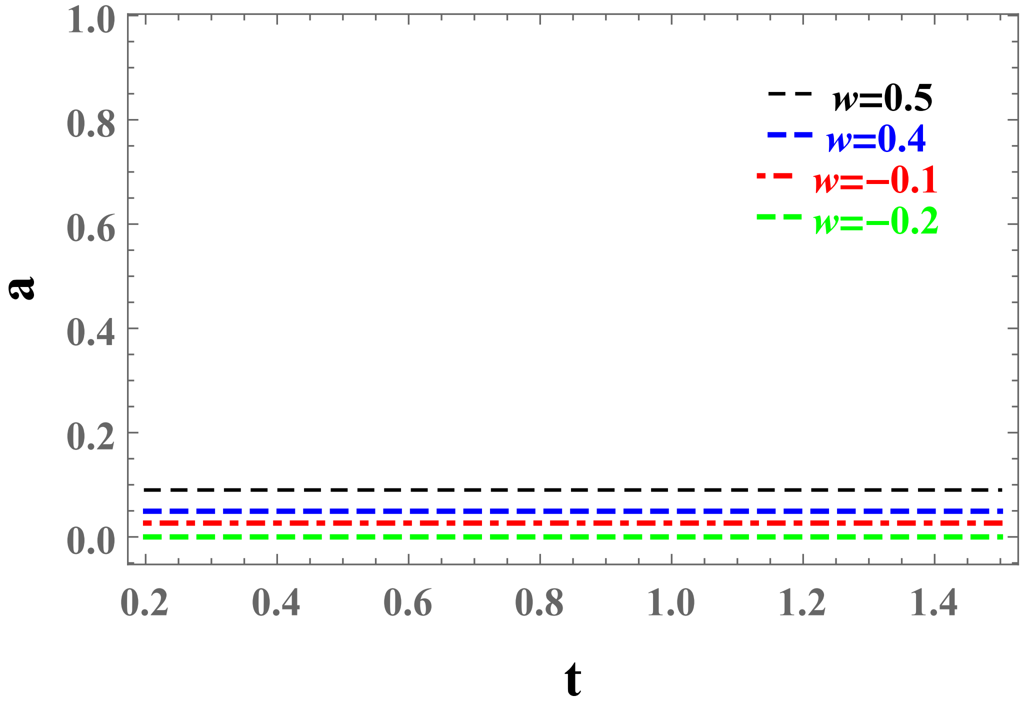

Figure 5. Evaluation of scale factor$a(t)$ for different values of $w > -1/3$ has been shown.

and $t = \tau$

To investigate the impact of dark energy on gravitational collapse, we take into account firstly the situation for which

$$\rho_{DM} = 0 \quad and \quad p = w\rho \neq 0. \tag{3.7}$$

here,a non-zero constant is w. Although $w < -1/3$ does not meet the strong energy condition[1], and it has said that the fluid is composed of dark energy. So there is a short analysis of the principal properties of the collapse in the above-mentioned setting thus we're seeing easily, it is partly played by the DE whilst the collapse.

Through Eq.(3.5), we have determine the value of $\rho = \frac{\rho_0}{a^3(1+w)}$, here the constant of integration is ρ_0. Therefore,Eqs(3.2) and (3.6)in return gives

$$a(t) = \left(\frac{\rho_0}{3m_0}\right)^{-\frac{1}{3w}} \tag{3.8}$$

However, in such context, the other physically relevant values could be shown in the form as

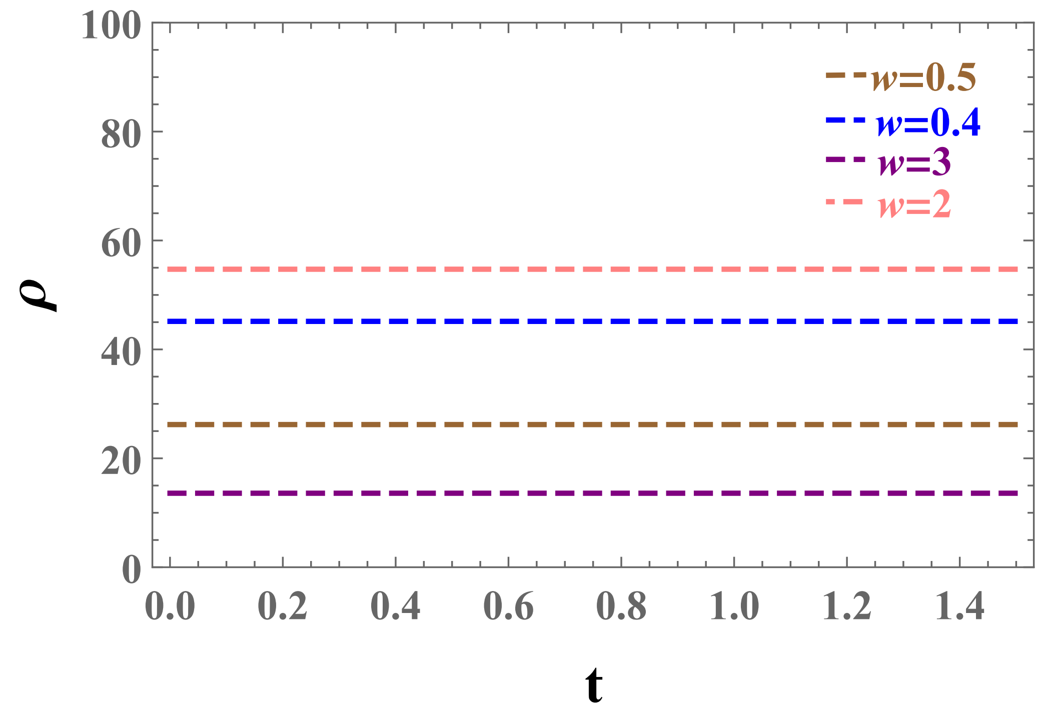

Figure 6. For different values of w for dark energy collapse ρ has shown in this figure.

$$\rho(t) = 3m_0\left(\frac{\rho_0}{3m_0}\right)^{-\frac{1}{w}} \tag{3.9}$$

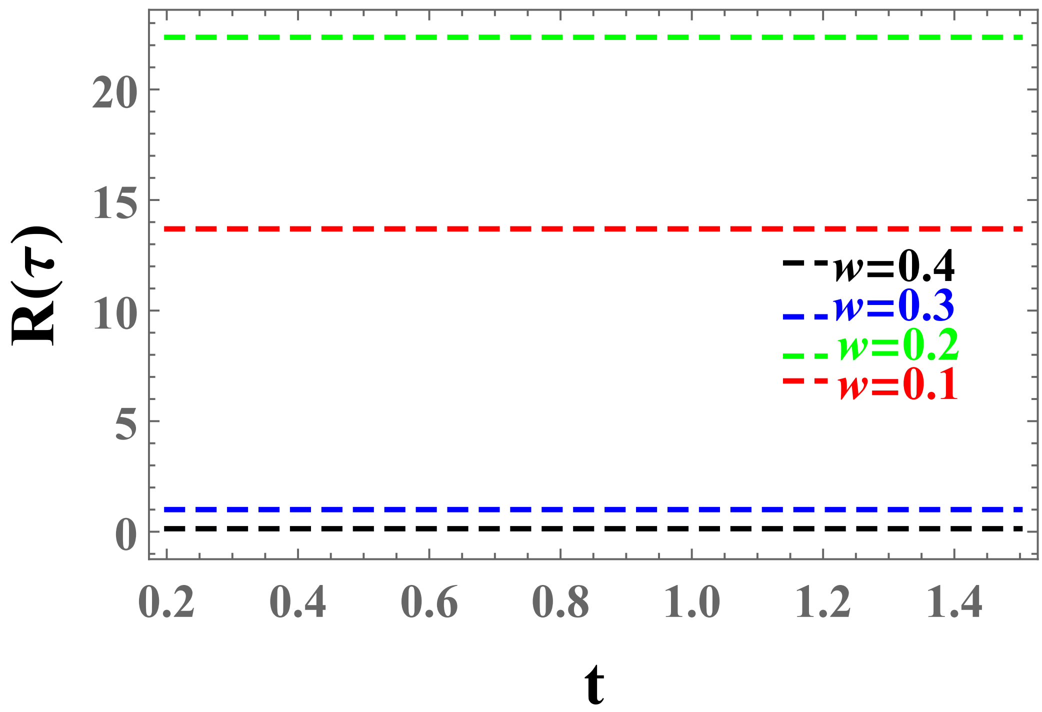

Figure 7. Evaluation $r(t)$ for $w > -1/3$ has given in this figure.

$$R(\tau) = r\left(\frac{\rho_0}{3m_0}\right)^{-\frac{1}{3w}} \tag{3.10}$$

4. Conclusion

In the structural formation of the Universe, gravitational collapse plays a vital role. In this study the gravitational collapse of a newly suggested 4D limit for Einstein-Gauss-Bonnet gravity, consisting of homogeneous and isotropic fluid was examined. While the gravity theory of Einstein-Gauss-Bonnet is the low energy limit of the hereotic superstring theory of gravity [4,5]. In this paper, we have investigate the features that the collapse still produces black holes for $w > -1/3$, even the situation of the dust cloud in which $w = 0$. Within Sec. III, we investigate the collapse phenomena of the fluid in two cases, one of the is dust cloud ρ_{DM}, and the second one is $p = w\rho$ the dark energy. In the first case, we have studied the effect of GB term on trapped surface as well as we demonstrated that the situation observed in the theory suggested in [6] did not actually vary substantially through the dynamic dust collapse structure in the gravity of Einstein. For the first case, a tempting collapse role is that the scale factor with zero velocity approaches the singularity, and causing the singularity "weaker" at the final stage of the collapse relative to relativistic context. The metric Eq. (1.18) is considered to stay regular as $r \to 0$ as the tends to the singularity takes place by de Sitter-like space[21]. It's indeed probably for this reason also that singularity would be weaker in collapse as well as with zero velocity, the collapsing cloud attain the singularity.

While the process of collapse in the context of dark energy is examined in second scenario, where evaluation of scale factors and other relevant physical quantities have been examined with the help of field equations and conservation of energy momentum tensor through which we are able to investigate the formation singularity and the black hole on the surface.

In addition, the principle of 4D-EGB supports new branches of collapsing solutions which do not provide for $\lambda = 0$ a relativistic limit and long distances. In future work, such ideas would be examined.Over the past few years, the interesting and modified theories of gravitational collapse have emerged which has led towards the attraction among the researchers for carrying out the studies in evaluating the Einstein Gauss Bonnet with respect to gravitational collapse. The authors have demonstrated the importance of investigating the gravitational collapse which would enable in understanding the Einstein Gauss Bonnet in depth. Thus, this becomes the gap in the research where the Gauss Bonnet theory would be further investigated with respect to gravitational collapse. While for the solution branch, which reproduces the nature of singularity as well as AH is qualitatively identical with the situation of Einstein's theory in GR at the limit with $\lambda = 0$ with asymptotically exterior flat vacuum, hence indicating that black holes might develop in four-dimensional Einstein Gauss-Bonnet theory in the same manner as it would do in general relativity.

It has been clearly obvious here that the use of inhomogeneities in the collapse of dust will change the trapped surfaces formation significantly, resulting in the formation of naked singularities [27]. It would also be fascinating to see whether those results stay true in the theory of 4D-EGB regarding to inhomogeneous dust collapse.

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