INFN, Sezione di Napoli

Introducing quantum mechanics in high schools: a proposal based on Heisenberg's "Umdeutung"

Introducing quantum mechanics in high schools: a proposal based on Heisenberg's "Umdeutung"

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Outline

- Introduction: motivations.
- The proposal: 1) the inspiring source: the 1925 seminal paper by W. Heisenberg; 2) playing with two-state systems.
- Preliminary results and conclusions: building an effective approach in teaching Quantum Mechanics.

Introduction: motivations

- **At high school as well as undergraduate level, Quantum Mechanics is usually introduced through an overview of the main crucial experiments and theoretical attempts which took place at the beginning of the 20-th century.**

- **QM implies major changes in understanding the world and the physical reality.**

- **Drawbacks (mainly at high school level): students lack advanced mathematical tools and show difficulties in accepting nondeterminism. The consequence is a fall back to classical reasoning and a misunderstanding of quantum concepts.**

- **Teaching quantum physics in high school is a challenging task: it requires to deal with abstract concepts without resorting to advanced mathematics. Further crucial issue: how to devise simple experiments to see QM at work in a classroom.**

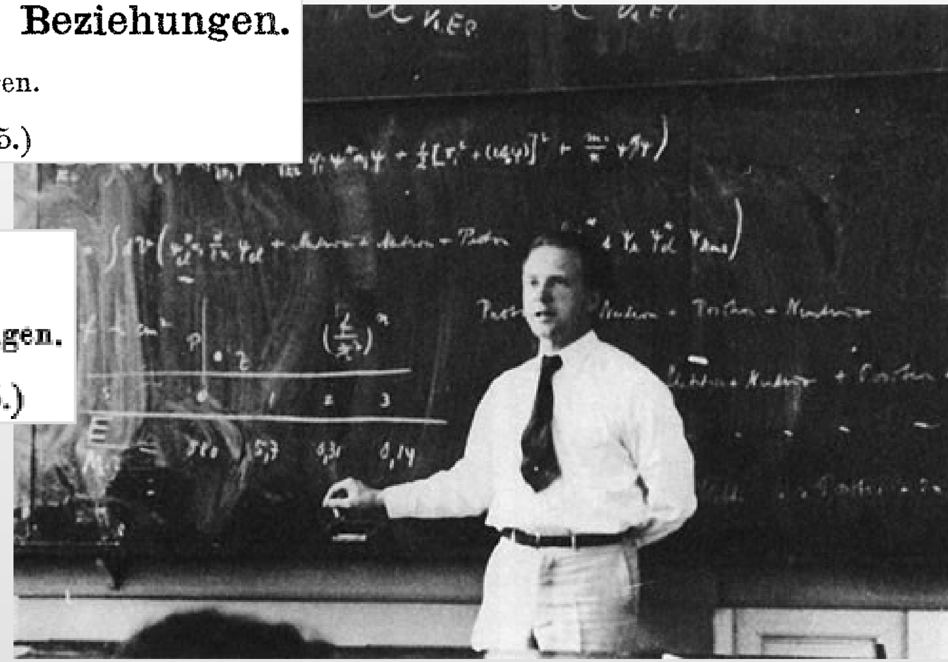
The proposal: step 1

History: the inspiring source

Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

(Eingegangen am 29. Juli 1925.)



Zur Quantenmechanik.

Von **M. Born** und **P. Jordan** in Göttingen.

(Eingegangen am 27. September 1925.)

Zur Quantenmechanik. II.

Von **M. Born**, **W. Heisenberg** und **P. Jordan** in Göttingen.

(Eingegangen am 16. November 1925.)

1. *W. Heisenberg's "Umdeutung" provides a simple calculational method to deal with quantum mechanical states and observables, based on the identification of the physical quantities of interest with transition frequencies and amplitudes.*
2. *Such frequencies and amplitudes form matrices.*

$$\boxed{x \to \{a(n,n-\alpha)e^{i\omega(n,n-\alpha)t}\}.}$$

Set of transition components

$$\boxed{x^2 \to \{b(n,n-\beta)e^{i\omega(n,n-\beta)t}\},}$$

$$\boxed{b(n,n-\beta) = \sum_\alpha a(n,n-\alpha)a(n-\alpha,n-\beta)}$$

Transition amplitude

Symbolic algebra of atomic processes

Newton's second law, with x replaced by the set of transition components

$$\boxed{\ddot{x} + f(x) = 0,}$$

$$h = 4\pi m \sum_{\alpha=0}^{\infty} \{|a(n+\alpha,n)|^2 \omega(n+\alpha,n) - |a(n,n-\alpha)|^2 \omega(n,n-\alpha)\}.$$

"Amplitude condition": it relates the amplitudes of different lines within an atomic spectrum

Born, Heisenberg and Jordan contribution

1. $\mathbf{q} = (q(nm)e^{2\pi i\nu(nm)t})$, $\mathbf{p} = (p(nm)e^{2\pi i\nu(nm)t})$,
2. $\nu(jk) + \nu(kl) + \nu(lj) = 0$,
3. $\dot{\mathbf{q}} = \partial \mathbf{H}/\partial \mathbf{p}$, $\dot{\mathbf{p}} = -\partial \mathbf{H}/\partial \mathbf{q}$,
4. $E_n = H(nn)$, and
5. $(\mathbf{pq} - \mathbf{qp})_{\text{diagonal}} = h/2\pi i$,

1. $(\mathbf{pq} - \mathbf{qp})_{\text{nondiagonal}} = 0$,
2. $\dot{\mathbf{g}} = (2\pi i/h)(\mathbf{Hg} - \mathbf{gH})$,
3. $\dot{\mathbf{H}} = 0$, and
4. $h\nu(nm) = H(nn) - H(mm)$.

After having sent Heisenberg's paper to the Zeitschrift für Physik for publication, I began to ponder about his symbolic multiplication, and was soon so involved in it…For I felt there was something fundamental behind it…And one morning, about 10 July 1925, I suddenly saw the light: Heisenberg's symbolic multiplication was nothing but the matrix calculus, well known to me since my student days from the lectures of Rosanes in Breslau.

"MATRIX MECHANICS"

A simple example: harmonic oscillator

PROBLEM: given a conservative force $F(x)$ that binds the electron in an atom, find the quantum mechanical properties (observables) ω_{nm} and A_{nm} associated with the transitions between stationary states. For a simple harmonic oscillator $F(x) = -kx$, so that the solution to the equation of motion is:

$$x(t) = a \cos \omega_0 t,$$

$$\omega_0 \equiv \sqrt{k/m}$$

Fundamental amplitude

General quantum condition

$$\oint m\dot{x}^2\, dt = nh$$

$$\Rightarrow \quad m a^2 \omega_0 \pi = nh.$$

$$\Rightarrow \quad a(n) = \sqrt{\frac{2\hbar n}{m\omega_0}}, \quad \omega(n) = \omega_0.$$

Amplitude and frequency in the state *n*

Let's substitute the quantized amplitude $a(n)$ into the classical energy function, and obtain the **quantum energy spectrum**:

$$E = m\omega_0^2 a^2/2, \quad \Rightarrow \quad E_n = n\hbar\omega_0$$

There exists a single Fourier harmonic term, so that the selection rule is $\Delta n = 1$ (only transitions between adjacent states are allowed).

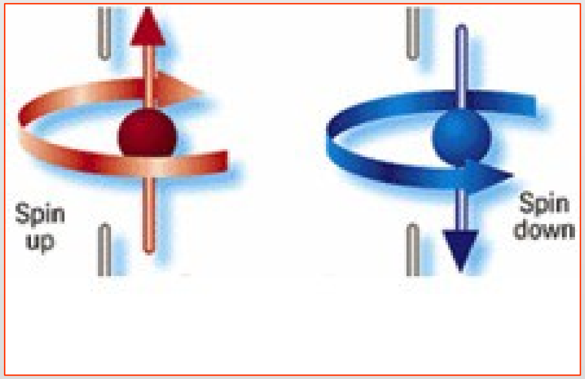
Finally, correspondence theorems allow us to compute the **radiation frequency** and **transition probability** from expressions given above:

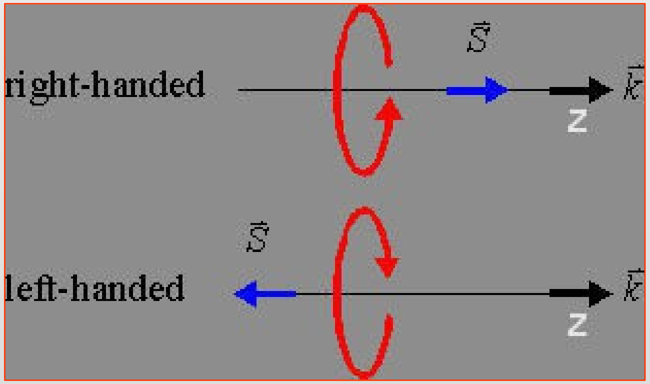
$$\omega_{n,n-1} = \omega_0, \quad A_{n,n-1} = \frac{e^2\omega_0^2}{6\pi\varepsilon_0 c^3 m} n.$$

Step 2: playing with two-state systems

SIMPLE FRAMEWORK: 2x2 matrix formulation of QM – quantum states are 2-vectors belonging to a finite vector space and observables are 2x2 matrices!

SIMPLE MODEL: quantum 2-state system!





Preliminary results and conclusions

1. *Teachers and students have the possibility to become familiar with quantum issues such as entanglement and non locality without an advanced mathematical background.*
2. *It is possible to fill the gap between high-school curricula and the actual scientific and technogical advances in physics (e.g. qubits, quantum computer, quantum teleportation)*
3. *The proposal is also suitable within pre-service and in-service training programs for physics teachers.*
4. *Teachers are much more interested in learning basic principles and practical teaching strategies then in deepening their knowledge of formalism.*
5. *A promising strategy in teacher education could be to shape teaching-learning proposals by relying on the historical path, which led to a concept, as well as on the social and philosophical contexts in which the concept itself developed. In this way significant changes in teachers' and students' conceptions regarding the Nature of Science are expected.*

THANK YOU !