T and C symmetry breaking in Algebraic Quantum Field Theory

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Mathematical theories of spinors

- 1. **Covariant spinors** (matrix columns or rows) -
 Élie Cartan.
- 2. **Algebraic spinors** - approach is based on theory of Clifford algebras. Matrix representation in a 2m dimensional
complex space in the form of square matrices $2^m \cdot 2^m$.
- 3. **Superalgebraic spinors** – extension of the theory of algebraic spinors and of axiomatic algebraic QFT. Theory of C*-algebras. Grassmann variables and derivatives with respect to them. CAR-algebra of second quantization of fermions (CAR – Canonical Anticommutation Relations)

Theory of superalgebraic spinors

1. M. Pavšič. A theory of quantized fields based on orthogonal and symplectic Clifford algebras. *Advances in Applied Clifford Algebras*, 2012, v.22, p.449-481.
2. V. Monakhov. Superalgebraic representation of Dirac matrices. *Theoretical and Mathematical Physics*. 2016. v. 186. p.70–82.
3. V. Monakhov. Dirac matrices as elements of superalgebraic matrix algebra. *Bulletin of the Russian Academy of Sciences: Physics*, 2016, v.80, p. 985–988.
4. V. Monakhov. Superalgebraic structure of Lorentz transformations. *J. of Physics: Conf. Series*, 2018, v.1051, 012023.
5. V.Monakhov. Generalization of Dirac conjugation in the superalgebraic theory of spino rs *Theoretical and Mathematical Physics*, 2019, v.200, p.1026-1042.
6. V. Monakhov. Vacuum and spacetime signature in the theory of superalgebraic spinors. *Universe*, 2019,v.5 (7), 162.
7. V. Monakhov. Spacetime and inner space of spinors in the theory of superalgebraic spinors. *Journal of Physics: Conference Series,* 2020, v.1557(1), 12031.
8. V. Monakhov. Generation of Electroweak Interaction by Analogs of Dirac Gamma Matrices Constructed from Operators of the Creation and Annihilation of Spinors. *Bulletin of the Russian Academy of Sciences: Physics*, 2020, Vol. 84, No. 10, pp. 1216–1220.

4-component superalgebraic spinors

$$\Psi = \int d^3 p \left(\psi^{\alpha}(p) \frac{\partial}{\partial \theta^{\alpha}(p)} + \psi^{\tau}(p) \theta^{\tau}(p) \right)$$

$$\theta^{a}(p)^{+} = \frac{\partial}{\partial \theta^{a}(p)}; \ \{ \frac{\partial}{\partial \theta^{k}(p)}, \theta^{l}(p') \} = \delta_{k}^{l} \, \delta(p - p')$$

$$\frac{\partial}{\partial \theta^{1}(p)} \cong \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \frac{\partial}{\partial \theta^{2}(p)} \cong \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \ \theta^{3}(p) \cong \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \ \theta^{4}(p) \cong \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\theta^{1}(p) \cong \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \quad \theta^{2}(p) \cong \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\frac{\partial}{\partial \theta^{3}(p)} \cong \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}, \ \frac{\partial}{\partial \theta^{4}(p)} \cong \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

Gamma-operators (analogs of matrices): two additional compared to Dirac's theory!

$$\hat{A} = [A, \bullet] \Rightarrow \hat{A}\Psi = [A, \Psi] = A\Psi - \Psi A$$

$$\hat{\gamma}^0 = \int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\theta^1(p) + \frac{\partial}{\partial\theta^2(p)}\theta^2(p) + \frac{\partial}{\partial\theta^3(p)}\theta^3(p) + \frac{\partial}{\partial\theta^4(p)}\theta^4(p), \bullet]$$

$$\hat{\gamma}^1 = \int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^4(p)} - \theta^4(p)\theta^1(p) + \frac{\partial}{\partial\theta^2(p)}\frac{\partial}{\partial\theta^3(p)} - \theta^3(p)\theta^2(p), \bullet]$$

$$\hat{\gamma}^2 = i\int \mathrm{d}^3 p \, [-\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^4(p)} - \theta^4(p)\theta^1(p) + \frac{\partial}{\partial\theta^2(p)}\frac{\partial}{\partial\theta^3(p)} + \theta^3(p)\theta^2(p), \bullet]$$

$$\hat{\gamma}^3 = \int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^3(p)} - \theta^3(p)\theta^1(p) - \frac{\partial}{\partial\theta^2(p)}\frac{\partial}{\partial\theta^4(p)} + \theta^4(p)\theta^2(p), \bullet]$$

$$\hat{\gamma}^4 = i\hat{\gamma}^5 = i\int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^3(p)} + \theta^3(p)\theta^1(p) + \frac{\partial}{\partial\theta^2(p)}\frac{\partial}{\partial\theta^4(p)} + \theta^4(p)\theta^2(p), \bullet]$$

$$\hat{\gamma}^6 = i\int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^2(p)} + \theta^2(p)\theta^1(p) - \frac{\partial}{\partial\theta^3(p)}\frac{\partial}{\partial\theta^4(p)} - \theta^4(p)\theta^3(p), \bullet]$$

$$\hat{\gamma}^7 = \int \mathrm{d}^3 p \, [\frac{\partial}{\partial\theta^1(p)}\frac{\partial}{\partial\theta^2(p)} - \theta^2(p)\theta^1(p) + \frac{\partial}{\partial\theta^3(p)}\frac{\partial}{\partial\theta^4(p)} - \theta^4(p)\theta^3(p), \bullet]$$

Operators of annihilation and creation of spinor. Operator of generalized Dirac conjugation

$$b_\alpha(p_i) = \exp(\hat{\gamma}^{0k}\varphi_k)\frac{\partial}{\partial\theta^\alpha(0)}|_{p=0\to p=p_i}$$

$$\overline{b_\alpha}(p_i) = \exp(\hat{\gamma}^{0k}\varphi_k)\theta^l(0)|_{p=0\to p=p_i}$$

$$\overline{\Psi} = (M\Psi)^+$$

$$\text{Signature} = (+---) \Rightarrow M = \hat{\gamma}^0$$

$$\overline{b_\alpha}(p) = (\hat{\gamma}^0 b_\alpha(p))^+$$

$$\overline{\Psi} = (\hat{\gamma}^0\Psi)^+ = (\bullet)^+\hat{\gamma}^0\Psi$$

Discretization of momentum space. Spinor vacuum

$$\{\frac{\partial}{\partial\theta^k(p_i)},\theta^l(p_j)\} = \delta_k^l \frac{1}{\Delta^3 p_i}\delta_j^i; \quad \delta(p_i - p_j) = \frac{1}{\Delta^3 p_i}\delta_j^i$$

$$\{\frac{\partial}{\partial\theta^k(p_i)},\frac{\partial}{\partial\theta^l(p_j)}\} = \{\theta^k(p_i),\theta^l(p_j)\} = 0$$

$$\Psi_V = \prod_i \Psi_V(p_i)$$

$$\Psi_V(0) = (\Delta^3 p|_{p=0})^4 \frac{\partial}{\partial\theta^1(0)}\theta^1(0)\frac{\partial}{\partial\theta^2(0)}\theta^2(0)\frac{\partial}{\partial\theta^3(0)}\theta^3(0)\frac{\partial}{\partial\theta^4(0)}\theta^4(0)$$

$$\Psi_V(p_i) = (\Delta^3 p_i)^4\, b_1(p_i)\bar{b}_1(p_i)b_2(p_i)\bar{b}_2(p_i)b_3(p_i)\bar{b}_3(p_i)b_4(p_i)\bar{b}_4(p_i)$$

Properties of the spinor vacuum

$$\Psi_V(p_i)^+ = \Psi_V(-p_i) \quad => \Psi_V{}^+ = \Psi_V$$

$$(\Psi_V)^2 = \Psi_V$$

$b_1(p_i)\Psi_V = 0$, annihilation operator

$\bar{b}_1(p_i)\Psi_V \neq 0$, creation operator

Ψ_V is primitive Hermitian idempotent.

Alternative spinor vacuum

$\hat{\gamma}^1, \hat{\gamma}^2, \hat{\gamma}^3, \hat{\gamma}^5, \hat{\gamma}^6, \hat{\gamma}^7$ – change Ψ_V to Ψ_{alt}

$\hat{\gamma}^0$ – keeps Ψ_V

$$\Psi_{altV}(0) = (\Delta^3 p\,|_{p=0})^4\, \theta^1(0)\frac{\partial}{\partial\theta^1(0)}\theta^2(0)\frac{\partial}{\partial\theta^2(0)}\theta^3(0)\frac{\partial}{\partial\theta^3(0)}\theta^4(0)\frac{\partial}{\partial\theta^4(0)}$$

$$\Psi_{altV}(p_i) = (\Delta^3 p_i)^4\, \bar{b}_1(p_i)b_1(p_i)\bar{b}_2(p_i)b_2(p_i)\bar{b}_3(p_i)b_3(p_i)\bar{b}_4(p_i)b_4(p_i)$$

$$\Psi_{altV} = \prod_i \Psi_{altV}(p_i)$$

$\bar{b}_k(p_i)$ – annihilation operator

$b_k(p_i)$ – creation operator

Clifford algebra: operators of reflection

Operator A transforms Clifford vector X as

$$X' = AXA^{-1} = (\lambda A)X(\lambda A)^{-1},$$

i.e. A is defined up to numerical factor λ

Operator A transforms spinor Ψ as

$$\Psi' = A\Psi,$$

$$(\Psi', \Psi') = (\Psi, \Psi) \Rightarrow \lambda = e^{i\varphi}$$

$$A = i\hat{\gamma}^0 \Rightarrow \hat{\gamma}^0{}' = \hat{\gamma}^0,\ \hat{\gamma}^k{}' = -\hat{\gamma}^k, k = 1,2,3,6,7,5 - \text{reflects } \hat{\gamma}^k$$

$$A = \hat{\gamma}^{ab} \Rightarrow \hat{\gamma}^a{}' = -\hat{\gamma}^a,\ \hat{\gamma}^b{}' = -\hat{\gamma}^b,\ \hat{\gamma}^c{}' = \hat{\gamma}^c, a \neq c \neq b$$

$- \text{reflects } \hat{\gamma}^a$ and $\hat{\gamma}^b$

CAR algebra: operators of reflection

Operator A transforms Clifford vector X as

$$X' = AXA^{-1} = (\lambda A)X(\lambda A)^{-1}, \text{ numerical factor } \lambda$$

Operator A transforms spinor Ψ as

$$\Psi' = A\Psi, \quad \text{numerical factor } \lambda = e^{i\varphi}.$$

New : Operator A transforms antispinor $\overline{\Psi}$ as

$$\Psi' = A\overline{\Psi}.$$

New : CAR algebra $\{\lambda \frac{\partial}{\partial\theta^k(p)}, \lambda\theta^l(p')\} = \delta_k^l \delta(p - p')$

$=> \lambda^2 = 1 => \lambda = \pm 1$

R-operators

$$d\hat{G} = [dG, \bullet]$$

$$(1 + d\hat{G})\Psi_1\Psi_2 \ldots \Psi_k = 1 + [dG, \Psi_1]\Psi_2 \ldots \Psi_k + \Psi_1[dG, \Psi_2] \ldots \Psi_k + \ldots =$$

$$= (e^{d\hat{G}}\Psi_1)(e^{d\hat{G}}\Psi_2) \ldots (e^{d\hat{G}}\Psi_k)$$

$$e^{\hat{G}}\Psi_1\Psi_2 \ldots \Psi_k = (e^{\hat{G}}\Psi_1)(e^{\hat{G}}\Psi_2) \ldots (e^{\hat{G}}\Psi_k)$$

$R_{\hat{G}} = e^{\hat{G}}$ – it is R - operator

Other R - operators:

Complex conjugation $(\bullet)^*$, transposition $(\bullet)^T$,

Hermitian conjugation $(\bullet)^+ = (\bullet)^T (\bullet)^*$

Operators Q and P

$$\hat{Q} = \mathrm{i}\hat{\gamma}^6\hat{\gamma}^7 =$$

$$\int d^3p\,[\frac{\partial}{\partial\theta^1(p)}\theta^1(p) + \frac{\partial}{\partial\theta^2(p)}\theta^2(p) - \frac{\partial}{\partial\theta^3(p)}\theta^3(p) - \frac{\partial}{\partial\theta^4(p)}\theta^4(p),\bullet]$$

– generator of rotations in the plane $\hat{\gamma}^6$, $\hat{\gamma}^7$.

Operator of charge in the theory of second quantization.

$$\hat{Q}\Psi = \Psi, \qquad \hat{Q}\overline{\Psi} = -\overline{\Psi},$$

$$\mathrm{e}^{\mathrm{i}\hat{Q}\varphi}\Psi = \mathrm{e}^{\mathrm{i}\varphi}\Psi, \quad \mathrm{e}^{\mathrm{i}\hat{Q}\varphi}\overline{\Psi} = \mathrm{e}^{-\mathrm{i}\varphi}\overline{\Psi}.$$

$P = R_{-x}R_{i\hat{\gamma}^0}R_{\hat{\gamma}^{67}} = R_{-x}R_{\hat{\gamma}^0\hat{Q}}$ – spatial reflection

Operator T of time reflection

$$T_1 = R_{-x^0} R_{\hat{\gamma}^1 \hat{\gamma}^3} (\bullet)^*, \qquad \Psi_V \to \Psi_V$$

"Rewinding the film", annihilation operator must become creation one, and vice versa

$$R = R_{\hat{\gamma}^{05}} R_{\hat{\gamma}^{26}} (\bullet)^T \ - \text{reverse}\ R\Psi_1\Psi_2 \ldots \Psi_k = \Psi_k \ldots \Psi_2 \Psi_1$$

$$R\Psi_\mathrm{V} = \Psi_\mathrm{alt}, R\Psi = \Psi, R\overline{\Psi} = \overline{\Psi}$$

$$T = RT_1 = R_{-x^0} R_{\hat{\gamma}^7} (\bullet)^+, \quad \Psi_V \to \Psi_{alt}$$

Charge conjugation C

CPT operator must be antiunitary. Operator P is unitary, T is antiunitary and reversing vacuum. Therefore, charge conjugation operator C must be unitary and reversing vacuum.

$$C_1 = R_{-q} R_{i\hat{\gamma}^{56}}, \qquad \Psi_V \to \Psi_V$$

$$C = RC_1 = R_{-q} R_{-i\hat{\gamma}^{02}} (\bullet)^T, \qquad \Psi_V \to \Psi_{alt}$$

Conclusion 1

$$P = R_{-x^k} R_{\hat{\gamma}^0 \hat{Q}}, \qquad \Psi_V \to \Psi_V,$$

$$T = R_{-x^0} R_{\hat{\gamma}^7} (\bullet)^+, \qquad \Psi_V \to \Psi_{alt}, \text{breaks symmetry}$$

$$C = R_{-q} R_{-i\hat{\gamma}^{02}} (\bullet)^T, \qquad \Psi_V \to \Psi_{alt}, \text{breaks symmetry}$$

$$CPT = R_{-q} R_{-x^\mu} J_+, \quad \Psi_V \to \Psi_V$$

$$J_+ = R_{\hat{\gamma}^{26}} (\bullet)^* - \text{operator of real structure}$$

(charge conjugation) in Krein spaces.

Conclusion 2

- Operators T and C are not consistent with vacuum of the Universe.
- They can only be approximate symmetry operators.
- The symmetry breaking is small when spinor is independent paeticle.
- Vacuum is multiparticle state.
- P, TC, CPT can be exact symmetry operators of spinors.