Monopole solutions in SU(2) Yang-Mills+ spinor field theory

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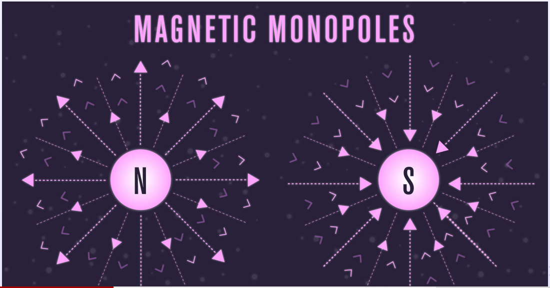
Introduction

In particle physics, magnetic monopole is a hypothetical elementary particle with only one magnetic pole. As yet there is no evidence for the existence of magnetic monopoles, but they are interesting theoretically. Modern interest in the concept stems from particle theories, notably the grand unified and superstring theories, which predict their existence. They find their applications to a wide variety of topics in theoretical physics, including various problems in the standard model , astrophysics, and cosmology.



Motivation of the Research

Firstly magnetic monopoles were proposed by P.A.M. Dirac In 1931 [1]. Later, magnetic monopoles in non-Abelian gauge theories were discovered by 't Hooft-Polyakov [2]-[3]. In theoretical physics, the 't Hooft–Polyakov monopole is similar to the Dirac monopole but without any singularities. Monopole solutions also were investigated in the non-Abelian Proca theory, which interacts with a scalar Higgs field and nonlinear spinor fields [4]. In this research, monopole solutions within SU(2) Yang-Mills theory containing a doublet of nonlinear spinor fields will be discussed. These solutions describe a magnetic monopole created by a spherical lump of nonlinear spinor fields.

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Lagrangian

The Lagrangian of non-Abelian SU(2) field A^a_μ interacting with nonlinear spinor field ψ has the form:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\hbar c \bar{\psi} \gamma^\mu D_\mu \psi - m_f c^2 \bar{\psi}\psi + \frac{\Lambda}{2} g\hbar c \left(\bar{\psi}\psi\right)^2 , \qquad (1)$$

here m_f - mass of the spinor field; $a, b, c = 1, 2, 3$ - color indices and $\mu, \nu = 0, 1, 2, 3$ - spacetime indices. The corresponding field equations have the form:

$$D_\nu F^{a\mu\nu} = \frac{g\hbar c}{2} \bar{\psi} \gamma^\mu \sigma^a \psi, \qquad (2)$$

$$i\hbar \gamma^\mu D_\mu \psi - m_f c \psi + \Lambda g \hbar \psi \left(\bar{\psi}\psi\right) = 0. \qquad (3)$$

Ansätze

The standard Ansätze for SU(2) monopole and spinor filed:

$$A_i^a = \frac{1}{g}\left[1 - f(r)\right]\begin{pmatrix} 0 & \sin\varphi & \sin\theta\cos\theta\cos\varphi \\ 0 & -\cos\varphi & \sin\theta\cos\theta\sin\varphi \\ 0 & 0 & -\sin^2\theta \end{pmatrix}, \quad i = r, \theta, \varphi, \tag{4}$$

$$A_t^a = 0, \tag{5}$$

$$\psi^T = \frac{e^{-i\frac{Et}{\hbar}}}{gr\sqrt{2}}\left\{\begin{pmatrix} 0 \\ -u \end{pmatrix}, \begin{pmatrix} u \\ 0 \end{pmatrix}, \begin{pmatrix} iv\sin\theta e^{-i\varphi} \\ -iv\cos\theta \end{pmatrix}, \begin{pmatrix} -iv\cos\theta \\ -iv\sin\theta e^{i\varphi} \end{pmatrix}\right\}, \tag{6}$$

where $E/\hbar$ is the spinor frequency and the functions u and v depend on the radial coordinate r only.

Research methods

Equations for the unknown functions f, u, and v can be obtained by substituting the expressions (4)-(6) into the field equations (2) and (3).

$$-f'' + \frac{f\left(f^2-1\right)}{x^2} + \tilde{g}^2\,\frac{\tilde{u}\tilde{v}}{x} = 0, \tag{7}$$

$$\tilde{v}' + \frac{f\,\tilde{v}}{x} = \tilde{u}\left(-\tilde{m}_f + \tilde{E} + \tilde{\Lambda}\frac{\tilde{u}^2-\tilde{v}^2}{x^2}\right), \tag{8}$$

$$\tilde{u}' - \frac{f\,\tilde{u}}{x} = \tilde{v}\left(-\tilde{m}_f - \tilde{E} + \tilde{\Lambda}\frac{\tilde{u}^2-\tilde{v}^2}{x^2}\right). \tag{9}$$

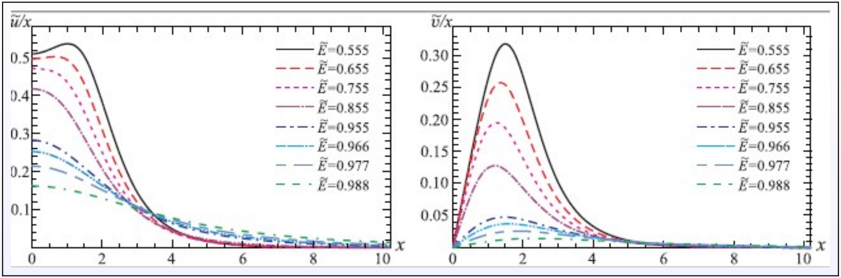
Here $x = r/r_0$(r_0 is a constant corresponding to the characteristic size of the system under consideration).

Boundary conditions

Boundary conditions near the origin $x = 0$ where solutions are sought in the form of the Taylor series:

$$f = 1 + \frac{f_2}{2} x^2 + \ldots, \quad \tilde{u} = \tilde{u}_1 x + \frac{\tilde{u}_3}{3!} x^3 + \ldots, \quad \tilde{v} = \frac{\tilde{v}_2}{2} x^2 + \frac{\tilde{v}_4}{4!} x^4 + \ldots, \quad (10)$$

where $\tilde{v}_2 = 2\tilde{u}_1 \left(\tilde{E} - \tilde{m}_f + \tilde{\Lambda} \tilde{u}_1^2 \right) /3$ and the expansion coefficients f_2 and $\tilde{u}_1$ are free parameters. Eqs. (7)-(9) are solved numerically as a nonlinear problem for the eigenvalues f_2 and $\tilde{u}_1$ and the eigenfunctions $\tilde{u}, \tilde{v}$, and f, whose typical behavior is shown in Fig. 1. The corresponding computed values of the system parameters are given in Table 1.



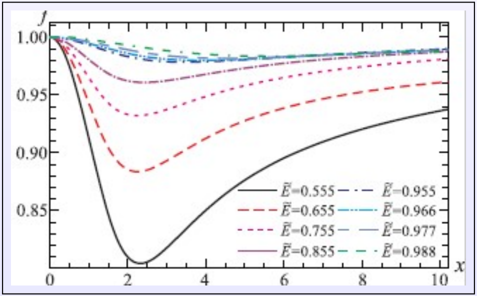


Figure: The functions $\tilde{u}(x)/x$, $\tilde{v}(x)/x$, and $f(x)$ for different values of the parameter $\tilde{E}$ ($\tilde{\Lambda} = 8$, $\tilde{m}_f = 1$, $\tilde{g} = 1$).

Eigenvalues $\tilde{u}_1$ and f_2 and the total energy $\tilde{W}_t$ for different values of the parameter $\tilde{E}$:

The ground state:

$\tilde{E}$	0.555	0.655	0.755	0.855	0.955	0.966	0.977	0.988
f_2	-0.21167	-0.1438	-0.092587	-0.0519	-0.016338	- 0.012473	- 0.00854	- 0.00466
$\tilde{u}_1$	0.510757	0.497238	0.47145	0.41848	0.2834	0.25342	0.2151	0.163
$\tilde{W}_t$	15.6339	11.7187	8.6202	6.4621	5.8124	6.04989	6.5242	7.3827

The first excited state, one node solutions:

$\tilde{E}$	0.755	0.855	0.955	0.977	0.988
f_2	-0.43377	-0.23663	-0.087295	-00526425	-0.03217
$\tilde{u}_1$	0.6005	0.57331012	0.494131	0.43494	0.37143
$\tilde{W}_t$	76.182	62.582	53.748	57.3803	65.957

Magnetic Yang-Mills field

The radial magnetic field:

$$H^a_r \sim \frac{2f_\propto}{gr^3}, \tag{11}$$

Nonzero tangential components of the magnetic field are:

$$H^a_\theta \sim \frac{1}{g} f', \quad H^b_\varphi \sim \frac{1}{g} f', \tag{12}$$

where $a = 1, 2, 3$ and $b = 1, 2$. The corresponding graphs for these components are shown in Fig. 4.

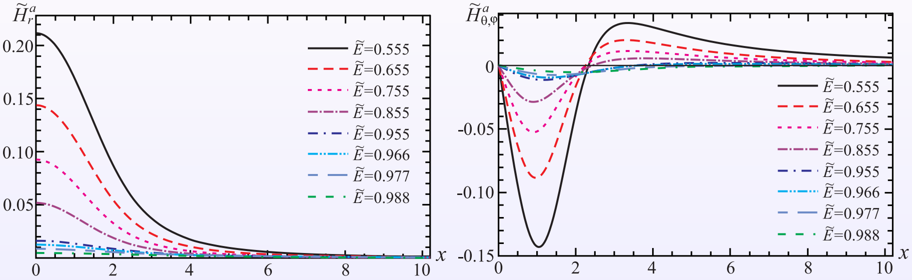


Figure: The distributions of the color magnetic fields for different values of the parameter $\tilde{E}$: the radial component $\tilde{H}_r^a \equiv g r_0^2 H_r^a$ is given by Eq. (11) and the tangential components $\tilde{H}_{\theta,\varphi}^a \equiv g r_0 H_{\theta,\varphi}^a$ – by Eq. (12).

Energy density

The total energy density of the monopole-plus-spinor-fields system under consideration is

$$\tilde{\epsilon} = \tilde{\epsilon}_m + \tilde{\epsilon}_s = \frac{1}{\tilde{g}^2}\left[\frac{f'^2}{x^2} + \frac{\left(f^2-1\right)^2}{2x^4}\right] + \left[\tilde{E}\,\frac{\tilde{u}^2+\tilde{v}^2}{x^2} + \frac{\tilde{\Lambda}}{2}\frac{\left(\tilde{u}^2-\tilde{v}^2\right)^2}{x^4}\right], \tag{13}$$

where $\tilde{\epsilon}_m \equiv \left(r_0^4/\hbar c\right)\epsilon_m$- energy density of the monopole,
$\tilde{\epsilon}_s \equiv \left(r_0^4/\hbar c\right)\epsilon_s$-energy density of spinor field.

Energy density

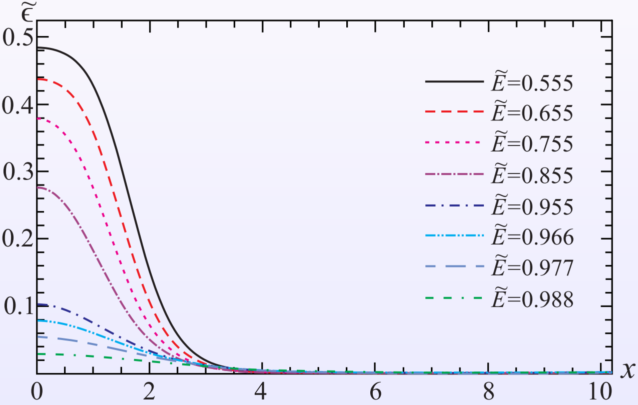


Figure: The energy density $\tilde{\epsilon}$ from Eq. (13) for different values of the parameter $\tilde{E}$.

Energy spectrum

Total energy of the system:

$$\tilde{W}_t = 4\pi \int\limits_0^\infty x^2 \tilde{\epsilon} dx = \left(\tilde{W}_t\right)_m + \left(\tilde{W}_t\right)_s . \qquad (14)$$

$\left(\tilde{W}_t\right)_m$ - energy of the monopole, $\left(\tilde{W}_t\right)_s$ - energy of the spinor field. The calculated data for $\tilde{W}_t$ given in Table 1.

Energy spectrum

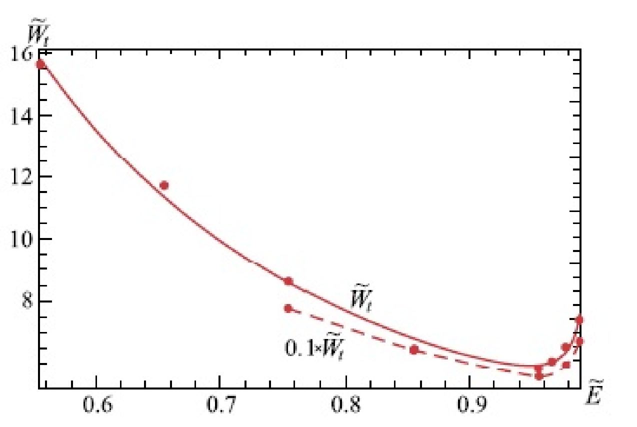


Figure: The spectrum of the total energy $\tilde{W}_t$ for the ground (solid line) and excited(dashed line) as functions of the parameter $\tilde{E}$.

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4. It is shown that the monopole solution obtained differs in principle from the 't Hooft-Polyakov monopole.

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Thanks for your attention !