Growth of matter perturbations in an interacting dark energy scenario emerging from metric-scalar-torsion couplings

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Abstract

We study the growth of linear matter density perturbations in a modified gravity approach of scalar field couplings with metric and torsion. In the equivalent scalar-tensor formulation, the matter fields in the Einstein frame interact as usual with an effective dark energy component, whose dynamics is presumably governed by a scalar field that sources a torsion mode. As a consequence, the matter density ceases to be self-conserved, thereby making an impact not only on the background cosmological evolution but also on the perturbative spectrum of the local inhomogeneities. In order to estimate the effect on the growth of the linear matter perturbations, with the least possible alteration of the standard parametric form of the growth factor, we resort to a suitable Taylor expansion of the corresponding exponent, known as the growth index, about the value of the cosmic scale factor at the present epoch. In particular, we obtain an appropriate fitting formula for the growth index in terms of the coupling function and the matter density parameter. While the overall parametric formulation of the growth factor is found to fit well with the latest redshift-space-distortion (RSD) and the observational Hubble (OH) data at low redshifts, the fitting formula enables us to constrain the growth index to well within the concordant cosmological limits, thus ensuring the viability of the formalism.

Keywords: Cosmological perturbations, dark energy theory, modified gravity, torsion, cosmology of theories beyond the SM.

1 Introduction

The effect of the evolving dark energy (DE) on the rate of the large-scale structure (LSS) formation has been a prime area of investigation in modern cosmology, particularly from the point of view of asserting the characteristics of the respective DE component [1–5]. While the observations grossly favour such a component to be a cosmological constant Λ [6–12], a stringent *fine-tuning* problem associated with the corresponding model, viz. ΛCDM (where CDM stands for cold dark matter), has prompted extensive explorations of a dynamically evolving DE from various perspectives. Moreover, certain observational results do provide some scope of a plausible dynamical DE evolution, albeit upto a significant degree of mildness. In this context, it is worth noting that however mild the DE dynamics may be, at the standard Friedmann-Robertson-Walker (FRW) background cosmological level, there may be substantial effects of such dynamics on the spectrum of the linear matter density perturbations. Hence, the analysis of the observational data on the evolution of such perturbations, or the LSS growth data, is crucial for constraining dynamical DE models of all sort.

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Apart from the commonly known dynamical DE models involving scalar fields (such as quintessence, kessence, and so on [13–21]), a considerable interest has developed in recent years on the cosmological scenarios emerging from scalar-tensor equivalent modified gravity (MG) theories [22–26] that stretch beyond the standard principles of General Relativity (GR). Such scenarios are particularly useful for providing plausible resolutions to the issue of cosmic *coincidence* which one usually encounters in scalar field DE models and in the concordant ΛCDM model. One resolution of course comes from the consideration of plausible contact interaction(s) between a scalar field induced DE component and the matter field(s) [1, 3, 27–47], which the scalar-tensor formulations naturally lead to, under conformal transformations [48–57]. A DE-matter (DEM) interaction makes the background matter density $\rho^{(m)}(z)$ drifting from its usual (*dust*-like) evolution with redshift z, thereby affecting the drag force on the matter perturbations. The evolution of the matter density contrast $\delta^{(m)}(z) := \delta\rho^{(m)}(z)/\rho^{(m)}(z)$ and the growth factor $f(z)$ of the matter perturbations are therefore not similar to those in the non-interacting models, in which the field perturbations decay out in the sub-horizon regime, while oscillating about a vanishing mean value. Actually, the decaying nature persists in the interacting scenarios as well, however with the oscillations about a value proportional to the amount of the interaction, measured by the strength of the scalar field and matter coupling. As such, the field perturbations contribute to the velocity divergences of the matter, affecting in turn the evolution of $\delta^{(m)}(z)$ [3, 58]. Strikingly enough, a DEM interaction can make the growth factor $f(z)$ acquiring a value > 1 at large z, which necessitates the modifications of the commonly known $f(z)$ parametrizations in the literature [59–64], such as the well-known parametrization $f(z) = \left[\Omega^{(m)}(z)\right]^{\gamma(z)}$, where $\Omega^{(m)}(z)$ is the matter density parameter and $\gamma(z)$ is the so-called *growth index* [60–72]. Our objective in this paper is to attempt such a modification and demonstrate its utilization in constraining a DEM scenario emerging from a typical scalar-tensor equivalent 'geometric' alternative of GR, viz. the metric-scalar-torsion (MST) cosmological theory, formulated recently by one of us (SS) [73–75], on the basis of certain considerations drawn from robust argumentations that have been prevailing for a long time [76–80].

MST essentially forms a class of modified (or 'alternative') gravity theories that contemplates on the appropriate gravitational coupling(s) with scalar field(s) in the Riemann-Cartan (U_4) space-time geometry, endowed with curvature as well as *torsion*. The latter being an inherent aspect of a general metric-compatible affine connection, is considered as the entity that naturally extends the geometric principles of GR, not only from a classical viewpoint, but also from the perspective a plausible low energy manifestation of a fundamental (quantum gravitational) theory[1]. Nevertheless, conventional U_4 theories (of Einstein-Cartan type) are faced with a stringent uniqueness problem while taking the minimal couplings with scalar fields into consideration [76–80]. Such couplings are simply not conducive to any unambiguous assertion of equivalent Lagrangians upon eliminating boundary terms in the usual manner. The obvious wayout is the consideration of explicit non-minimal (or, contact) couplings of the scalar field(s) with, most appropriately, the entire U_4 Lagrangian given by the U_4 curvature scalar $\widetilde{R}$ [73]. For any particular non-minimal coupling of a scalar field ϕ with $\widetilde{R}$, the resulting (MST) action turns out to be equivalent to the scalar-tensor action, as the trace mode of torsion, $\mathcal{T}_\mu$, gets sourced by the field ϕ, by virtue of the corresponding (auxiliary) equation of motion. On the other hand, torsion's axial (or, pseudo-trace) mode $\mathcal{A}_\mu$ can lead to an effective potential, for e.g. a mass term $m^2\phi^2$ (with $m =$ constant) in that scalar-tensor equivalent action, upon implementing a norm-fixing constraint ($\mathcal{A}_\mu\mathcal{A}^\mu =$ constant) as in the Einstein-aether theories [151–153], or incorporating a ϕ-coupled higher order term $(\mathcal{A}_\mu\mathcal{A}^\mu)^2$ [73]. Such a mass term is shown to play a crucial role in giving rise to a viable cosmological scenario marked by a ϕ-induced DE component with a weak enough dynamical evolution amounting to cosmological parametric estimations well within the corresponding observational error limits for ΛCDM. This also corroborates to the local gravitational bounds on the effective Brans-Dicke (BD) parameter $\mathfrak{w}$, which turns out to be linear in the inverse of the MST coupling parameter β [73].

Particularly intriguing is the MST cosmological scenario that emerges under a conformal transformation

[1]See the hefty literature on the vast course of development of the torsion gravity theories in various contexts, the physical implications and observable effects of torsion thus anticipated, as well as searched extensively over several decades [80–150].

from the Jordan frame to the Einstein frame, in which the effective DE component interacts with the cosmological matter (*a priori* in the form of *dust*). Nevertheless, the crude estimate of β (or of the parameter $s = 2\beta$, that appears in the exact solution of the Friedmann equations), obtained under the demand of a small deviation from the background ΛCDM evolution [73], requires a robust reconciliation at the perturbative level. On the other hand, the methodology adopted here can in principle apply to any scalar-tensor cosmological scenario, once we resort to the dynamics in the Einstein frame.

Now, the methodology of our analysis purports to fulfill our objective mentioned above. Specifically, we take the following course, and organize this paper accordingly: in section 2, we review the basic tenets of MST cosmology in the standard FRW framework, and in particular, the exact solution of the cosmological equations in the Einstein frame that describes a typical interacting DE evolution. Then in the initial part of section 3, we obtain the differential equations for $\delta^{(m)}(z)$ and $f(z)$, and get their evolution profiles by numerically solving those equations for certain fiducial settings of the parameters $s = 2\beta$ and $\Omega_0^{(m)} \equiv \Omega^{(m)}\big|_{z=0}$. Thereafter, in subsection 3.1, we resort to a suitable growth factor parametrization, demanding that an appropriate expansion of the growth index $\gamma(z)$ about the present epoch ($z = 0$) should adhere to the observational constraints on the growth history predictions at least upto $z \simeq 1$ or so. Next, in subsection 3.2, we attain the pre-requisites for the growth data fitting with the quantity $f\sigma^{(8)}(z)$, where $f(z)$ is as given by its chosen parametrization, and $\sigma^{(8)}(z)$ is the root-mean-square amplitude of matter perturbations within a sphere of radius $8\,\mathrm{Mph}^{-1}$. Finally, in section 4, we estimate the requisite parameters s, $\Omega_0^{(m)}$ and $\sigma_0^{(8)} \equiv \sigma^{(8)}\big|_{z=0}$, and hence constrain the model by fitting $f\sigma^{(8)}(z)$ with a refined sub-sample of the redshift-space-distorsion (RSD) data, and its combination with the observational Hubble data [176]. In section 5, we conclude with a summary of the work, and an account on some open issues.

Conventions and Notations: We use metric signature $(-,+,+,+)$ and natural units (with the speed of light $c = 1$), and denote the metric determinant by g, the Planck length parameter by $\kappa = \sqrt{8\pi G_N}$ (where G_N is the Newton's gravitational constant) and the values of parameters or functions at the present epoch by an affixed subscript '0'.

2 MST Cosmology in the Einstein frame, and the emergent DEM interacting scenario

As mentioned above, an intriguing scenario of an effective DEM interaction emerges from a typical scalar-tensor equivalent MG formulation, viz. the one involving a non-minimal metric-scalar-torsion (MST) coupling, in the Einstein frame [73]. Let us first review briefly the main aspects of such a formalism, and the emergent cosmological scenario in the standard FRW framework.

Torsion, by definition, is a third rank tensor $T^{\alpha}_{\ \mu\nu}$ which is anti-symmetric in two of its indies (μ and ν), because of being the resultant of the anti-symmetrization of a general asymmetric affine connection ($\widetilde{\Gamma}^{\alpha}_{\mu\nu} \neq \widetilde{\Gamma}^{\alpha}_{\nu\mu}$), that characterizes the four-dimensional Riemann-Cartan (or U_4) space-time geometry. The latter however demands the metric-compatibility, viz. the condition $\widetilde{\nabla}_{\alpha} g_{\mu\nu} = 0$, where $\widetilde{\nabla}_{\alpha}$ is the U_4 covariant derivative defined in terms of the corresponding connection $\widetilde{\Gamma}^{\alpha}_{\mu\nu}$. Such a condition leads to a lot of simplification in the expression for the U_4 curvature scalar equivalent, $\widetilde{R}$, which is usually considered as the free U_4 Lagrangian in analogy with the free gravitational Lagrangian in GR, viz. the Riemannian (or R_4) curvature scalar R. Specifically, $\widetilde{R}$ gets reduced to a form given by R, plus four torsion-dependent terms proportional to the norms of irreducible modes, viz. the trace vector $\mathcal{T}_{\mu} \equiv T^{\alpha}_{\ \mu\alpha}$, the pseudo-trace vector $\mathcal{A}^{\mu} := \epsilon^{\alpha\beta\gamma\mu} T_{\alpha\beta\gamma}$ and the (pseudo-)tracefree tensor $\mathcal{Q}^{\alpha}_{\ \mu\nu} := T^{\alpha}_{\ \mu\nu} + \frac{1}{3}(\delta^{\alpha}_{\mu}\mathcal{T}_{\nu} - \delta^{\alpha}_{\nu}\mathcal{T}_{\mu}) - \frac{1}{6}\epsilon^{\alpha}_{\ \mu\nu\sigma}\mathcal{A}^{\alpha}$, as well as the covariant divergence of $\mathcal{T}_{\mu}$ [80]. In absence of sources (or the generators of the so-called *canonical spin density*), all the torsion terms drop out, and hence the U_4 theory effectively reduces to GR. The situation remains the same for minimal couplings with scalar fields as well. However, such minimal couplings are themselves problematic, when it comes to assigning the effective Lagrangian uniquely upon eliminating the boundary terms [76–80].

An easy cure is to resort to distinct non-minimal couplings of a given scalar field ϕ, in general, with each of the constituent terms in $\widetilde{R}$ [80]. However, this implies the involvement of more than one arbitrary coupling parameters, which may affect the predictability and elegance of the theory. Hence, it is much reasonable to consider a non-minimal ϕ-coupling with the entire $\widetilde{R}$, so that there is a unique (MST) coupling parameter (to be denoted by β, say) [73].

Eliminating boundary terms, we obtain the auxiliary equation (or constraint) $\mathcal{T}_\mu = 3\phi^{-1}\partial_\mu\phi$, which implies that the (presumably primordial, and *a priori* massless) scalar field ϕ acts as a source of the trace mode of torsion. Considering further, a mass term $m^2\phi^2$ induced by torsion's axial mode $\mathcal{A}_\mu$, via one of the possible ways mentioned above (in the Introduction), we get the effective MST action[2] [73]:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{\beta\phi^2}{2}R - \frac{1-6\beta}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\,m^2\phi^2 + \mathcal{L}^{(m)}(g_{\mu\nu},\{\psi\})\right], \tag{1}$$

which is nothing but the scalar-tensor action in presence of minimally coupled matter fields ($\{\psi\}$) described by the Lagrangian $\mathcal{L}^{(m)}$, in the Jordan frame.

Under a conformal transformation $g_{\mu\nu} \to \widehat{g}_{\mu\nu} = (\phi/\phi_0)^2 g_{\mu\nu}$ and field redefinition $\varphi := \phi_0 \ln(\phi/\phi_0)$, with $\phi_0 = (\kappa\sqrt{\beta})^{-1}$ — the value of ϕ at the present epoch $t = t_0$, one obtains the Einstein frame MST action

$$\widehat{\mathcal{S}} = \int d^4x\sqrt{-\widehat{g}}\left[\frac{\widehat{R}}{2\kappa^2} - \frac{1}{2}\,\widehat{g}^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \Lambda e^{-2\varphi/\phi_0} + \widehat{\mathcal{L}}^{(m)}(\widehat{g}_{\mu\nu},\varphi,\{\psi\})\right], \tag{2}$$

where $\widehat{R}$ is the corresponding (Ricci) curvature scalar, and $\kappa = \sqrt{8\pi G_N}$ denotes the gravitational coupling factor[3]. The parameter $\Lambda = \frac{1}{2}m^2\phi_0^2$, which amounts to the effective field potential at $t = t_0$, and

$$\widehat{\mathcal{L}}^{(m)}(\widehat{g}_{\mu\nu},\varphi,\{\psi\}) = e^{-4\varphi/\phi_0}\,\mathcal{L}^{(m)}(g_{\mu\nu},\{\psi\}), \tag{3}$$

is the transformed matter Lagrangian, which depends on the field φ both explicitly as well as implicitly (since $g_{\mu\nu} = g_{\mu\nu}(\widehat{g}_{\mu\nu},\varphi)$). It is in fact this φ-dependence which leads to the DEM interaction in the standard cosmological setup, as we shall see below. Note also that, by definition, $\varphi\big|_{t=t_0} = 0$.

Dropping the hats ($\widehat{\ }$), we express the gravitational field equation and the individual matter and field (non-)conservation relations in the Einstein frame as follows:

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R = \kappa^2\left[T^{(m)}_{\mu\nu} + T^{(\varphi)}_{\mu\nu}\right], \tag{4}$$

$$\nabla_\alpha\left(g^{\alpha\nu}\,T^{(m)}_{\mu\nu}\right) = -\nabla_\alpha\left(g^{\alpha\nu}\,T^{(\varphi)}_{\mu\nu}\right) = -\frac{T^{(m)}\partial_\mu\varphi}{\phi_0}, \tag{5}$$

where $T^{(m)}_{\mu\nu}$ and $T^{(\varphi)}_{\mu\nu}$ are the respective energy-momentum tensors for matter and scalar field and $T^{(m)} \equiv g^{\mu\nu}\,T^{(m)}_{\mu\nu}$ denotes the trace of $T^{(m)}_{\mu\nu}$.

Considering the matter to be *a priori* in the form of a pressure-less fluid (viz. 'dust'), we have in the standard spatially flat FRW framework, $T^{\mu}_{\nu}{}^{(m)} = \mathrm{diag}\left[-\rho^{(m)},\,0,\,0,\,0\right]$, so that $-T^{(m)} = \rho^{(m)}$ is just the matter density, which is purely a function of the cosmic time t. Because of the interaction (5), the matter density ceases to have its usual dust-like evolution (i.e. $\rho^{(m)}(t) \not\propto a^{-3}(t)$, where $a(t)$ is the FRW scale factor). Nevertheless, the above Eqs. (4) and (5) are shown to be solvable in an exact analytic way, for the configuration

$$\varphi(t) = s\,\phi_0\,\ln[a(t)], \qquad \rho^{(m)}(t) \propto a^{-(3+s)}(t), \tag{6}$$

[2]Ignoring of course, any external source for the tensorial mode $\mathcal{Q}^{\alpha}_{\ \mu\nu}$, which therefore vanishes identically. This is particularly relevant to what we intend to study here, viz. a homogeneous and isotropic cosmological evolution in presence of torsion, which is plausible only when the latter's modes are severely constrained. One such constraint is $\mathcal{Q}^{\alpha}_{\ \mu\nu} = 0$ [97].

[3]This can be actually be retrieved from the relationship $\kappa = (\phi/\phi_0)\,\kappa_{\rm eff}(\phi)$, where $\kappa_{\rm eff}(\phi) = (\phi\sqrt{\beta})^{-1}$ is the effective (running) gravitational coupling one has in the Jordan frame.

provided one sets the constant parameter $s = 2\beta$ [73]. Consequently, the matter density parameter $\Omega^{(m)}(a)$ is expressed as

$$\Omega^{(m)}(a) := \frac{\rho^{(m)}(a)}{\rho(a)} = \frac{(3-s)\,\Omega_0^{(m)} a^{-(3-s)}}{3\Omega_0^{(m)}(a^{-(3-s)}-1)+(3-s)}\,, \tag{7}$$

where $\rho(a)$ is the total (or critical) density of the universe and $\Omega_0^{(m)}$ is the value of $\Omega^{(m)}$ at the present epoch ($t = t_0$, whence $a = 1$). Using the Friedmann and Raychaudhuri equations we can then express the Hubble parameter and total EoS parameter of the system, respectively, as

$$H(a) := \frac{\dot{a}}{a} = H_0 \left(1-\frac{s}{3}\right)^{-1/2} \left[\Omega_0^{(m)} a^{-(3+s)} + \left(1-\frac{s}{3}-\Omega_0^{(m)}\right) a^{-2s}\right]^{1/2}, \tag{8}$$

$$\mathsf{w}(a) := \frac{p(a)}{\rho(a)} = -1 + \Omega^{(m)}(a) + \frac{2s}{3}\,, \tag{9}$$

where $H_0 = H(a=1)$ is the Hubble constant, and $p(a)$ denotes the total pressure. Note that in the limit $s \to 0$, the above equations reduce to the corresponding ones for ΛCDM. Therefore one can directly estimate the extent to which the MST cosmological scenario can deviate from ΛCDM, by demanding that such a deviation should not breach the corresponding 68% parametric margins for ΛCDM. This would in turn provide an estimation of the parameter s, which has actually been carried out in [73], using the Planck 2015 and the WMAP 9 year results. The upper bound on s, thus obtained, is of the order of 10^{-2}. Nevertheless, a rather robust reconciliation is required from an independent analysis, for instance, using the RSD and $H(z)$ observations, which we endeavor to do in this paper.

3 Growth of matter density perturbations

In this section, we discuss the evolution of linear matter density perturbations in the deep sub-horizon regime for the aforementioned Einstein frame background MST cosmological scenario. The perturbations can be studied in the well-defined conformal Newtonian gauge. The metric in this gauge is given as [3]

$$ds^2 = e^{2N}[-(1-2\Phi)\mathcal{H}^{-2}dN^2 + (1+2\Phi)\delta_{ij}dx^i dx^j]\,, \tag{10}$$

where $N := \ln a(t)$ is the number of e-foldings, $\mathcal{H}$ is the conformal Hubble parameter and Φ is the Bardeen potential. Note that we have taken the same potential Φ in both temporal and spatial part of the metric under the assumption of a vanishing anisotropic stress.

The evolution of the matter density contrast $\delta^{(m)}$ depends on the divergence or convergence of the peculiar velocity $\vec{\boldsymbol{v}}^{(m)}$ via the perturbed continuity equation

$$\frac{d\delta^{(m)}}{dN} = -\theta^{(m)}\,, \quad \text{where} \quad \theta^{(m)} := \nabla\cdot\vec{\boldsymbol{v}}^{(m)}\,. \tag{11}$$

On the other hand, the *Euler* equation for matter perturbations is given by

$$\frac{d\theta^{(m)}}{dN} = -\left[\frac{\theta^{(m)}}{2}\left(1-3\,\mathsf{w}-\kappa\sqrt{2s}\,\frac{d\varphi}{dN}\right) + \widehat{\lambda}^{-2}\left(\Phi+\kappa\sqrt{\frac{s}{2}}\,\delta\varphi\right)\right], \tag{12}$$

where $\widehat{\lambda} \equiv \mathcal{H}/k$ (with k being the comoving wavenumber), and

$$\Phi \simeq \frac{3}{2}\widehat{\lambda}^2\Omega^{(m)}\delta^{(m)}\,, \quad \text{and} \quad <\delta\varphi> \simeq 3\widehat{\lambda}^2\sqrt{\frac{s}{2}}\,\Omega^{(m)}\delta^{(m)}\,, \tag{13}$$

considering only the mean value of $\delta\varphi$, as it shows a damped oscillatory behavior in the sub-horizon regime.

Φ and $\delta\varphi$ both being proportional to $\widehat{\lambda}^2$, become negligible in the deep sub-horizon limit ($\widehat{\lambda}^2 \ll 1$). However, their contribution may not be negligible in the evolution of $\theta^{(m)}(N)$, because of the $\widehat{\lambda}^{-2}$ pre-factor in the second term of Eq. (12). As a consequence, the DE perturbation $\delta\varphi$ which itself is negligible in the sub-horizon regime (despite being scale-dependent) may, by virtue of its coupling with matter, lead to a significant effect on the growth of matter density perturbations.

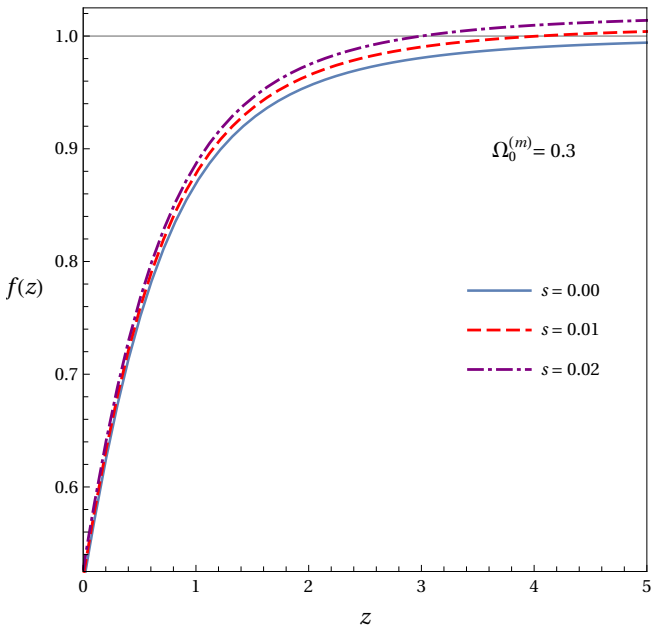
Eqs. (9), (11) and (12) yield the second-order differential equation

$$\frac{d\delta^{(m)}}{dN} + \left[2(1-s) - \frac{3\Omega^{(m)}}{2}\right]\frac{d\delta^{(m)}}{dN} = \frac{3(1+s)}{2}\,\Omega^{(m)}\delta^{(m)}\,, \tag{14}$$

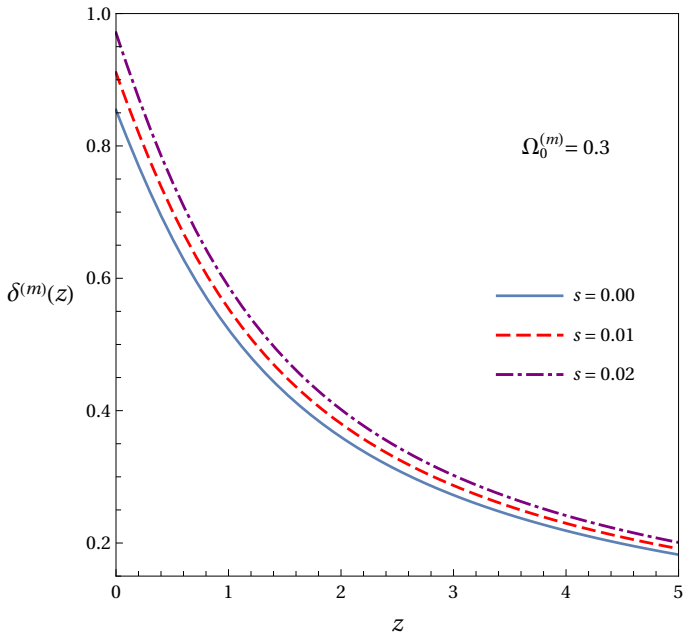
which can be reduced to the following first-order differential equation:

$$\frac{df}{dN} + f^2 + \left[2(1-s) - \frac{3\Omega^{(m)}}{2}\right] f = \frac{3(1+s)}{2}\,\Omega^{(m)}\,, \tag{15}$$

by defining the so-called growth factor $f(N) := d[\ln \delta^{(m)}]/dN$ [154–158]. Due to the pre-factor $(1+s)$ in the r.h.s. of Eq. (15), the function $f(N)$ can cross the unity barrier at high redshifts (whence $\Omega^{(m)} \to 1$). This is illustrated in Fig. (1a), where we have plotted $f(z)$ for a fixed $\Omega_0^{(m)} = 0.3$ and certain fiducial values of s, including $s = 0$ (the ΛCDM case). Fig. (1b), on the other hand, depicts the evolution of $\delta^{(m)}(z)$, which tends to increase with s for a fixed $\Omega_0^{(m)} = 0.3$.



(a) Growth factor evolution for fiducial $\Omega_0^{(m)}$ and s.



(b) Density contrast evolution for fiducial $\Omega_0^{(m)}$ and s.

Figure 1: Functional variations of the growth factor and the matter density contrast, $f(z)$ and $\delta^{(m)}(z)$ respectively, in the redshift range $z \in [0, 5]$, for certain fiducial parametric settings, viz. $\Omega_0^{(m)} = 0.3$ (fixed) and $s = 0$, 0.01, and 0.02.

3.1 Growth factor parametrization

As mentioned earlier, following the well-known prescription of [59, 60] we may consider parametrizing the growth factor $f(z)$ as $[\Omega^{(m)}(z)]^{\gamma(z)}$. However, such a parametrization does not explain the crossing of $f(z)$ from < 1 to > 1 at large-redshifts, as illustrated in Fig. (1a). More precisely, this parametrization $f(z)$ always

restricted within the range $[0,1]$ at all redshifts which in our case is not true. So to alleviate this limitation, we propose the ansatz:

$$f(z) = (1+s)\left[\Omega^{(m)}(z)\right]^{\gamma(z)}\,, \tag{16}$$

which evidently implies $f(z)$ approaching $1+s$ at large redshifts (whence $\Omega^{(m)} \to 1$). Now, to determine the growth rate of matter perturbations from Eq. (16), it is necessary to find a suitable functional form of $\gamma(z)$. In particular, choosing to express the growth index as a function of the scale factor a, we in this paper resort to the following truncated form of its Taylor expansion about $a = 1$ (which corresponds to the present epoch):

$$\gamma(a) = \gamma_0 + \gamma_1\,(1-a)\,, \qquad \text{with} \;\; \gamma_0, \gamma_1 := \text{constants}\,, \tag{17}$$

as in [62,70]. Note that this parametrization is valid atleast upto a redshift $z \simeq 1$ and is therefore suitable for the analysis using the RSD observational dataset [159–161], as most of the data points in that set lie within $z = 1$. In fact, it is rather convenient for us to re-write Eq. (17) as

$$\gamma(N) = \gamma_0 + \gamma_1(1-e^N) \quad \text{with} \quad \gamma_0 = \gamma(N)\big|_{N=0}\,, \quad \gamma_1 = \frac{d\gamma(N)}{dN}\Big|_{N=0}\,, \tag{18}$$

where

$$\gamma_0 = \gamma(N)\big|_{N=0} = \frac{1}{\ln\Omega_0^{(m)}}\ln\left(\frac{f_0}{1+s}\right) \qquad [f_0 = f|_{N=0}]\,, \tag{19}$$

$$\gamma_1 = \frac{d\gamma(N)}{dN}\Big|_{N=0} = \frac{1}{\ln\Omega_0^{(m)}}\Big[\gamma_0(s-3+3\Omega_0^{(m)}) + (1+s)(\Omega_0^{(m)})^{\gamma_0} + 2(1-s) - \frac{3}{2}\Big(\Omega_0^{(m)} + (\Omega_0^{(m)})^{1-\gamma_0}\Big)\Big]\,, \tag{20}$$

by Eqs. (15) and (16).

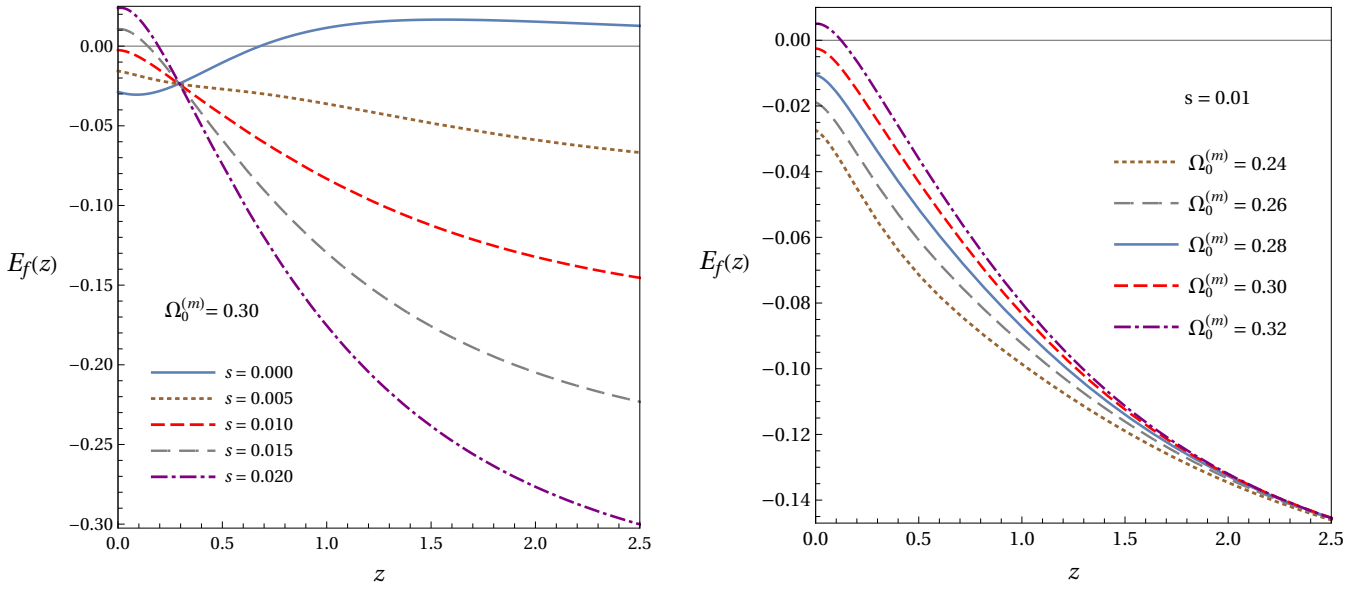
For the ΛCDM case ($s = 0$), assuming $\Omega_0^{(m)} = 0.3$, one gets $\gamma_0 \simeq 0.555$ and $\gamma_1 \simeq -0.016$. Moreover, the signature of γ_1 can discriminate between various DE models and modified gravity theories. For instance, the minimal level Dvali-Gabadadze-Porrati (DGP) model predicts ($0.035 < \gamma_1 < 0.042$) which is in sharp contrast with the GR predictions [62].

3.2 Numerical fitting of growth index

Let us now focus on determining the parametric set $p(\theta) = \{s, \Omega_0^{(m)}, \sigma_0^{(8)}, \gamma_0, \gamma_1\}$. While the form of the parameter γ_1 is already obtained in terms of s, $\Omega_0^{(m)}$ and γ_0, we require to assert the form of γ_0 in the first place. However, as we see from Eq. (19), γ_0 depends on s and $\Omega_0^{(m)}$ as well. Hence we resort to solving numerically the differential equation (15), by taking $s \in [0, 0.1]$ and $\Omega_0^{(m)} \in [0.2, 0.4]$ (which are of course fairly wide range of values), and for a step-size of 0.01. Using Eq. (19) thereafter, we obtain the following fit:

$$\gamma_0 \simeq \frac{0.547}{[\Omega_0^{(m)}]^{0.012}} - 1.118\,s\,\Omega_0^{(m)}\,. \tag{21}$$

In order to verify the validity of this fitting, let us take the $\Omega_0^{(m)} = 0.3$, say, and the limit $s \to 0$. Eq. (21) then gives $\gamma_0 \simeq 0.555$ which is precisely what we had estimated theoretically, for the ΛCDM case, in the last subsection, by using Eqs. (15) and (19). The goodness of the fit is illustrated in Figs. (2a) and (2b), in which we have plotted the fractional error in the fitting, viz. $E_f(z) (= [f_F(z) - f(z)]/f(z)$ with $z \in [0, 2.5]$, for a fixed $\Omega_0^{(m)} = 0.3$ and a range of fiducial values of s, and for a fixed $s = 0.01$ and a range of fiducial values of $\Omega_0^{(m)}$, respectively. In both the cases, the error turns out to be $\simeq 0.2\%$ at $z \simeq 1$, indicating a fair amount of the accuracy of the fit.



(a) Growth factor fitting error for fixed $\Omega_0^{(m)}$ and variable s. (b) Growth factor fitting error for fixed s and variable $\Omega_0^{(m)}$.

Figure 2: Functional variations of the growth factor fitting error, $E_f(z)$, in the redshift range $z \in [0, 2.5]$, for certain fiducial parametric settings.

4 Parametric estimations from RSD and Hubble observations

After formulating γ_0 and γ_1 in terms of s and $\Omega_0^{(m)}$, we are left with only three parameters s, $\Omega_0^{(m)}$ and $\sigma_0^{(8)}$ in hand. So in order to estimate them from the observations use the $f\sigma^{(8)}(z)$ observations from various galaxy data surveys [159–169], we will now proceed to perform the statistical analysis, in particular MCMC simulation to estimate our model parameters. Theoretically, $(f\sigma^{(8)})_{th}(z)$ can be written as [170–175]

$$(f\sigma^{(8)})_{th}(z) = f(z)\sigma_0^{(8)}\frac{\delta^{(m)}(z)}{\delta_0^{(m)}} \ , \quad \text{where} \quad \sigma_0^{(8)} = \sigma^{(8)}|_{z=z_0} \ . \tag{22}$$

which can be explicitly written as

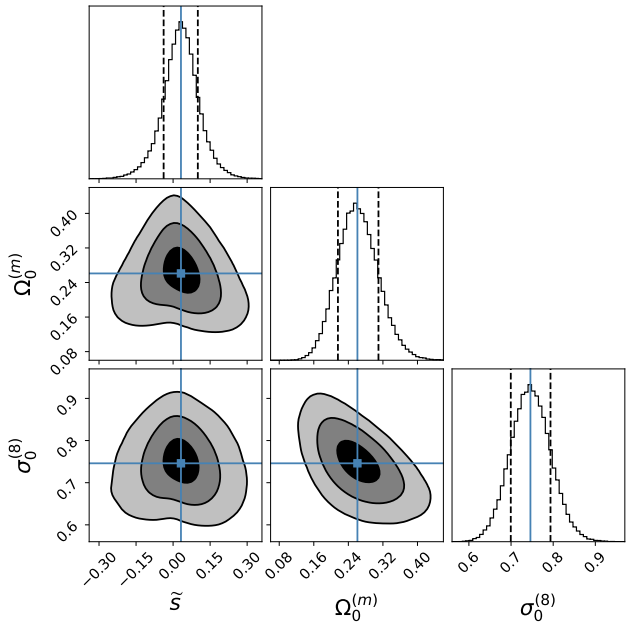
$$(f\sigma^{(8)})_{th}(N) = \sigma_0^{(8)}(1+s)(\Omega^{(m)})^{\gamma(N)} \exp\left[(1+s)\int_0^N (\Omega^{(m)})^{\gamma(N)} dN\right] \ , \tag{23}$$

where we have used Eq. (16). Since our parameter s is presumably positive definite and small, it is convenient for us to write $s = |\widetilde{s}|$, where $\widetilde{s}$ can take both positive and negative values.

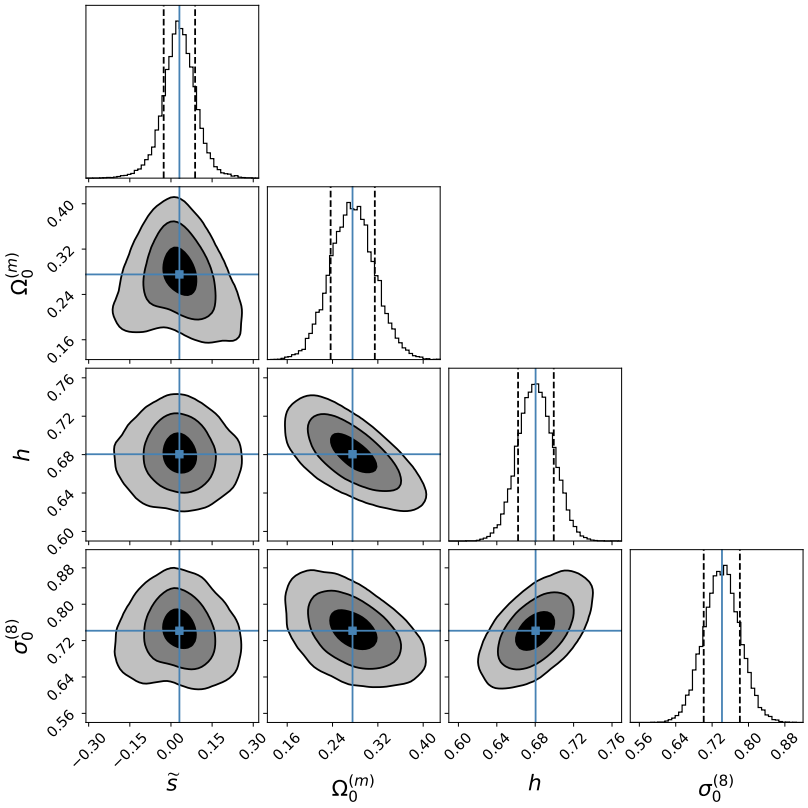
In order to perform the standard χ^2 minimization, we use the growth data observations: $A_{obs} \equiv (f\sigma^{(8)})_{obs}$ along with the theoretical predicted values: $A_{th} \equiv (f\sigma^{(8)})_{th}$ in the standard definition of the χ^2 function

$$\chi^2 := V^m C_{mn}^{-1} V^n, \tag{24}$$

where $V := A_{obs} - A_{th}$ and C_{mn}^{-1} is the inverse of the covariance matrix between three WiggleZ data points [175]. As we have already shown in fig. (2) that the parametric form (18) tends to diverge in case of interacting DE from its numerical solution (16) at high redshifts, we therefore restrict ourselves for the observations upto $z = 1$ for the datasts: GOLD-2017 [172] , and $H(z)$ data set [176]. Also, we set the range of priors as follows: (i) $-1 \leq \widetilde{s} \leq 1$, (ii) $0.1 \leq \Omega_0^{(m)} \leq 0.6$, (iii) $0.5 \leq \sigma_0^{(8)} \leq 1.2$ and (iv) $0.4 \leq h \leq 0.9$, where $h := H_0/[100\, Km\, s^{-1}\, Mpc^{-1}]$. The obtained contour plots between parameters upto 3σ level are shown in Figs. 3a and 3b.



(a) 2-D posterior distribution for Gold data.



(b) 2-D posterior distribution for GOLD+$H(z)$ data.

Figure 3: The 1σ-3σ contour levels for Gold dataset (left), and its combination with the Hubble dataset (right). The solid blue line denotes the best-fit and dashed lines correspond to the 1σ level.

Observational datasets	Parametric estimations (best fit & 68% limits)				χ^2/dof
	$\Omega_0^{(m)}$	$\sigma_0^{(8)}$	h	$\widetilde{s}$	
1. GOLD	$0.2610^{+0.0487}_{-0.0451}$	$0.7460^{+0.0476}_{-0.0466}$	-	$0.0319^{+0.0683}_{-0.0702}$	0.8390
2. GOLD+$H(z)$	$0.2753^{+0.0393}_{-0.0387}$	$0.7417^{+0.0393}_{-0.0402}$	$0.6804^{+0.0189}_{-0.0183}$	$0.0308^{+0.0573}_{-0.0573}$	0.6125

Table 1: Best fit values with 1σ confidence limits of parameters $\Omega_0^{(m)}$, $\sigma_0^{(8)}$, h and $\widetilde{s}$ together with their corresponding χ^2/dof for GOLD and GOLD $+H(z)$ dataset.

The estimations are shown in table (1) in which one can see that the best-fit of $\widetilde{s}$ for both sets of data (GOLD and GOLD+$H(z)$) is insignificant (as expected, since observations mostly prefer the ΛCDM model), but even then within 1σ limits its domain can reach upto significantly large value i.e. $\mathcal{O}(10^{-2})$ which shows a reasonable large deviation from the ΛCDM model. This indicates from the low-redshift data we can still observe a convincing amount of DEM interaction even at the 1σ level.

5 Conclusion

We have formulated the growth of linear matter density perturbations in a parametric form for a DE model which stems out from a modified gravity approach consists of metric and torsion as two basic entities of the space-time geometry. In the formalism, we have briefly demonstrated that a non-minimal coupling of metric and torsion with scalar field can give rise to a scalar-tensor action of DE in the Jordan frame which upon conformal transformation to the Einstein frame naturally makes scalar field non-minimally coupled with the matter sector. Due to this coupling, matter and scalar field exchange their energies between each other which ceases their individual energy densities to be self-conserved. The latter, thus, has direct influence on the underline matter density contrast and its evolution, which we have explored in this work.

We have demonstrated that in the perturbed FRW space-time, the scalar field and matter coupling enhances the growth of matter density perturbations in the sub-horizon regime, allowing it to cross the upper barrier of unity at large redshifts. Since this effect is unique in the interacting DEM scenarios it requires a slight modifications in the standard parametric ansatz of growth factor. With suitable modification we propose a slightly different growth factor ansatz to make the parametric formulation compatible with the theoretical predictions. Also, in view of the time evolving growth index, which is even encountered for the ΛCDM model, we have chosen an appropriate functional form i.e. first order Taylor expansion about present-day value of the scale factor $a(t)$. This simple but well defining form of the growth index indeed illustrates the parametric formulation of growth factor close to its actual evolution atleast upto $z \simeq 1$. Since the present-day value of growth index itself depends on the background model parameters, therefore in order to choose its explicit function form we have numerically obtained its fitting formula in terms of coupling as well as energy density parameter which we have shown to be a well approximation for a wide range of parameters.

As to the parametric estimations, we have constrained parameters $\widetilde{s}$, $\Omega_0^{(m)}$ and $\sigma_0^{(8)}$ by using the RSD as well as its combination with the Hubble data. We have found that for the GOLD datset the $\widetilde{s}$ and hence s parameter can show mildly large deviation from the ΛCDM model upto 1σ, which is comparatively smaller for the combined dataset, as expected. The consistency in our estimations with the theoretical predictions confirms the validity of our fitting function. However, to explain growth history for redshifts > 1, the above parametrization still requires further modifications to deal with various DE models as well as modified gravity theories, which we will shall endeavor to report in near future.

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