# Models of Gauss-Bonnet gravity leading to cosmological attractors predictions

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- The inflation was supposed to solve problems related with the hot big-bang model[1]

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A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. D **23**, 347 (1981);
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V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuation and Nonsingular Universe. (In Russian)," JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)];

- The $R^2$ inflationary predictions [2] in the leading approximation in terms of inverse e-folding numbers $1/N$ for spectral index $n_s$ and tensor-to-scalar ratio $r$:

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12}{N^2} \tag{1}$$

are in the best agreement with Planck 2018 data [3]

[2] A. A. Starobinsky, "Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations, Phys. Lett. **B117** 175 (1982).
A. Starobinsky, "The Perturbation Spectrum Evolving from a Nonsingular Initially de Sitter Cosmology and the Microwave Background Anisotropy," Sov. Astron. Lett. **9**, 302 (1983).

[3] Y. Akrami *et al.* [Planck], "Planck 2018 results. X. Constraints on inflation," [arXiv:1807.06211 [astro-ph.CO]].

- There exist two variant for interpretation of relation between time derivative and e-folding number derivative:
  1. $\frac{d}{dt} = H\frac{d}{dN_e}$ and
  2. $\frac{d}{dt} = -H\frac{d}{dN}$.

  In the case of the first type formulation, the inflation interval in the e-folding formulation is $-65 < N_e < 0$.
  In the case of the second type formulation, inflation interval in e-folding formulation $0 < N < 65$.
  The second formation was applied in cosmological attractor approximation [4] and we follow to the second formulation with $N = -\ln\left(\frac{a}{a_{end}}\right)$.

[4] M. Galante, R. Kallosh, A. Linde and D. Roest, "Unity of Cosmological Inflation Attractors," Phys. Rev. Lett. **114** (2015) no.14, 141302 [arXiv:1412.3797 [hep-th]]. R. Kallosh and A. Linde, "Universality Class in Conformal Inflation," JCAP **1307**, 002 (2013) [arXiv:1306.5220 [hep-th]].

- The $\alpha$ – attractors model generalizes the prediction of $R^2$ Starobinsky inflation.
- The cosmological attractor models predict the same values of observable parameters $n_{\mathrm{s}}$ and $r$ in the leading $1/N$ approximation:

$$n_s \simeq 1 - \frac{2}{N + N_0}, \quad r \simeq \frac{12 C_\alpha}{(N + N_0)^2}, \tag{2}$$

where $C_\alpha$ and $N_0 \ll 60$ are constants.

- here we have additional freedom in choice of constant $C_\alpha$

Models of Einstein-Gauss-Bonnet inflation are actively studied [5]

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S. Koh, B.H. Lee and G. Tumurtushaa, Phys. Rev. D **95**, 123509 (2017) [arXiv:1610.04360];
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Z. Yi, Y. Gong, and M. Sabir, Phys. Rev. D **98**, 083521 (2018)[arXiv:1804.09116];
N. Rashidi and K. Nozari, Astrophys. J. **890**, 58 (2020) [arXiv:2001.07012];
S. Odintsov and V. Oikonomou, Phys. Lett. B **805**, 135437 (2020) [arXiv:2004.00479];
E. O. Pozdeeva, M. R. Gangopadhyay, M. Sami, A. V. Toporensky and S. Y. Vernov, Phys. Rev. D **102** (2020) no.4, 043525,[arXiv:2006.08027 [gr-qc]]

- We construct a gravity model with the Gauss-Bonnet term multiplied to a function of a scalar field which allows to reconstruct expressions for spectral index and tensor-to-scalar ratio from cosmological attractor models in the slow-roll regime.This model includes several constants with variable values. Therefore, we construct a family of models with different values of the constants. We consider the scalar power spectral amplitude and estimate possible values of model parameters using modern observational data .

# Plan

1. We reformulate the problem of the slow-roll regime in Einstein-Gauss-Bonnet gravity in terms of e-folding numbers
2. We apply our reformulation to construct model with variable values of parameters which leads to the cosmological attractor approximation for inflationary parameters.
3. To satisfy observational data we introduce restriction to model parameters.
4. In Conclusions we summaries our results.

# Einstein-Gauss-Bonnet gravity

- We consider the model with the Gauss-Bonnet term multiplied to a function of the scalar field $\phi$:

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2}-\frac{\partial^\nu\phi\partial_\nu\phi}{2}-V(\phi)-\frac{\xi(\phi)}{2}\mathcal{G}\right], \qquad (3)$$

where $\mathcal{G}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$.

- Application of the variation principle leads to the following system of equations in spatially flat FLRW metric with $ds^2=-dt^2+a^2(t)(dx^2+dy^2+dz^2)$:

$$6H^2=\dot{\phi}^2+2V+24\dot{\xi}H^3, \qquad (4)$$

$$2\dot{H}=\,-\dot{\phi}^2+4\ddot{\xi}H^2+4\dot{\xi}H\left(2\dot{H}-H^2\right), \qquad (5)$$

$$\ddot{\phi}+3H\dot{\phi}+V_{,\phi}+12\xi_{,\phi}H^2\left(\dot{H}+H^2\right)=0, \qquad (6)$$

where $H=\dot{a}/a$, the dot means the derivative of time: $\dot{A}=dA/dt$ .

- We consider the model with the Gauss-Bonnet term multiplied to a function of the scalar field in FLRW metric in the slow-roll regime [6]:

$$\dot{\phi}^2 \ll V, \quad |\ddot{\phi}| \ll 3H|\dot{\phi}|, \quad 4|\dot{\xi}|H \ll 1, \quad |\ddot{\xi}| \ll |\dot{\xi}|H, \tag{7}$$

in which of the equations of motion are:

$$H^2 \simeq \frac{V}{3}, \; \dot{H} \simeq -\frac{\dot{\phi}^2}{2} - 2\dot{\xi}H^3, \; \dot{\phi} \simeq -\frac{V_{,\phi} + 12\xi_{,\phi}H^4}{3H}. \tag{8}$$

- The slow-roll parameters are:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_{i+1} = \frac{d\ln|\epsilon_i|}{d\ln a}, \quad i \geq 1, \tag{9}$$

$$\delta_1 = 4\dot{\xi}H, \quad \delta_{i+1} = \frac{d\ln|\delta_i|}{d\ln a}, \quad i \geq 1. \tag{10}$$

[6] Z. Guo and D. J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897 [hep-th]].

- To get cosmological attractor generalization we consider the model in slow-roll regime using the e-folding number representation and designation $A' = dA/dN$:

$$(\phi')^2 \simeq \frac{V'}{V} + \frac{4}{3}\xi' V = \frac{\left(H^2\right)'}{H^2} + 4H^2\xi'. \qquad (11)$$

- We present the first slow-roll parameters in terms of $H^2$, $\xi$:

$$\epsilon_1 = \frac{1}{2}\frac{(H^2)'}{H^2}, \delta_1 = -4H^2\xi'. \qquad (12)$$

- The second slow-roll parameters are related with first slow-roll parameters:

$$\epsilon_2 = -\epsilon_1'/\epsilon_1, \quad \delta_2 = -\delta_1'/\delta_1. \qquad (13)$$

- The slow-roll approximation requires $|\epsilon_i| \ll 1$, $|\delta_i| \ll 1$.

- The numbers of inflation scenarios in Einstein-Gauss-Bonnet can be restricted such as the speed of sound should be real [7].
- However there is a wonderful properties : In slow-roll regime the speed of sound is real. Such as the square of speed of sound can be presented in the form $c_A^2 = 1 + \Delta c_A^2$, where

$$\Delta c_A^2 = -\frac{2\delta_1^2 \epsilon_1}{3\delta_1^2 + 2(2\epsilon_1 - \delta_1)(1 + \delta_1)}. \quad (14)$$

In general case of slow-roll regime

$$\Delta c_A^2 \simeq -(\delta_1^2 \epsilon_1)/(2\epsilon_1 - \delta_1) \ll 1.$$

If $2\epsilon_1 \approx \delta_1$, then $\Delta c_A^2 \simeq -2\epsilon_1/3 \ll 1$. Thus, we can conclude $c_A^2 > 0$ in slow-roll regime.

[7] G. Hikmawan, J. Soda, A. Suroso and F. P. Zen,Phys. Rev. D **93** (2016) no.6, 068301 [arXiv:1512.00222 [hep-th]]
S. Odintsov and V. Oikonomou, Phys. Lett. B **805** (2020), 135437 [arXiv:2004.00479 [gr-qc]]

- The spectral index of scalar perturbations and the tensor-to-scalar ratio can be presented in terms e-folding numbers derivatives:

$$n_s = 1 - 2\epsilon_1 + \frac{r'}{r}\,, \tag{15}$$

$$r = 8|2\epsilon_1 - \delta_1| = 8\left(\frac{(H^2)'}{H^2} + 4H^2\xi'\right) = 8(\phi')^2\,. \tag{16}$$

using $\epsilon_2 = -\epsilon_1'/\epsilon_1$, $\delta_2 = -\delta_1'/\delta_1$.

- The expression for amplitude in the leading order is :

$$A_s \simeq \frac{2H^2}{\pi^2 r} \simeq \frac{V}{6\pi^2 U^2 r}. \tag{17}$$

# Generalization of the cosmological attractor method

- Accordingly to inflationary parameters of cosmological-attractor models without the Gauss-Bonnet term spectral index includes only logarithmic derivative of tensor-to-scalar ratio

$$\frac{r'}{r} = -\frac{2}{N+N_0}, \quad \text{and} \quad n_s \approx 1 + \frac{r'}{r}. \tag{18}$$

  in the leading order of $1/N$ approximation.
- The model without the Gauss-Bonnet term and exponential potential leading to cosmological-attractor prediction was considered in [8]
- We generalize this model to the Einstein-Gauss-Bonnet gravity.

[8] V. Mukhanov, Eur. Phys. J. C **73** (2013), 2486 [arXiv:1303.3925 [astro-ph.CO]].

# Exponential form

To generalize cosmological attractor approximation to inflationary models with the Gauss-Bonnet term we compare (16) with (2):

$$\frac{r}{8} = \frac{(H^2)'}{H^2} + 4H^2\xi' = \frac{3C_\alpha}{2(N+N_0)^2}. \tag{19}$$

For simplicity we suppose that all terms in this equation are proportional to $1/(N+N_0)^2$ and get the same approximation of slow-roll parameter $\epsilon_1$ in leading $1/N$ order:

$$H^2 = H_0^2 \exp\left(-\frac{3C_\beta}{2(N+N_0)}\right), \quad \xi = \xi_0 \exp\left(\frac{3C_\beta}{2(N+N_0)}\right), \tag{20}$$

where $C_\beta$ is a constant. We substitute (20), (20) to (19) and get:

$$\frac{r}{8} = \frac{3C_\beta}{2(N+N_0)^2}\left(1 - 4\xi_0 H_0^2\right), \tag{21}$$

fixing a relation between $C_\alpha$ and $C_\beta$:

$$C_\beta = \frac{C_\alpha}{1-4\xi_0 H_0^2}, \quad H_0^2 \neq \frac{1}{4\xi_0}. \tag{22}$$

Accordingly (16) the derivative of field is related with e-folding number:

$$(\phi')^2 = \frac{3C_\alpha}{2(N+N_0)^2}; \; \phi' = \frac{\omega_\phi\sqrt{\frac{3C_\alpha}{2}}}{N+N_0}, \; \omega_\phi = \pm 1 \tag{23}$$

from here

$$\phi = \omega_\phi\sqrt{\frac{3C_\alpha}{2}}\ln\left(\frac{N+N_0}{N_\phi}\right), \quad N+N_0 = N_\phi \exp\left(\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right). \tag{24}$$

Using (8), (20) and (24) we construct family of the models with the Gauss-Bonnet interaction and potential with variable parameter $C_\alpha$:

$$V = 3H_0^2 \exp\left(-\frac{3}{2}\frac{C_\beta}{N_\phi}\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right), \tag{25}$$

$$\xi = \xi_0 \exp\left(\frac{3}{2}\frac{C_\beta}{N_\phi}\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right) \tag{26}$$

leading to appropriate inflationary scenarios. This model is generalization of the general relativity model obtained in [9]

[9] V. Mukhanov, Eur. Phys. J. C **73** (2013), 2486 [arXiv:1303.3925 [astro-ph.CO]].

# Inflationary parameters in terms of field

To estimate the inflationary parameters (tensor-to-scalar ratio, spectral index of scalar perturbations) for both models we use following expressions

$$r = 8Q^2, \quad n_s = 1 - Q\frac{V_\phi}{V} + 2Q_{,\phi}, \quad Q = V_{,\phi}/V + 4\xi_{,\phi}V/3. \qquad (27)$$

For the exponential models we get

$$r = \frac{64\left(1 - 4H_0^2\xi_0\right)^2}{3C_\alpha}\left(\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right)^2,$$

$$n_s = 1 - \frac{8\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\left(1 + \exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right)\left(1 - 4H_0^2\xi_0\right)}{3C_\alpha}$$

# Inflationary parameters in terms of e-folding number

Now we apply relation between field and e-folding number (24) and obtain:

$$r = \frac{64\left(1-4H_0^2\xi_0\right)^2}{3C_\alpha}\left(\frac{N_\phi}{N+N_0}\right)^2 \tag{28}$$

$$n_s = 1-\frac{8\left(1-4H_0^2\xi_0\right)}{3C_\alpha}\left(\frac{N_\phi}{N+N_0}+\left(\frac{N_\phi}{N+N_0}\right)^2\right) \tag{29}$$

To present this expressions in more regular form we apply (22)

$$r = \frac{64N_\phi^2\left(1-4H_0^2\xi_0\right)}{3C_\beta\left(N+N_0\right)^2} = \frac{64}{3}\frac{C_\alpha N_\phi^2}{\left(N+N_0\right)^2} \tag{30}$$

$$n_s = 1-\frac{8}{3C_\beta}\left(\frac{N_\phi}{N+N_0}+\left(\frac{N_\phi}{N+N_0}\right)^2\right) \tag{31}$$

Substituting $N_\phi = \frac{3C_\beta}{4}$, $C_\beta = 1$ we get

$$r = \frac{64N_\phi^2\left(1 - 4H_0^2\xi_0\right)}{3C_\beta\left(N+N_0\right)^2} = \frac{12C_\alpha}{\left(N+N_0\right)^2} \tag{32}$$

$$n_s = 1 - \left(\frac{2}{N+N_0} + \frac{9}{16(N+N_0)^2}\right) \tag{33}$$

Evidently that in the beginning of inflation then $N + N_0 \approx 60$ we can roughly suppose

$$r \approx \frac{12C_\alpha}{\left(N+N_0\right)^2}, \quad n_s \approx 1 - \frac{2}{N+N_0} \tag{34}$$

The obtained approximations are coincide with inflationary parameters of attractors approximation and in the case of $C_\alpha \approx 1$ with parameters of $R^2$ inflation. The case $C_\alpha = 1$ leads to switch of Gauss-Bonnet interaction and to coinciding with exponential scenario early obtained in Einstein Gravity.

In the large field $\phi$ approximation the expressions for $r$ and $\tilde{r}$ are coincide up to second order, the expressions for $n_s$ and $\tilde{n}_s$ are coincide up to first order of $\exp\left(-\sqrt{\frac{2}{3C_\alpha}}\phi\right)$. The precision coincides with sensibility of cosmological attractor approximation (2). To satisfy the proposal sensibility we can write

$$n_s \simeq 1 + \frac{8\left(4H_0^2\xi_0 - 1\right)}{3C_\alpha}\,\exp\left(-\sqrt{\frac{2}{3C_\alpha}}\phi\right),$$

$$r \simeq \frac{64\left(4H_0^2\xi_0 - 1\right)^2}{3C_\alpha}\exp\left(-2\sqrt{\frac{2}{3C_\alpha}}\phi\right).$$

Accordingly to (22) these relations can be presented in the forms:

$$n_s \simeq 1 - \frac{2}{N_\phi}\exp\left(-\sqrt{\frac{2}{3C_\alpha}}\phi\right),$$

$$r \simeq \frac{12C_\alpha}{N_\phi^2}\exp\left(-2\sqrt{\frac{2}{3C_\alpha}}\phi\right)$$

which are fully correspond to (2).

# Restriction to the model parameters

- Accordingly to the Planck data
  - value of scalar spectral index and the restriction to tensor-to-scalar ratio are:
    $$n_s \approx 0.965 \pm 0.004 \text{ and } r < 0.056$$
  - value of scalar power spectrum amplitude is $A_s \approx 2 \cdot 10^{-9}$.

• The considered inflationary models with the Gauss-Bonnet interaction can be presented more precisely, namely, to satisfy condition $\epsilon_1(N \simeq 0) \approx 1$ we should suppose $N_0 = \sqrt{3C_\beta/4}$.

Accordingly to (2) the highest value of constant $C_\alpha$ is related with modern observations restriction to the tensor-to-scalar ratio $r$ and the value of e-folding number at the beginning of inflation. At the same time a start point of inflation defines the value of spectral index of scalar perturbations.

We numerically estimate the value of the model parameters using (2) and supposing that the inflation begin:

1. at $N \approx 55 - N_0$ before the end of inflation: $n_s \approx 0.964$ and $0 \leq C_\alpha < 14.1$;
2. at $N \approx 60 - N_0$ before the end of inflation: $n_s \approx 0.967$ and $0 \leq C_\alpha < 16.7$;
3. at $N \approx 65 - N_0$ before the end of inflation: $n_s \approx 0.969$ and $0 \leq C_\alpha < 19.6$.

To get expression for scalar power spectrum amplitude we substitute (19) and (20) to (17):

$$
\begin{aligned}
A_s &\simeq \frac{H_0^2 (N+N_0)^2}{6\pi^2 C_\alpha} \exp\left(-\frac{3C_\beta}{2(N+N_0)}\right) \qquad (35)\\
&= \frac{H_0^2 (N+N_0)^2}{6\pi^2 C_\alpha} \exp\left(-\frac{2N_0^2}{N+N_0}\right), \qquad (36)
\end{aligned}
$$

from where

$$
\frac{H_0^2}{C_\alpha} = \frac{6\pi^2 A_s}{(N+N_0)^2} \exp\left(\frac{2N_0^2}{N+N_0}\right). \qquad (37)
$$

To estimate $H_0^2/C_\alpha$ we suppose $N_0 \approx 1$ in three cases:

1. if the start point of inflation $N \approx 54$, then $H_0^2/C_\alpha \approx 4.09 \cdot 10^{-11}$
2. if the start point of inflation $N \approx 59$, then $H_0^2/C_\alpha \approx 3.40 \cdot 10^{-11}$
3. if the start point of inflation $N \approx 64$, then $H_0^2/C_\alpha \approx 2.90 \cdot 10^{-11}$

# Conclusion

- We get the generalization to the cosmological attractor of exponential form to gravity with the Gauss-Bonnet term. The generalization lead to analytical expressions for inflationary parameters coinciding with inflationary parameters of cosmological attractor models in the leading order approximation.
- Within the framework of the model we obtain an analytical expression for scalar power spectrum amplitude. We estimate the models constants using observation data.
- We considered the exponential inflationary scenario in Einstein-Gauss-Bonnet gravity and get that the direct calculations of spectral index for this model includes second order inverse e-folding number term. However such as in the beginning of inflation $N + N_0 \approx 60$ this term is negligible in relation to first order inverse e-folding number term. And roughly we can suppose that the cosmological attractor approximation for inflationary parameters are satisfied for considering model. Moreover the switch of Gauss-Bonnet interaction can leads $R^2$ gravity prediction for inflationary parameters in leading order approximation in inverse e-folding number.

**Thank for your attention**