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Behavior of various scalar field potentials under tracking parameter of Quintessence class of scalar field models

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Abstract: Scalar field models are known for their dynamical nature i.e, dynamical Equation of state parameter to explain the late-time cosmic acceleration of the universe. Quintessence canonical scalar field with a potential one such field which is introduced to account for the late time acceleration of the universe. In the present work, different kinds of scalar field potentials (under quintessence cosmological model) are mathematically investigated using tracking parameter to examine whether these potentials are showing thawing or tracking behavior. Tracking parameter is considered because during cosmic evolution the dynamics of tracking depends susceptibily on the variation of this particular parameter. Each potential is analyzed using this parameter and accordingly the behavior is shown. It is found that among the four potentials used, three are showing tracking properties and only one is showing thawing property as per the tracking parameter. The tracking and thawing properties/ behavior are discussed in result and discussion section of the paper.

Keywords: Tracker behavior; Scalar fields; Quintessence; Thawing behavior; Cosmology

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1. Introduction

The accelerating expansion of the universe predicted by the observations of Type Ia Supernovae, published by the High-Z Supernova Search Team in 1998 [1] and the Supernova Cosmology Project in 1999 [2] suggested the concept of Dark Energy. Dark energy is a hypothetical form of energy pervading the whole space. It is responsible for the accelerated expansion of the universe as it has strong negative pressure leading to gravitational repulsion (repulsive force). To understand this dark energy concept, various candidates come in existence. They are : Cosmological Constant (leading to coincidence problem [3], Quantum field theory (leading to the λ problem [4]), Scalar field models, Brane World Models etc. Though there are many candidates (so are their theories) of dark energy, none of them are captivating enough to decode the mystery of dark energy.

Scalar fields come to light through particle physics and string theory. Regarded as one of the candidate of dark energy, it consists of wide variety of scalar field dark energy models (quintessence,tachyonic,k-essence etc). The simplest field mimicking dark energy is a canonical scalar field with a potential known as quintessence [5,6,7]. The main difference between quintessence and cosmological constant is that the quintessence is dynamic in nature. This field is capable of reproducing the late-time accelerated behavior. It has Equation of state given by $w_\varphi = \frac{p_\varphi}{\rho_\varphi}$, where is p_φ pressure, ρ_φ is energy density. Quintessence scalar field is broadly classified into two classes[8,9,10]: (I) Thawing Models, and (II) Freezing Models. Thawing models are slow roll models which depends on initial value conditions whereas freezing models are fast roll models, insensitive to the initial conditions but over the entire range of scalar field, they depend sensitively on the shape of the potential. P.Steinhardt [11] was the first to introduce tracker fields. Tracker field is basically a class of quintessence model consisting of a scalar field rolling down a potential. It is introduced to avoid the problem of initial value conditions.

Tracker model consists of tracker solution, tracker field and tracker Equation (with quintessence as an Equation). For detailed analysis of tracker fields see the literature [12,13,14]. 'Γ' is the tracking parameter and the properties of Γ determine whether the tracking solutions exist or not. For any kind of potential Γ can be >, < or = 1.

The paper is methodized as follows. In sec.2, mathematical background for quintessence cosmological model is set up for tracking parameter. In sec.3, we mathematically examined the tracking parameter with various potentials to check for their tracking behavior and the results are highlighted in table 1. In sec.4, the results are concluded with brief discussion. Finally, we conclude the paper in sec.5.

2. Mathematical Background

The action proposed for quintessence [15] scalar field 'φ' gives the action as

$$S = \int d^4x \sqrt{-g} \left[\frac{-1}{2} (⟦\nabla\varphi)⟧^2 - V(\varphi)\right] \tag{1}$$

The energy density and pressure for spatially homogeneous quintessence scalar field can be obtained from the Lagrangian

$$L_{quin} = \frac{1}{2}\partial^i\varphi\partial^i\varphi - V(\varphi) \tag{2}$$

As

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi); \tag{3}$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{4}$$

where, ρ_φ is the energy density of scalar field, p_φ is the pressure density of scalar field and $V(\varphi)$ is the scalar field potential.

Equation (1) is varied w.r.t scalar field to get the Equation of motion as

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0 \tag{5}$$

where 'φ' is a function of time and V'(φ) is the derivative w.r.t φ. This Equation of motion describe how the scalar field evolve with time. Here, $H = \frac{\dot{a}}{a}$ is the hubble parameter which denotes the expansion rate of universe.

The Equation of state parameter ($w_{\varphi,}$) of quintessence is

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \quad \geq 1 \tag{6}$$

Also, density parameter

$$\Omega_\varphi = \frac{\rho_\varphi}{3H^2M_{pl}^2} = \frac{\rho_\varphi}{\rho_\varphi + \rho_m} \tag{7}$$

Where, ρ_m is matter density and M_{pl} is planck mass.
Also,

$$\dot{\rho_m} + 3H\rho_m = 0 \tag{8}$$

Let's define the dimensionless parameters,

$$\lambda = \frac{-m_p V''}{V} \tag{9}$$

$$\Gamma = \frac{VV''}{(V')^2} \tag{10}$$

Which are functions of the field $\lambda = \lambda(\varphi)$ and $\Gamma = \Gamma(\varphi)$.

Where 'λ' parametrizes the slope of the potential and 'Γ' is called the tracking parameter. where V'' and V' are differentiation w.r.t scalar field φ and V is the scalar field potential.

The potential is shallow for small value of λ (A tracker solution leads to small λ), and if the field dominates the universe, it could become quintessence.

Taking time derivative of (9) and using (10), we obtain

$$\frac{\dot{\lambda}}{\lambda^2} = \frac{-\dot{\varphi}}{m_p}(\Gamma - 1) \tag{11}$$

From Equations (5)–(8), we obtain the following Equations for w and Ω_φ (see [16,17,18])

$$w' = (1 - w)\left[-3(1 + w) + \lambda\sqrt{3(1 + w)\Omega_\varphi}\right] \tag{12}$$

$$\Omega'_\varphi = -3w\Omega_\varphi\left(1 - \Omega_\varphi\right) \tag{13}$$

$$\lambda' = -\sqrt{3(1 + w)\Omega_\varphi}(\Gamma - 1)\lambda^2 \tag{14}$$

Where a prime represents a derivative with respect to $N = ln\, a$. There are several different cases for the evolution of w, depending on the initial conditions and field potential.

3. Results

The four potentials of different forms (power law, exponential and hyperbolic form) are defined as:

a) Exponential form : $V(\varphi) = V_0\alpha e^{(1-\varphi)^u}$

b) Power law form : $V(\varphi) = V_0\alpha(1 - \varphi)^u$ and V(φ)=$V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u$

c) Hyperbolic form [19] : V(φ) = $V_0 cosh(\alpha\varphi^u)$

where, V_0 , α and u are real constants in which α and u are dimensionless and V_0 is dimensionful. Here, first the analysis is done keeping u ∈ R, for all values in the range R. Then, a particular value of u (-1 and 1) is chosen to study the behavior of these potentials . Further,other values are equally appealing provided they must be in a range of R. So,here we considered u = -1 and 1 respectively, for the potentials described above to proceed further towards its mathematical analysis.

3.1. Analysis

a) V(φ) = $\boldsymbol{V_0\alpha e^{(1-\varphi)^u}}$

$$\boldsymbol{V'(\varphi) = -V_0\alpha u(1 - \varphi)^{u-1}e^{(1-\varphi)^u}}$$

$$V''(\varphi) = V_0\alpha u^2(1-\varphi)^{2u-2}e^{(1-\varphi)^u} + (u-1)u(1-\varphi)^{u-2}e^{(1-\varphi)^u}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha e^{(1-\varphi)^u}V_0\alpha u^2(1-\varphi)^{2u-2}e^{(1-\varphi)^u} + (u-1)u(1-\varphi)^{u-2}e^{(1-\varphi)^u}}{(-V_0\alpha u(1-\varphi)^{u-1}e^{(1-\varphi)^u})^2}$$

$$\Gamma = 1 + \frac{(u^2-u)}{u^2(1-\varphi)^u} \tag{15}$$

For u = 0, Γ= undefined; From (15), u=1, Γ= 1
For –ve and +ve values of u, Γ > 1.

b) V(φ)=$V_0\alpha(1-\varphi)^u$

$$V'(\varphi) = -V_0\alpha u(1-\varphi)^{u-1}$$

$$V''(\varphi) = V_0\alpha u(u-1)(1-\varphi)^{u-2}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha(1-\varphi)^u V_0\alpha u(u-1)(1-\varphi)^{u-2}}{(-V_0\alpha u(1-\varphi)^{u-1})^2}$$

$$\Gamma = 1 - \frac{1}{u} \tag{16}$$

For u = 0, Γ is undefined. From (16), u= -1, Γ > 1.

For –ve values of u, Γ > 1.
For +ve values of u, Γ < 1.

c) V(φ)=$V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u$

$$V'(\varphi) = -V_0\alpha u\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)}$$

$$V''(\varphi) = V_0\alpha u(u+1)\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)^2}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha u(u+1)\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)^2} V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u}{(-V_0\alpha u\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)})^2}$$

$$\Gamma = 1+\frac{1}{u} \tag{17}$$

For u = 0, Γ is undefined. From (17), u=1, Γ > 1.
For –ve values of u, Γ < 1.
For +ve values of u, Γ > 1.

d) V(φ)= $V_0\cosh(\alpha\varphi^u) = \frac{V_0}{2}\left(e^{\alpha\varphi^u}+e^{-\alpha\varphi^u}\right)$

For u>0, $V(\varphi) = \frac{V_0}{2}e^{\alpha\varphi^u}$

$$V'(\varphi) = \frac{V_0}{2}u\alpha e^{\alpha\varphi^{u-1}}$$

$$V''(\varphi) = \frac{V_0}{2}u\alpha^2(u-1)e^{\alpha\varphi^{u-2}}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{\frac{V_0}{2}e^{\alpha\varphi^u}\ \frac{V_0}{2}u\alpha^2(u-1)e^{\alpha\varphi^{u-2}}}{(\frac{V_0}{2}u\alpha e^{\alpha\varphi^{u-1}})^2}$$

$$\Gamma = 1-\frac{1}{u} \tag{18}$$

For u<0, $V(\varphi) = \frac{V_0}{2}e^{-\alpha\varphi^u}$

Similarly, as u>0,

$$\Gamma = 1-\frac{1}{u} \tag{19}$$

which implies for +ve or -ve values of u, Γ< 1.

Table 1. Behavior shown by potentials used for quintessence cosmological model.

S.No.	Potential (V(φ))	Tracking Parameter(Γ)	Value of 'u' (u∈R)
1.	$V_0\alpha e^{(1-\varphi)}$	Γ = 1	1
2.	$V_0\alpha(1-\varphi)^{-1}$	Γ > 1	-1
3.	$V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})$	Γ > 1	1
4.	$V_0\cosh(\alpha\varphi^u)$	Γ < 1	For all +ve and –ve values of u.

4. Discussion

It is clearly shown in the table that only three potentials are showing tracking behavior($\Gamma \geq 1$). The potentials showing tracking behavior are basically fast-roll (freezing) potentials. The hyperbolic potential is showing thawing behavior ($\Gamma < 1$) and is basically called slow-roll (thawing) potential. Let's discuss their behavior.

$\Gamma \geq 1$: The potential $V_0\alpha e^{(1-\varphi)}$ is a scaling attractor for $\Gamma = 1$, it is a special case of tracker, known as scaling freezing.There exists an attractor solution to the evolution of the field, but the attractor is not a tracker. In these type of models, during most of the cosmological evolution scalar field density and its Equation of state remains close to that of the dominant background matter.

Γ > 1 : The potential $V_0\alpha(1-\varphi)^{-1}$ and $V_0\alpha\left(\frac{\varphi}{\varphi-\varphi^2}\right)$ are attractors for $\Gamma > 1$ (know as tracking condition). This kind of attractor is called tracker. From Equation (12), as $\mid\lambda\mid$ diminishes with time, $\Omega_\varphi$grows, which infers that quintessence dominate the universe as required. As time passes, the potential becomes shallower. In this case, during most of the cosmological evolution, scalar field density and its Equation of state remains less than to that of dominant background matter. This condition is equivalent to the case, as $\left|\frac{V\prime}{V}\right|$ be decreasing as V decreases.

$\Gamma < 1$: The potential $V_0\cosh(\alpha\varphi^u)$ is a thawing potential for $\Gamma < 1$. Thawing potentials does not enjoy the privilege of attractor behavior and should have initial conditions explained. At late times, the potential becomes shallow resulting in the gradual slow down of the field. Thawing potentials (that indulge slow- roll conditions) come up naturally to produce w (Equation of state) near -1, converging to a single, universal behavior.

From future work perspective, possible analytic solutions of these potentials can be done for the dark energy Equation of state , expressed in terms of several free parameters and using cosmological observations one can also put cosmological constraints on these various forms of potentials discussed to get some new insights.

5. Conclusions

In the present work, we investigate various (power law, exponential and hyperbolic form) scalar field potentials under quintessence cosmological model to examine whether these potentials are showing tracking behavior or not. We analyzed this tracking behavior using tracking parameter for each potential and accordingly we get our results. It is found that among the four potentials used for examining tracking behavior, three (power law and exponential) are showing tracking properties and one (hyperbolic) is showing thawing property.

Conflicts of Interest: The author declare no conflict of interest.

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