It is clearly shown in the table that only three potentials are showing tracking behavior $(\Gamma \geq 1)$. The potentials showing tracking behavior are basically fast-roll (freezing) potentials. The hyperbolic potential is showing thawing behavior ($\Gamma<1$) and is basically called slow-roll (thawing) potential. Let's discuss their behavior.
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Behavior of various scalar field potentials under tracking parameter of Quintessence class of scalar field models

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Abstract

Scalar field models are known for their dynamical nature i.e, dynamical equation of state parameter to explain the late-time cosmic acceleration of the universe. Quintessence canonical scalar field with a potential one such field which is introduced to account for the late time acceleration of the universe. In the present work, different kinds of scalar field potentials (under quintessence cosmological model) are mathematically investigated using tracking parameter to examine whether these potentials are showing thawing or tracking behavior. Tracking parameter is considered because during cosmic evolution the dynamics of tracking depends susceptibily on the variation of this particular parameter. Each potential is analyzed using this parameter and accordingly the behavior is shown. It is found that among the four potentials used, three are showing tracking properties and only one is showing thawing property as per the tracking parameter.

Background

The simplest field mimicking dark energy is a canonical scalar field with a potential known as quintessence. This field is capable of reproducing the late-time accelerated behavior. It has equation of state given by :

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi}$$

where is p_φ pressure, ρ_φ is energy density. Quintessence scalar field is broadly classified into two classes :

(I) Thawing Models : They are slow roll models which depends on initial value conditions.

(II) Freezing Models : They are fast roll models, insensitive to the initial conditions but over the entire range of scalar field, they depend sensitively on the shape of the potential.

'Γ' is the tracking parameter and the properties of Γ determine whether the tracking solutions exist or not. For any kind of potential Γ can be >, < or = 1.

$$\Gamma = \frac{VV''}{(V')^2}$$

'Γ' is called the tracking parameter. where V'' and V' are differentiation w.r.t scalar field φ and V is the scalar field potential.

Mathematical Background

The four potentials of different forms (power law, exponential and hyperbolic form) are defined as:

a) Exponential form : $V(\varphi) = V_0\alpha e^{(1-\varphi)^u}$

b) Power law form : $V(\varphi) = V_0\alpha(1-\varphi)^u$ and $V(\varphi)=V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u$

c) Hyperbolic form : $V(\varphi) = V_0 cosh(\alpha\varphi^u)$

where, V_0 , α and u are real constants in which α and u are dimensionless and V_0 is dimensional. Here, first the analysis is done keeping $u \in R$, for all values in the range R. Then, a particular value of u (-1 and 1) is chosen to study the behavior of these potentials . Further,other values are equally appealing provided they must be in a range of R. So,here we considered u = -1 and 1 respectively, for the potentials described above to proceed further towards its mathematical analysis,

Analysis

a) $V(\varphi) = V_0\alpha e^{(1-\varphi)^u}$

$$V'(\varphi) = -V_0\alpha u(1-\varphi)^{u-1}e^{(1-\varphi)^u}$$

$$V''(\varphi) = V_0\alpha u^2(1-\varphi)^{2u-2}e^{(1-\varphi)^u} + (u-1)u(1-\varphi)^{u-2}e^{(1-\varphi)^u}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha e^{(1-\varphi)^u}V_0\alpha u^2(1-\varphi)^{2u-2}e^{(1-\varphi)^u} + (u-1)u(1-\varphi)^{u-2}e^{(1-\varphi)^u}}{(-V_0\alpha u(1-\varphi)^{u-1}e^{(1-\varphi)^u})^2}$$

$$\Gamma = 1 + \frac{(u^2-u)}{u^2(1-\varphi)^u} \qquad (1)$$

For u = 0, Γ= undefined; From (15), u=1, Γ= 1

For –ve and +ve values of u, Γ > 1.

b) $V(\varphi) = V_0\alpha(1-\varphi)^u$

$$V'(\varphi) = -V_0\alpha u(1-\varphi)^{u-1}$$

$$V''(\varphi) = V_0\alpha u(u-1)(1-\varphi)^{u-2}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha(1-\varphi)^u V_0\alpha u(u-1)(1-\varphi)^{u-2}}{(-V_0\alpha u(1-\varphi)^{u-1})^2}$$

$$\Gamma = 1 - \frac{1}{u} \qquad (2)$$

For u = 0, Γ is undefined. From (16), u= -1, Γ > 1.

For –ve values of u, Γ > 1.

For +ve values of u, Γ < 1.

c) $V(\varphi)=V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u$

$$V'(\varphi) = -V_0\alpha u\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)}$$

$$V''(\varphi) = V_0\alpha u(u+1)\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)^2}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{V_0\alpha u(u+1)\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)^2}V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})^u}{(-V_0\alpha u\frac{(\frac{\varphi}{\varphi-\varphi^2})^u}{(\varphi-1)})^2}$$

$$\Gamma = 1 + \frac{1}{u} \qquad (3)$$

For u = 0, Γ is undefined. From (17), u=1, Γ > 1.

For –ve values of u, Γ < 1.

For +ve values of u, Γ > 1.

d) $V(\varphi)= V_0\cosh(\alpha\varphi^u) = \frac{V_0}{2}(e^{\alpha\varphi^u} + e^{-\alpha\varphi^u})$

For u>0, $V(\varphi) = \frac{V_0}{2}e^{\alpha\varphi^u}$

$$V'(\varphi) = \frac{V_0}{2}u\alpha e^{\alpha\varphi^{u-1}}$$

$$V''(\varphi) = \frac{V_0}{2}u\alpha^2(u-1)e^{\alpha\varphi^{u-2}}$$

$$\Gamma = \frac{VV''}{(V')^2}$$

$$\Gamma = \frac{\frac{V_0}{2}e^{\alpha\varphi^u}\frac{V_0}{2}u\alpha^2(u-1)e^{\alpha\varphi^{u-2}}}{(\frac{V_0}{2}u\alpha e^{\alpha\varphi^{u-1}})^2}$$

$$\Gamma = 1 - \frac{1}{u} \qquad (4)$$

For u<0, $V(\varphi) = \frac{V_0}{2}e^{-\alpha\varphi^u}$

Similarly, as u>0, $\Gamma = 1 - \frac{1}{u}$ (5)

which implies for +ve or -ve values of u, Γ< 1.

Results

S.No.	Potential (V(φ))	Tracking parameter(Γ)	Value of 'u' (u∈R)
1.	$V_0\alpha e^{(1-\varphi)}$	Γ = 1	1
2.	$V_0\alpha(1-\varphi)^{-1}$	Γ > 1	-1
3.	$V_0\alpha(\frac{\varphi}{\varphi-\varphi^2})$	Γ > 1	1
4.	$V_0\cosh(\alpha\varphi^u)$	Γ < 1	For all +ve and –ve values of u.

Table 1 : Behavior shown by potentials used for quintessence cosmological model

Discussion

It is clearly shown in the table that only three potentials are showing tracking behavior(Γ ≥ 1). The potentials showing tracking behavior are basically fast-roll (freezing) potentials. The hyperbolic potential is showing thawing behavior (Γ < 1) and is basically called slow-roll (thawing) potential. Let's discuss their behavior.

- **Γ ≥ 1** : The potential $V_0\alpha e^{(1-\varphi)}$ is a scaling attractor for Γ=1, it is a special case of tracker, known as scaling freezing. In these type of models, during most of the cosmological evolution scalar field density and its equation of state remains close to that of the dominant background matter.
- **Γ > 1** : The potential $V_0\alpha(1-\varphi)^{-1}$and $V_0\alpha(1-\varphi)^{-1}$are attractors for Γ >1 (know as tracking condition). This kind of attractor is called tracker. As time passes, the potential becomes shallower. In this case, during most of the cosmological evolution, scalar field density and its equation of state remains less than to that of dominant background matter.
- **Γ < 1** : The potential $V_0\cosh(\alpha\varphi^u)$ is a thawing potential for Γ<1. They does not enjoy the privilege of attractor behavior and should have initial conditions explained. At late times, the potential becomes shallow resulting in the gradual slow down of the field. Thawing potentials (that indulge slow- roll conditions) come up naturally to produce *w* (equation of state) near -1, converging to a single, universal behavior.

From future work perspective, possible analytic solutions of these potentials can be done and using cosmological observations one can also put cosmological constraints on these various forms of potentials discussed to get some new insights.

Conclusion

In the present work, we investigate various (power law, exponential and hyperbolic form) scalar field potentials under quintessence cosmological model to examine whether these potentials are showing tracking behavior or not. We analyzed this tracking behavior using tracking parameter for each potential and accordingly we get our results. It is found that among the four potentials used for examining tracking behavior, three (power law and exponential) are showing tracking properties and one (hyperbolic) is showing thawing property.

References

[1] Chiba, T. Slow-roll thawing quintessence, *Physical Review D*, **(2009)**, *79*(8), 083517.

[2] Steinhardt, P. J., Wang, L., & Zlatev, I. *Physical Review D*, **(1999)**, *59*(12), 123504.

[3] Copeland, E. J., Liddle, A. R., & Wands, D. **(1998).** Exponential potentials and cosmological scaling solutions. *Physical Review D*, **(1987)**, *57*(8), 4686