

Casimir effect as a probe for new physics phenomenology

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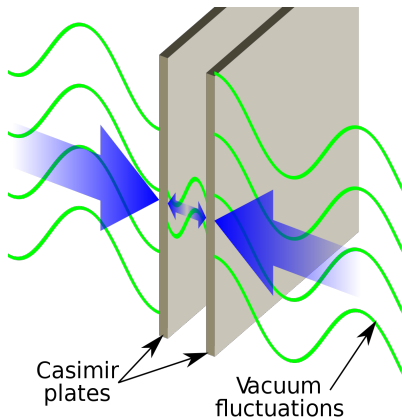
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The 1st Electronic Conference on Universe,
22nd – 28th February 2021, Online

- Brief excursus
- Casimir effect for mixed fields
- Casimir effect in curved backgrounds
- Casimir effect in minimal length theories
- Final remarks

Casimir effect



The Casimir effect occurs whenever a quantum field is bound in a finite region of space

The confinement gives rise to a net attractive force between the plates used to constrain the field

The first-ever physical manifestation of zero-point energy (Bohr's interpretation of Van der Waals forces)

How it works

Given a massless scalar field ϕ in the previous setting

$$\phi(t, \mathbf{x}_\perp, 0) = \phi(t, \mathbf{x}_\perp, D) = 0$$

One can evaluate the vacuum energy per unit transverse area*

$$\varepsilon = \langle 0 | T_{00} | 0 \rangle = \frac{1}{2} \sum_n \int \frac{d^2 k}{(2\pi)^2} \sqrt{k^2 + \frac{n^2 \pi^2}{D^2}}$$

The result can be achieved via dimensional regularization

*K. A. Milton, arXiv:hep-th/9901011 [hep-th] (1999)

How it works

In so doing, one gets

$$\varepsilon = -\frac{\pi^2}{1440} \frac{1}{D^3}$$

The force per unit area between the plates is

$$F = -\frac{\partial \varepsilon}{\partial D} = -\frac{\pi^2}{480} \frac{1}{D^4}$$

By multiplying for a factor 2 associated with the photon polarization

$$F_{em} = -\frac{\pi^2}{240} \frac{1}{D^4}$$

it is possible to reproduce Casimir original result*

*H. B. G. Casimir, Proc. K. Ned. Akad. Wet. (1948)

Generalizations

The aforesaid considerations can be extended to different spatial dimensions and geometry of the binding plates

Furthermore, different fields (i.e. scalar, spinor etc.) can be employed as well, either massless or massive

For instance, the massive scalar field in one spatial dimension yields*

$$F = -\frac{m^2}{\pi} \sum_n \left[K_2(2 D m n) - \frac{K_1(2 D m n)}{2 D m n} \right]$$

*S. Mobassem, Mod. Phys. Lett. A (2014)

- ~~Brief excursus~~
- Casimir effect for mixed fields

Mixed fields

In the previous setting, a massive scalar mixed field can be investigated*

$$\mathcal{L} = \sum_{\sigma} \left(\partial_{\mu} \phi_{\sigma}^{\dagger} \partial^{\mu} \phi_{\sigma} - m_{\sigma}^2 \phi_{\sigma}^{\dagger} \phi_{\sigma} \right) - m_{AB}^2 \left(\phi_A^{\dagger} \phi_B + \phi_B^{\dagger} \phi_A \right)$$

In QFT, mixing occurs at the level of fields, i.e. $\phi_{\sigma} = U \phi_m$

$$m_A^2 = m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta$$

$$m_B^2 = m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta$$

$$m_{AB}^2 = \left(m_2^2 - m_1^2 \right) \sin \theta \cos \theta$$

*M. Blasone, A. Capolupo, O. Romei and G. Vitiello, Phys. Rev. D (2001)

Mixed fields

At the level of states, the above mixing implies

$$|0\rangle_{AB} = G_\theta^{-1} |0\rangle_{12}$$

$$G_\theta = \exp \left[-i\theta \int d^3x \left(\pi_1 \phi_2 - \pi_2 \phi_1 + \phi_2^\dagger \pi_1^\dagger - \phi_1^\dagger \pi_2^\dagger \right) \right]$$

In the infinite volume limit, the flavor and the mass Fock spaces are unitarily non-equivalent

$$\lim_{V \rightarrow \infty} {}_{AB} \langle 0|0 \rangle_{12} = 0$$

The choice of the correct vacuum becomes crucial for the Casimir effect

Mixed fields

Indeed*

Mass vacuum

$$F_m = - \sum_j \frac{m_j^2}{\pi} \sum_n \left[K_2(2 D m_j n) - \frac{K_1(2 D m_j n)}{2 D m_j n} \right]$$

Flavor vacuum

$$F_f = F_m + \delta F$$

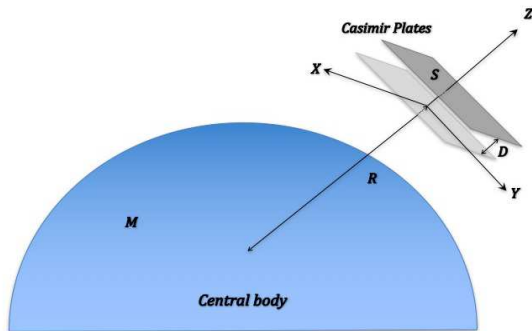
For $m_j D \ll 1$

$$\delta F = - \frac{3 \sin^2 \theta D^2 \zeta(3) (m_2^2 - m_1^2)^2}{2\pi^3}$$

*M. Blasone, G. G. Luciano, L. P. and L. Smaldone, Phys. Lett. B (2018)

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Casimir effect in curved background



The weak-field limit will be considered

$$g_{00} = 1 + 2\phi_{GR} + 2\phi_{ETG} \quad g_{ij} = -(1 - 2\phi_{GR} - 2\psi_{ETG}) \delta_{ij}$$

Mathematical description

If a massless scalar field is again considered*

$$(\square + \xi \mathcal{R}) \phi(\mathbf{x}, t) = 0$$

The mean vacuum energy density is defined as

$$\varepsilon = \frac{1}{V_p} \sum_n \int d^2 \mathbf{k}_\perp \int dx dy dz \sqrt{-g_\Sigma} (g_{00})^{-1} T_{00}(\phi_n)$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$$

General result in the weak-field limit for ETG

$$\varepsilon = \varepsilon_0 + \varepsilon_{GR} + \varepsilon_{ETG} \quad \varepsilon_{GR} = -\frac{\phi_{GR} D_p}{R} \varepsilon_0$$

*F. Sorge, Class. Quant. Grav. (2005)

Mathematical description

The measurable physical quantity is the pressure

$$P = F/S_p, \quad F = -\frac{\partial(\varepsilon V_p)}{\partial D_p}$$

$$P = P_0 + P_{GR} + P_{ETG} \quad P_{GR} = -\frac{2\phi_{GR}D_p}{3R}P_0$$

In order to put a constraint on the free parameters of extended models, it is possible to require

Requirement

$$|P_{ETG}| \lesssim \delta P_{\text{exp}}$$

Casimir effect with Lorentz-violating theories

If we consider the Standard Model Extension (SME)*

$$S_{LV} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(-u \mathcal{R} + s^{\mu\nu} R_{\mu\nu}^T + t^{\rho\lambda\mu\nu} C_{\rho\lambda\mu\nu} \right)$$

$$ds^2 = \left[1 - \frac{GM}{r} \left(2 + 3 \bar{s}^{00} \right) \right] dt^2 - \left[1 + \frac{GM}{r} \left(2 - \bar{s}^{00} \right) \right] \left(dr^2 + r^2 d\Omega^2 \right)$$

The bound that can be obtained from the pressure is†

Bound

$$\bar{s}^{00} \lesssim \frac{1}{3} \frac{\delta P}{P_0} \frac{R}{R_S}$$

*V. A. Kostelecky, Phys. Rev. D (2004)

†M. Blasone, G. Lambiase, L. P. and A. Stabile, Eur. Phys. J. C (2018)

Casimir effect with SEP

If quadratic theories of gravity are considered*

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ \mathcal{R} + \frac{1}{2} \left[\mathcal{R} \mathcal{F}_1(\square) \mathcal{R} + \mathcal{R}_{\mu\nu} \mathcal{F}_2(\square) \mathcal{R}^{\mu\nu} \right] \right\}$$

$$ds^2 = (1 + 2\Phi) dt^2 - (1 - 2\Psi) (dr^2 + r^2 d\Omega^2)$$

The gravitational potentials are

$$\Phi(r) = -\frac{4Gm}{\pi r} \int_0^\infty dk \frac{a - 2c}{a(a - 3c)} \frac{\sin(kr)}{k}$$
$$\Psi(r) = \frac{4Gm}{\pi r} \int_0^\infty dk \frac{c}{a(a - 3c)} \frac{\sin(kr)}{k}$$

*T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Phys. Rev. Lett. (2012)

Casimir effect with SEP

The outcome for all the analyzed theories is*

Bound

$$P_G = \left[3(\Phi_0 - \Psi_0) - \frac{2}{3}(2\Psi_1 - \Phi_1) D_P \right] P_0, \quad |P_G| \lesssim \delta P$$

Furthermore, focusing on the first-order approximation in R , one can show that

SEP violation

$$P_G = 3\eta_0 \Phi_0 P_0$$

*L. Buoninfante, G. Lambiase, L. P. and A. Stabile, Eur. Phys. J. C (2019)

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Generalized Uncertainty Principle (GUP)

Many theories of Quantum Gravity predict the existence of a minimal length scale that modifies the standard Heisenberg Uncertainty Principle (HUP)

$$\delta x \delta p \geq \frac{\hbar}{2}$$

In string theory, several works on superstring collisions* suggested that the presence of gravity should modify HUP according to ($c = 1$)

$$\delta x \geq \frac{1}{2\delta p} + \beta \hbar \frac{\delta p}{m_p^2}$$

*D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B (1987);
D. J. Gross and P. F. Mende, Phys. Lett. B (1987)

A similar outcome can be achieved also in the context of gedanken experiments on large* and micro† black holes.

The previous uncertainty relation can also be cast in terms of a deformed commutator‡

$$[x, p] = i\hbar \left[1 + \beta \left(\frac{p}{m_p} \right)^2 \right]$$

Typically, in some models of string theory the deformation parameter is assumed to be $\beta \sim \mathcal{O}(1)$.

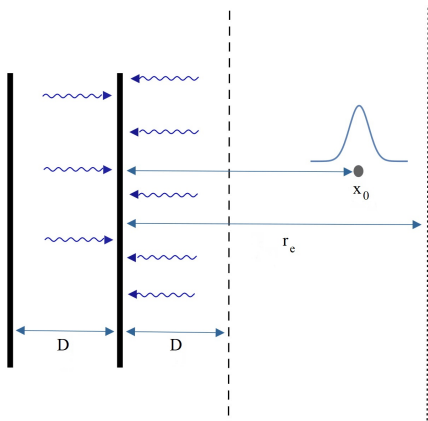
*M. Maggiore, Phys. Lett. B (1993)

†F. Scardigli, Phys. Lett. B (1999)

‡A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D (1995)

Casimir effect with GUP

There exists a method to heuristically derive the Casimir energy from HUP*



* J. Giné, Mod. Phys. Lett. A (2018)

Casimir effect with GUP

The imbalance of photons created from quantum vacuum between the inner and outer region of the binding objects is responsible for the attractive pressure

Results

$$\varepsilon_{\text{HUP}} \simeq \frac{1}{D^3} \qquad \varepsilon_{\text{GUP}} \simeq \frac{1}{D^3} \left[1 + \frac{\beta}{E_p D} \right]$$

Similar considerations can be applied also within different geometrical settings for the binding objects*

*M. Blasone, G. Lambiase, G. G. Luciano, L. P. and F. Scardigli, J. Phys. Conf. Ser. (2019); Int. J. Mod. Phys. D (2020)

Final remarks

Casimir effect turns out to be a remarkable tool to probe generalizations of SM and GR

In principle, one can extend the application of the Casimir effect to other scenarios and perform computation with more realistic fields (i.e. fermions and photons) and geometries

An improvement of the apparatus sensitivity may open the door to the possibility of detecting new physics phenomenology

