Article

An Overview of Black Hole Chemistry

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Abstract: Understanding the thermodynamic behaviour of black holes by using the concepts of chemistry will be discussed here. To establish the complete correspondence between the thermodynamics of an ordinary system and the thermodynamic of black holes, recent proposal suggests to identify the mass of a black hole as chemical enthalpy of an ordinary thermodynamic system. Similary, negative cosmological constant, surface gravity and horizon area of black holes are identified as pressure, temperature and entropy of a thermodynamic system. Consequently, black holes behave analougously to the variety of everyday phenomena. It allows one to understand the black holes by using the concepts chemistry such as Van der Waals fluids, phase transitions etc.

Keywords: Thermodynamic pressure; Chemical enthalpy; Van der Waals fluids; Phase transition.

1. Introduction

Black holes are one of the most fascinating predictions of general relativity. It was the work of Hawking given in Ref. [1] according to which black holes could emit radiations like almost black body using semiclsaasical approach. Further, Hawking and Page [2] studied the thermodynamical aspects of a black hole of an anti de Siiter background that it can undergo a phase transition between pure radiation and a stable black hole, known as Hawking-Page phase transition. This prediction makes black holes are of much theoretical interest. Another interesting perpespective [3] in this regard is the treatment of negative cosmological constant Λ of an anti de Sitter background as a thermodynamic variable related to the pressure. According to this perspective black holes behave analogously to the variety of everyday phenomena. Here an overview of this proposal will be discussed.

Thermodynamic quantities of a physical system such as energy E, temperature T and entropy S have well known counter-parts in black hole thermodynamic. Energy E of a physical system is related to mass M of a black hole. Similarly, temperature T is related to surface gravity κ and entropy S is related to the horizon area A of a black hole. Black hole thermodynamic is very important because it helps us to understand the underlying structure of the quantum gravity. Despite of the fact that the above correspondence exists between the thermodynamic of a physical system and the black holes thermodynamic, it is not well evident if one compare the forms of first law of thermodynamic for both. The form of first law of thermodynamic for physical system have the following form

$$dE = TdS - PdV + \text{work terms}. \tag{1}$$

Whereas, the form of the first law of thermodynamic for black holes have the form

$$dM = \frac{\kappa}{8\pi} dA + \omega dJ + \phi dQ, \tag{2}$$

where ωdJ and ϕdQ are the work terms. Note that in the above relation $G = c = \hbar = \kappa_\beta = 1$. It is now evident from the above two relations that the pressure-volume term which is present in the first law of thermodynamic of a physical system has no counter-part in the form of the first law of black hole thermodynamic. Recently, to address this correspondence new developments have taken place. These findings lead to a picture in which the mass M of a black hole is interpreted as the enthalpy of a spacetime. This idea originates from the consideration of the Smarr relation [4,5]. It was originally presented in Ref. [6] for $d = 4$ dimensions.

2. Methodology

To elaborate the correspondence, consider the example of following d-dimensional black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \tag{3}$$

where $f(r) = 1 - \frac{1}{d-2}\frac{16\pi}{\lambda_{d-2}}\frac{M}{r^{d-3}} + ...$, and $d\Omega_d^2$ is the line element of the unit sphere S^d with a volume $\lambda_d = 2\pi^{\frac{d-1}{2}}/\Gamma(\frac{d+1}{2})$. Smarr's relation for such black holes have the following form

$$(d-3)M = (d-2)TS. \tag{4}$$

If one include the cosmological constant in the metric then it will modify the Smarr relation [5]. Euler's theorem for $M = M(A, \Lambda)$ implies

$$(d-3)M = (d-2)\frac{\partial M}{\partial A}A - 2\frac{\partial M}{\partial \Lambda}\Lambda, \tag{5}$$

Since $T = 4\frac{\partial M}{\partial A}$, above equation suggests that one should regard $P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}$ as a thermodynamic variable [7,8]. Conjugate of thermodynamic pressure is the thermodynamic volume given by $V = -8\pi\frac{\partial M}{\partial \Lambda}$. Due to this consideration, one can have the modified Smarr relation given by

$$(d-3)M = (d-2)TS - 2PV. \tag{6}$$

Consequently, the first law of thermodynamic is also modified and the modified first law of thermodynamic have the form

$$dM = TdS + VdP. \tag{7}$$

This consideration leads to the complete thermodynamic correspondence. To include the case of charged and rotating black holes, one requires a more general form of Smarr relation given by

$$\frac{d-3}{d-2}M = TS + \sum_i (K^i - K^i)J^i - \frac{2}{d-2}PV + \frac{d-3}{d-2}\Phi Q. \tag{8}$$

3. Charged Black Holes

As vacuum pressure is induced by a negative cosmological constant, therefore, in this perspective it is considered as thermodynamic pressure term. Whereas, if one consider the gravitational version of enthalpy, this will correspond to the mass of a black hole, which means that, it is the total energy required to create a black hole and place it in a cosmological environment. This change of perspective opens new direction to understand the behaviour of black holes. This allows one to consider the black hole's behaviour analogous to a variety of everyday phenomena. One such realization was that the charged black holes behaves as Van der Waals fluids [3]. To approximate the behaviour of real fluids one need to modify the equation of state for an ideal gas [9]

$$(P + \frac{a}{v^2})(v - b) = T, \tag{9}$$

it gives

$$P = \frac{T}{v-b} - \frac{a}{v^2}. \tag{10}$$

Here a, b are constants and v represents the specific volume for the fluid under consideration. Locations of critical points can be found at $T = T_c$ where $P = P(v)$ will have a point of inflection at $P = P_c$ and $v = v_c$ and it satisfy the universal relation $\frac{P_c v_c}{\kappa T_c} = \frac{3}{8}$ for the fluid under consideration. For the temperature $T < T_c$ a liquid/gas phase transition will occur.

In order to see such behaviour for black hole, one can consider the simplest example of Reissner Nordstr"om-AdS black hole, whose metric have the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + sin^2\phi d\phi^2). \tag{11}$$

Here $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}$. One can have the following relations for its thermodynamic: $V = \frac{4}{3}\pi r_+^3$, where r_+ gives the event horizon of the black hole. Equation of state will have the form

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}, v = 2(\frac{3V}{4\pi})^{\frac{1}{3}}. \tag{12}$$

In the above relation, specific volume $v = 6V/N$ with $N = A/l_p^2, l_p = \sqrt{G\hbar/c^3}$. For any specific charge Q, the $P - V$ relation and the $P - T$ relation gives the behaviour of a Van der Waals fluids and liquid/gas phase transition is replaced by small/large black hole phase transition. Remarkably, critical points satisfies the relation $\frac{P_c v_c}{T_c} = \frac{3}{8}$ as was the case for Van der Waals fluids [10–13].

4. Discussion

An overview of black hole chemistry is presented in this article. Initially, black holes were only considered black, but after the work of Hawking it was realized that these are physical objects and then their thermodynamic properties were explored. The perspective which is reviewed here is one step farther in this direction. It allows one to discuss black holes behaviour analogous to many everyday chemical phenomena. It is new window to look at the black holes and one could expect variety of exciting new results about these ever mysterious objects in our Universe.

References

1. Hawking, S. W. Particle creation by black holes. *Commun. Math. Phys* **1975**, *43*, 199–220.
2. Hawking, S. W. The unpredictability of quantum gravity. *Commun. Math. Phys* **1982**, *87*, 395–415.
3. Kubiznak, D. and Mann, R. B. P-V criticality of charged AdS black holes. *JHEP* **2012**, *1207* 033.
4. Caldarelli, M. M. and Cognola, G. and Klemm, D. Thermodynamics of Kerr-Newman-AdS black holes and conformal field theories. *Class. Quant. Grav.* 2000, *17*, 399.
5. Kastor, D. , Ray, S. and Traschen, J. Enthalpy and the mechanics of AdS black holes. *Class. Quant. Grav.* **2009**, *26*, 195011.
6. Smarr, L. Mass formula for Kerr black holes. *Phys. Rev. Lett.* **1973**, *30*, 71.
7. Creighton, J. D. and Mann, R. B. Quasilocal thermodynamics of dilaton gravity coupled to gauge fields. *Phys. Rev. D* **1995**, *52*, 4569.
8. Padmanabhan, T. Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes. *Class. Quant. Grav.* **2002**, *19*, 5387.
9. Lovelock, D. The Einstein tensor and its generalizations. *J.Math.Phys.* **1971**, *12*, 498-501.
10. Gunasekaran, S. , Mann, R. B. and Kubiznak, D. Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization. *JHEP* **2012**, *1211*, 110.
11. Dolan, B. P. , Kostouki, A. , Kubiznak, D. and Mann, R. B. Isolated critical point from Lovelock gravity. *Class. Quant. Grav.* **2014**, *31*, 242001.

12. Altamirano, N. , Kubiznak, D. , Mann, R. B. and Sherkatghanad, Z. Kerr-AdS analogue of triple point and solid/liquid/gas phase transition. em Class. Quant. Grav. **2014**, *31*, 042001.
13. Kubiznak, D. , Mann, R. B. and Teo, M. Black hole chemistry: thermodynamics with Lambda. *Class. Quant. Grav.* **2017**, *34*, 063001.

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