Timescales for detecting magnetized white dwarfs in gravitational wave astronomy

Surajit Kalita
Banibrata Mukhopadhyay (IISc), Tushar Mondal (IISc)
& Tomasz Bulik (Warsaw)

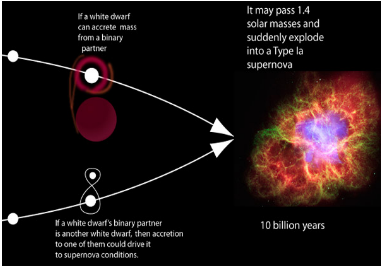
Indian Institute of Science, Bangalore

ApJ 896 (2020) 69; MNRAS 490 (2019) 2692

1st Electronic Conference on Universe
February $22 - 28$, 2021

Introduction: white dwarf and type Ia supernova

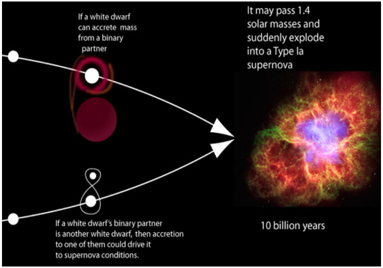
- If a progenitor star has mass $\lesssim (10 \pm 2) M_{\odot}$, at the end of its lifetime, it becomes a **white dwarf**.
- Inward gravitational force = force due to outward electron degeneracy pressure.



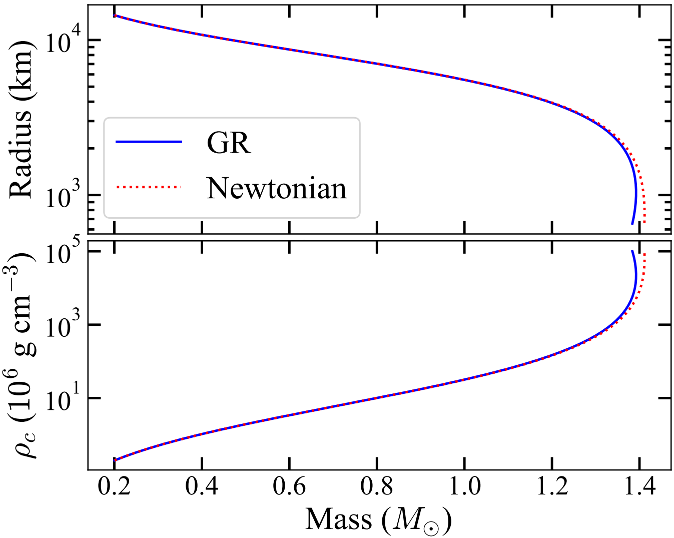
Introduction: white dwarf and type Ia supernova

- If a progenitor star has mass $\lesssim (10 \pm 2) M_\odot$, at the end of its lifetime, it becomes a **white dwarf**.
- Inward gravitational force = force due to outward electron degeneracy pressure.

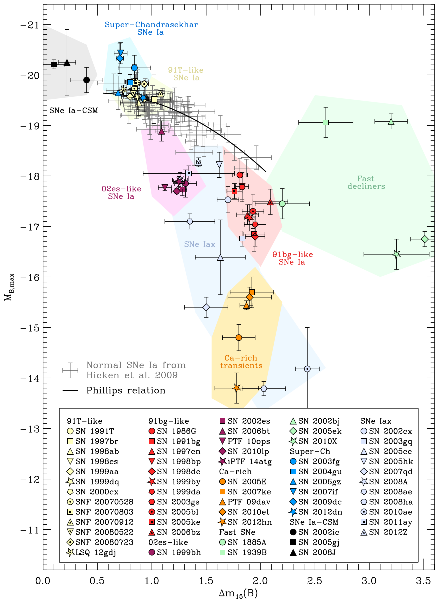
- If a white dwarf has a binary partner, it starts pulling matter out from the partner.
- At the **Chandrasekhar limit** ($\sim 1.4 M_\odot$ for a carbon-oxygen white dwarf), it burns out to produce type Ia supernova (SNIa).



Introduction: Chandrasekhar's theory



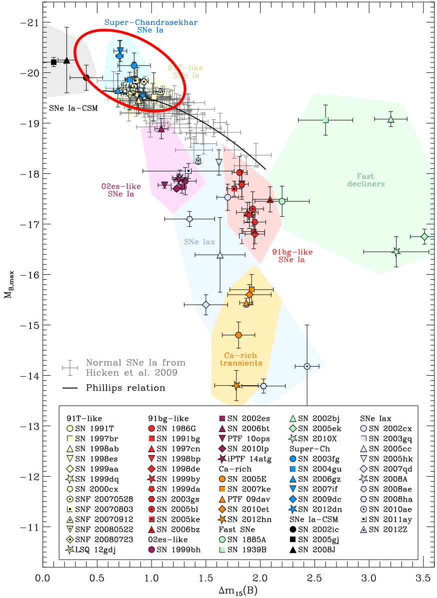
Peculiar type Ia supernovae



- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.

S. Taubenberger (2017)

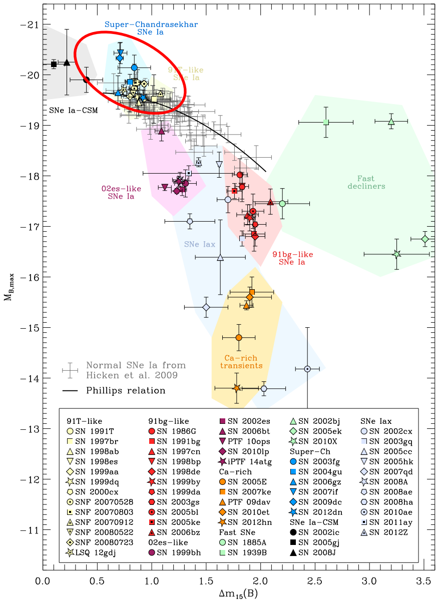
Peculiar type Ia supernovae



- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.

S. Taubenberger (2017)

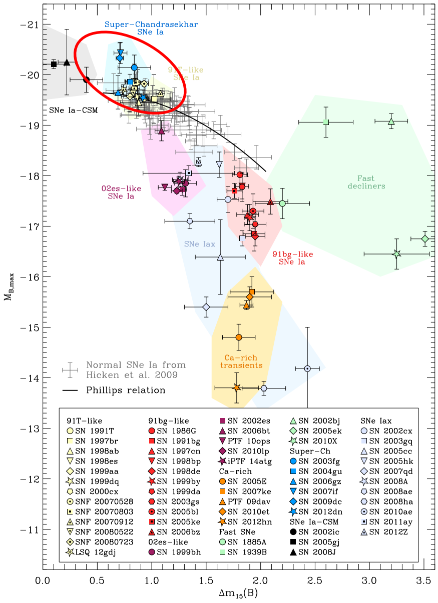
Peculiar type Ia supernovae



- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.
- $L \propto M_{WD}c^2 + mv^2 \implies M_{WD} \sim 2.1 - 2.8 M_\odot$.

S. Taubenberger (2017)

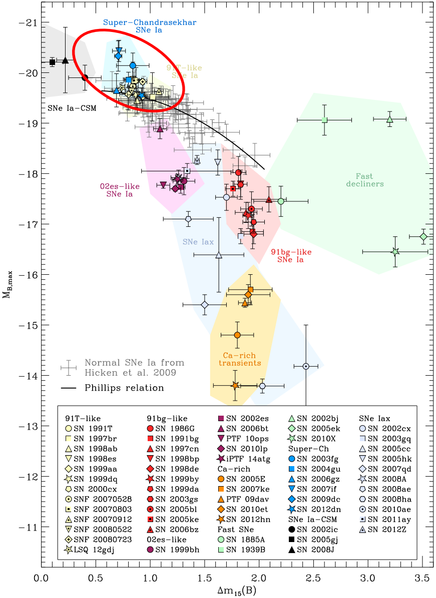
Peculiar type Ia supernovae



- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.
- $L \propto M_{\rm WD} c^2 + m v^2 \implies M_{\rm WD} \sim 2.1 - 2.8 M_\odot$.
- **Chandrasekhar mass limit is violated.**

S. Taubenberger (2017)

Peculiar type Ia supernovae

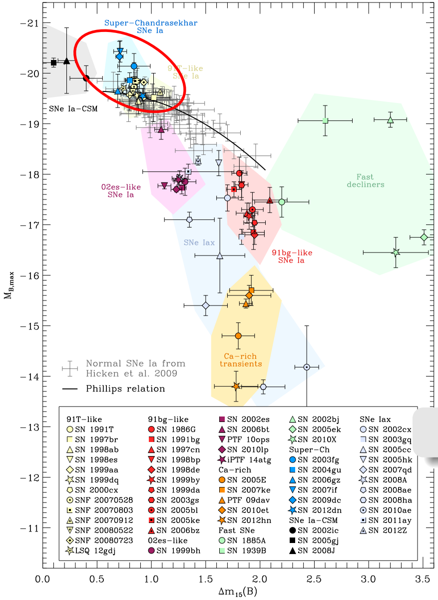


- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.
- $L \propto M_{\rm WD}c^2 + mv^2 \implies M_{\rm WD} \sim 2.1 - 2.8 M_\odot$.
- **Chandrasekhar mass limit is violated.**

  Rotation, magnetic field, modified theory of Einstein's gravity, massive gravity, etc.

S. Taubenberger (2017)

Peculiar type Ia supernovae



- Recent observations show some peculiar **SNeIa** with extremely **high luminosity**.
- $L \propto M_{\rm WD} c^2 + m v^2 \implies M_{\rm WD} \sim 2.1 - 2.8 M_\odot$.
- **Chandrasekhar mass limit is violated.**

  Rotation, magnetic field, modified theory of Einstein's gravity, massive gravity, etc.

Can we detect them directly?

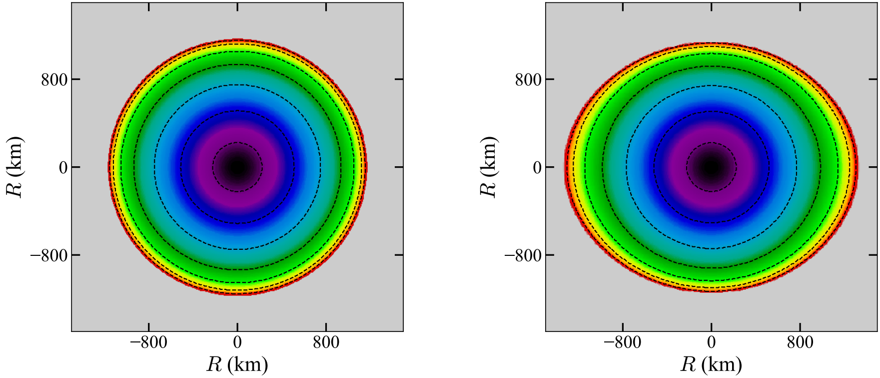
S. Taubenberger (2017)

Rotating white dwarfs

- Rotation can increase the mass of a WD.

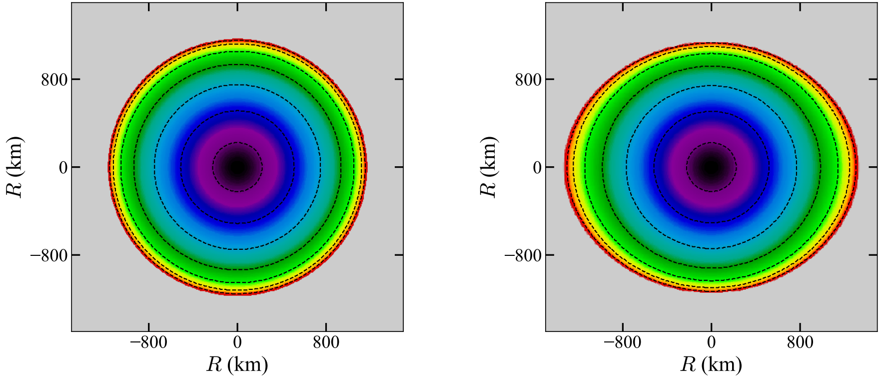
Rotating white dwarfs

- Rotation can increase the mass of a WD.
- Rotation turns a spherical WD to an oblate shaped WD.



Rotating white dwarfs

- Rotation can increase the mass of a WD.
- Rotation turns a spherical WD to an oblate shaped WD.



- Ostriker & Hartwick in 1968 showed that rotation alone can increase the mass of a WD up to $\sim 1.8 M_{\odot}$.

Magnetized WDs

Magnetic field has two effects:

1. Microscopic: Formation of Landau levels, etc.
2. Macroscopic: Shape, size, etc.

Magnetized WDs

Magnetic field has two effects:

1. Microscopic: Formation of Landau levels, etc.
2. Macroscopic: Shape, size, etc.

Target:

Direct detection of massive WDs

Magnetized WDs

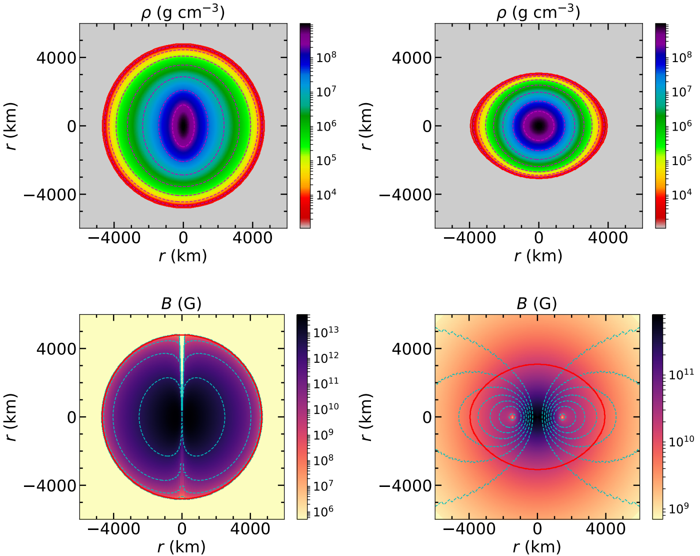
Magnetic field has two effects:

1. Microscopic: Formation of Landau levels, etc.
2. Macroscopic: Shape, size, etc.

Target:

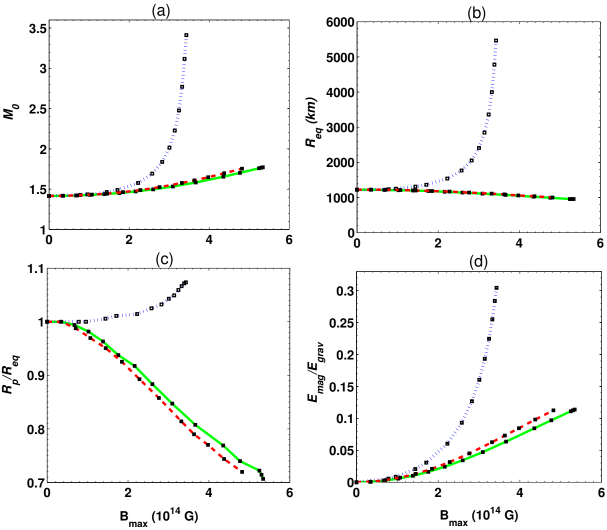
Direct detection of massive WDs

1. Exact mass-radius relation for WDs.
2. To invigilate the existence of limiting mass of WDs.
3. Possible corrections to the standard candle.



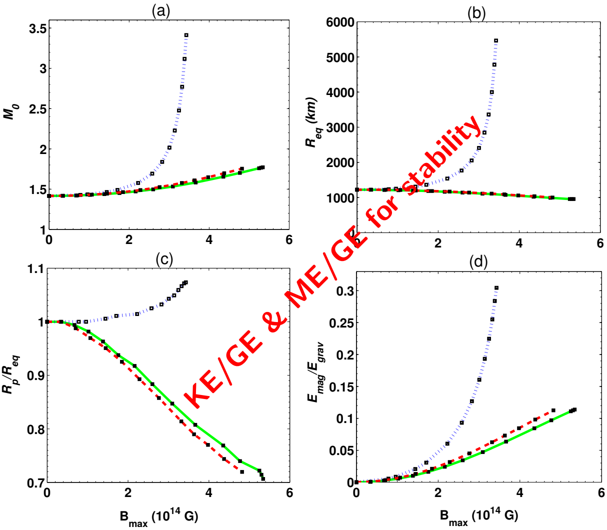
Toroidal magnetic field **Poloidal magnetic field**

Properties of magnetized WDs



Das & Mukhopadhyay, JCAP, 05 (2015) 016

Properties of magnetized WDs



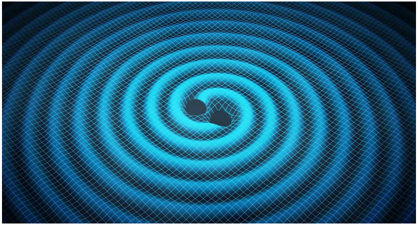
Das & Mukhopadhyay, JCAP, 05 (2015) 016

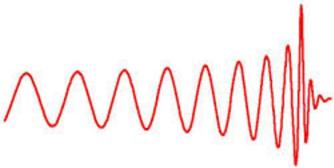
Introduction: gravitational wave

- Non-zero quadrupole moment $\implies$ **gravitational radiation**.

Introduction: gravitational wave

- Non-zero quadrupole moment $\Longrightarrow$ **gravitational radiation**.
- Gravitational wave from merger events have been detected.

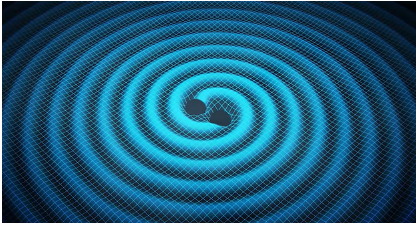


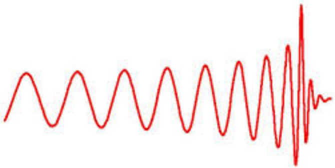


Google Image

Introduction: gravitational wave

- Non-zero quadrupole moment $\Longrightarrow$ **gravitational radiation**.
- Gravitational wave from merger events have been detected.



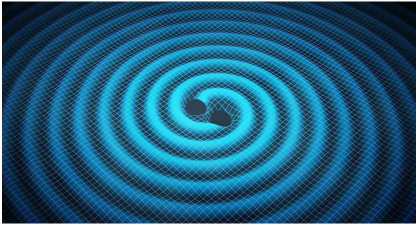


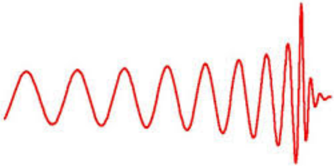
Google Image

- **Continuous gravitational wave (CGW)**: continuously emitted at certain frequency and amplitude.

Introduction: gravitational wave

- Non-zero quadrupole moment $\Longrightarrow$ **gravitational radiation**.
- Gravitational wave from merger events have been detected.

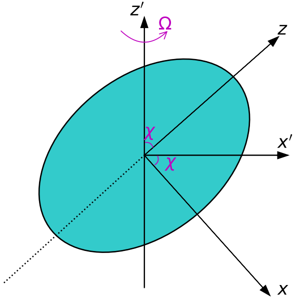




Google Image

- **Continuous gravitational wave (CGW)**: continuously emitted at certain frequency and amplitude.
- Rotating **white dwarfs** & **neutron stars** are prominent sources of CGW.

Deformed compact object



$$I_{nn} = I_{xx}\cos^2\alpha + I_{yy}\cos^2\beta + I_{zz}\cos^2\gamma$$

$$Q_{i'j'} = -I_{i'j'} + \tfrac{1}{3} I_{k'k'}\delta_{i'j'}$$

α, β, γ: direction cosines

GW amplitude

$$h_+ = h_0 \sin\chi \left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi\cos 2\Omega t\right],$$

$$h_\times = h_0 \sin\chi \left[\frac{1}{2}\sin i \cos\chi \sin\Omega t - \cos i \sin\chi \sin 2\Omega t\right],$$

$$h_0 = \frac{4G}{c^4}\frac{\Omega^2 |I_{zz} - I_{xx}|}{d}.$$

Deformed compact object

- Rotation $\Longleftrightarrow$ oblate.
  Toroidal magnetic field $\Longleftrightarrow$ prolate.
  Poloidal magnetic field $\Longleftrightarrow$ oblate.

Deformed compact object

- Rotation $\Longleftrightarrow$ oblate.
  Toroidal magnetic field $\Longleftrightarrow$ prolate.
  Poloidal magnetic field $\Longleftrightarrow$ oblate.
- *XNS* code is used *developed by Pili, Bucciantini & Del Zanna*.

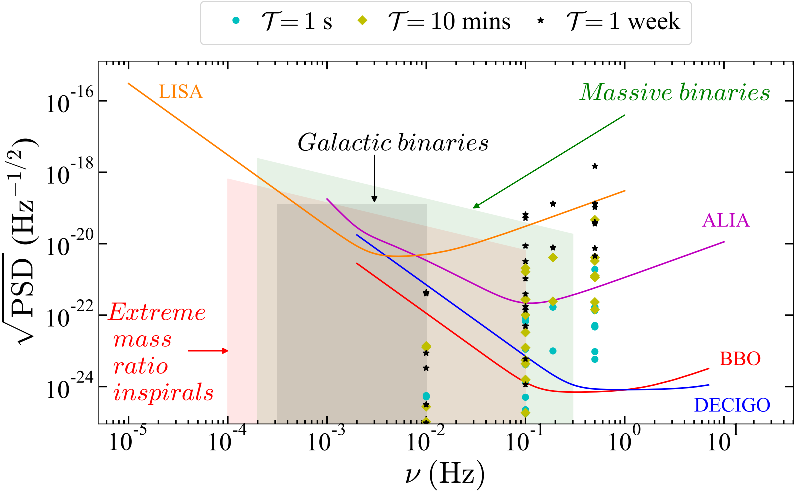
Deformed compact object

- Rotation $\Longleftrightarrow$ oblate.
  Toroidal magnetic field $\Longleftrightarrow$ prolate.
  Poloidal magnetic field $\Longleftrightarrow$ oblate.
- *XNS* code is used *developed by Pili, Bucciantini & Del Zanna*.
- **Advantages:** Toroidal/poloidal/mixed magnetic field with uniform/differential rotation.
- **Binary white dwarfs:**

$$h = 2.84 \times 10^{-22} \sqrt{\cos^4 i + 6\cos^2 i + 1} \left(\frac{M_c}{M_\odot}\right)^{5/3} \left(\frac{P_{orb}}{1\text{hr}}\right)^{-2/3} \left(\frac{d}{1\text{kpc}}\right)^{-1},$$

$$M_c = \left[\frac{m_1^3 m_2^3}{(m_1 + m_2)}\right]^{1/5}.$$

Gravitational radiation from magnetized WDs



$$d = 100\text{pc}, \qquad \chi = 30^\circ$$

GW amplitude

Dipole and quadrupolar radiation

- Pulsar like object can emit both dipole and quadrupolar radiation.

$$L_{\mathsf{D}} = \frac{B_p^2 R_p^6 \Omega^4}{2c^3} \sin^2\chi\ F(x_0),$$
$$L_{\mathsf{GW}} = \frac{2G}{5c^5}(I_{zz} - I_{xx})^2 \Omega^6 \sin^2\chi \left(1 + 15\sin^2\chi\right),$$

where $x_0 = R_0\Omega/c$ and $F(x_0) = \frac{x_0^4}{5\left(x_0^6 - 3x_0^4 + 36\right)} + \frac{1}{3\left(x_0^2+1\right)}$.

Dipole and quadrupolar radiation

- Pulsar like object can emit both dipole and quadrupolar radiation.

$$L_{\mathsf{D}} = \frac{B_p^2 R_p^6 \Omega^4}{2c^3} \sin^2\chi \ F(x_0),$$
$$L_{\mathsf{GW}} = \frac{2G}{5c^5}(I_{zz} - I_{xx})^2 \Omega^6 \sin^2\chi \left(1 + 15\sin^2\chi\right),$$

where $x_0 = R_0\Omega/c$ and $F(x_0) = \frac{x_0^4}{5\left(x_0^6 - 3x_0^4 + 36\right)} + \frac{1}{3\left(x_0^2+1\right)}$.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -L_{\mathsf{D}} - L_{\mathsf{GW}}.$$

Dipole and quadrupolar radiation

Energy conservation

$$\frac{\mathrm{d}(\Omega I_{z'z'})}{\mathrm{d}t} = -\frac{2G}{5c^5}\left(I_{zz} - I_{xx}\right)^2 \Omega^5 \sin^2\chi \left(1 + 15\sin^2\chi\right) - \frac{B_p^2 R_p^6 \Omega^3}{2c^3} \sin^2\chi \; F(x_0)$$

Dipole and quadrupolar radiation

Energy conservation

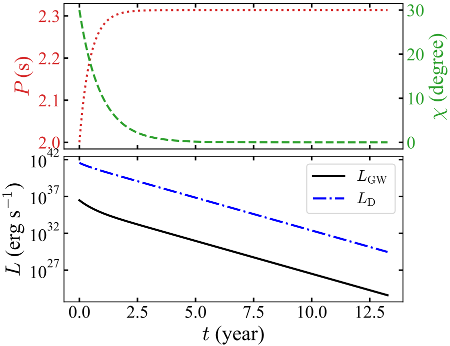
$$\frac{\mathrm{d}(\Omega I_{z'z'})}{\mathrm{d}t} = -\frac{2G}{5c^5}\left(I_{zz} - I_{xx}\right)^2 \Omega^5 \sin^2\chi \left(1 + 15\sin^2\chi\right) - \frac{B_p^2 R_p^6 \Omega^3}{2c^3} \sin^2\chi \; F(x_0)$$

Angular momentum conservation

$$I_{z'z'} \frac{\mathrm{d}\chi}{\mathrm{d}t} = -\frac{12G}{5c^5}\left(I_{zz} - I_{xx}\right)^2 \Omega^4 \sin^3\chi \cos\chi - \frac{B_p^2 R_p^6 \Omega^2}{2c^3} \sin\chi \cos\chi \; F(x_0)$$

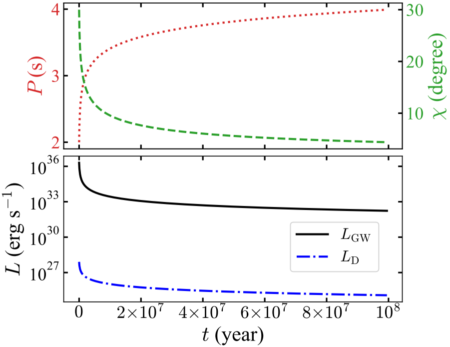
Timescales for dipole and quadrupolar radiation

$L_{\mathsf{D}} \gg L_{\mathsf{GW}}$



$$\Omega = \frac{\sqrt{3}}{2\cos\chi}\Omega_0$$

$L_{\mathsf{GW}} \gg L_{\mathsf{D}}$



$$\Omega = \frac{3^8 \sin\chi}{2^{15}\cos^{16}\chi}\Omega_0$$

Kalita et al. ApJ, 896 (2020) 69

Timescales for dipole and quadrupolar radiation

- For **strong poloidal fields**, a pulsar cannot emit dipole and quadrupolar radiations for a long time as χ becomes zero fast.

Timescales for dipole and quadrupolar radiation

- For **strong poloidal fields**, a pulsar cannot emit dipole and quadrupolar radiations for a long time as χ becomes zero fast.
- For **strong toroidal fields**, a pulsar can emit only quadrupolar radiation for a long time as χ and Ω decreases slowly.

Timescales for dipole and quadrupolar radiation

- For **strong poloidal fields**, a pulsar cannot emit dipole and quadrupolar radiations for a long time as χ becomes zero fast.
- For **strong toroidal fields**, a pulsar can emit only quadrupolar radiation for a long time as χ and Ω decreases slowly.
- The **birth rate of WD** is $\sim 10^{-12}$ pc^{-3} yr^{-1} $\implies$ which means within a 100 pc radius, on average, only one WD is formed in 10^6 yr.

Timescales for dipole and quadrupolar radiation

- For **strong poloidal fields**, a pulsar cannot emit dipole and quadrupolar radiations for a long time as χ becomes zero fast.
- For **strong toroidal fields**, a pulsar can emit only quadrupolar radiation for a long time as χ and Ω decreases slowly.
- The **birth rate of WD** is $\sim 10^{-12}$ pc^{-3} yr^{-1} $\implies$ which means within a 100 pc radius, on average, only one WD is formed in 10^6 yr.
- $\mathrm{SNR} = \frac{1}{\sqrt{5}}\sqrt{\frac{\mathcal{T}}{\mathtt{S}(\nu)}}h$

Timescales for dipole and quadrupolar radiation

- For **strong poloidal fields**, a pulsar cannot emit dipole and quadrupolar radiations for a long time as χ becomes zero fast.
- For **strong toroidal fields**, a pulsar can emit only quadrupolar radiation for a long time as χ and Ω decreases slowly.
- The **birth rate of WD** is $\sim 10^{-12}$ pc^{-3} yr^{-1} $\implies$ which means within a 100 pc radius, on average, only one WD is formed in 10^6 yr.
- $\mathrm{SNR} = \frac{1}{\sqrt{5}}\sqrt{\frac{\mathcal{T}}{\mathtt{S}(\nu)}}h$
- If the WD is super-Chandrasekhar with very less surface field, it should be able to be detected by the GW detectors with significant SNR.

Conclusions

- Magnetic field and rotation deform as well as increase the mass of compact objects.
- If the magnetic field and rotation axes are not aligned, the object can emit gravitational radiation.
- The gravitational radiation from isolated magnetized WDs can be detected in future with new detectors (e.g. LISA, DECIGO, ET etc.).
- Super-Chandrasekhar white dwarfs (and also massive neutron stars) can be detected directly.
- They can be detected for a long time depending on the geometry and strength of the magnetic field.

Conclusions

- Magnetic field and rotation deform as well as increase the mass of compact objects.
- If the magnetic field and rotation axes are not aligned, the object can emit gravitational radiation.
- The gravitational radiation from isolated magnetized WDs can be detected in future with new detectors (e.g. LISA, DECIGO, ET etc.).
- Super-Chandrasekhar white dwarfs (and also massive neutron stars) can be detected directly.
- They can be detected for a long time depending on the geometry and strength of the magnetic field.