

Differentially rotating relativistic stars beyond the j -constant law

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Motivation: Why equilibrium modelling?

- Numerical simulations are the primary weapons of choice to study complex phenomena, such as BNS mergers and their remnants.
- Performing realistic simulations comes at a price of several thousand core-hours.
- A thorough exploration of a problem's parameter space can become challenging.

see: Shibata et al (2005), Hotokezaka et al (2011), Sekiguchi et al (2011), Bauswein & Janka (2012), Bauswein et al (2012), Hotokezaka et al (2013), Bernuzzi et al (2014), Dietrich et al (2015), De Pietri et al (2016), Radice et al (2018), and many more ...

- Ignore some aspects of the realistic configuration (e.g. non-axisymmetric deformations, oscillations, time-dependence, thermal structure), to obtain a simplified model.
- What do we get in return?
 - allows for faster and wider exploration of the parameter space
 - configurations can be used as initial data for dynamical simulations
 - provides important insights on stellar stability

The line element for a stationary, axisymmetric star in full GR is

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{\gamma-\rho} r^2 \sin^2 \theta (d\phi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2)$$

where γ , ρ , ω and μ are metric functions depending only on the coordinates r and θ .

see: *Friedman & Stergioulas (2013)*, *Paschalidis & Stergioulas (2017)*

- Start from an initial guess for the metric potentials γ , ρ , ω , μ , the energy density ϵ and the angular velocity Ω .
- Use the first integral of the hydrostationary equilibrium and the specified EOS to obtain an updated matter distribution.
- Update the γ , ρ , ω and μ distributions.
- Resume steps until convergence to a solution is achieved.

see: *Komatsu et al (1989)*, *Cook et al (1992)*, *Nozawa et al (1998)*

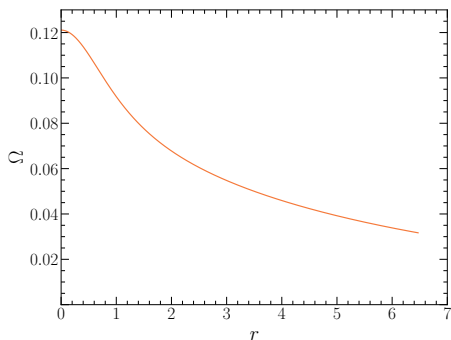
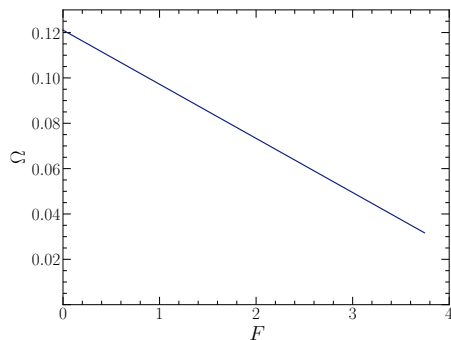
- A simple, computationally convenient, differential rotation law

$$F(\Omega) = A^2(\Omega_c - \Omega)$$

- A is a positive constant that determines the length scale over which the angular velocity Ω changes within the star.
- The KEH law is reduced to uniform rotation for $A \rightarrow \infty$ and to the j -constant law for $A \rightarrow 0$.

see: *Ansorg et al (2009)*, *Espino & Paschalidis (2019)*, *Espino et al (2019)*

KEH rotation law

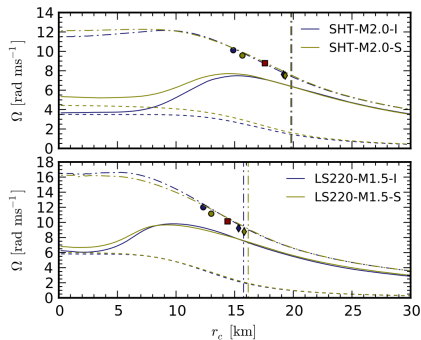


- The KEH law is widely used in the literature.
- The problem? For the case of a BNS merger remnant, the rotational profile doesn't look like this...

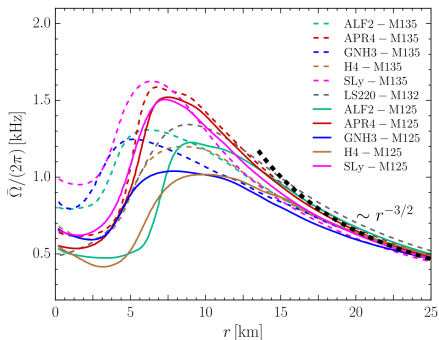
$\Omega(r)$ rotational profile from simulations

But like this...

Kastaun & Galeazzi (2015)



Hanauske et al (2017)



$\Omega(r)$ rotational profile from simulations

- The remnant's rotational profile has been studied extensively in a variety of setups.
- The rotational profile exhibits a maximum away from center and modest EOS dependence.

see: Kastaun & Galeazzi (2015), Bauswein & Stergioulas (2015), Kastaun et al (2016), Endrizzi et al (2016), Kastaun et al (2017), Ciolfi et al (2017), Hanauske et al (2017), Endrizzi et al (2018), Kiuchi et al (2018), Ciolfi et al (2019), East et al (2019), De Pietri et al (2020), ...

- Galeazzi et al (2012)
- Uryu et al (2016)
- Uryu et al (2017)
- Bauswein & Stergioulas (2017)
- Bozzola et al (2017)

- A new 4-parameter differential rotation law was proposed in Uryu et al (2017):

$$\Omega = \Omega_c \frac{1 + \left(\frac{F}{B^2 \Omega_c} \right)^p}{1 + \left(\frac{F}{A^2 \Omega_c} \right)^{q+p}}$$

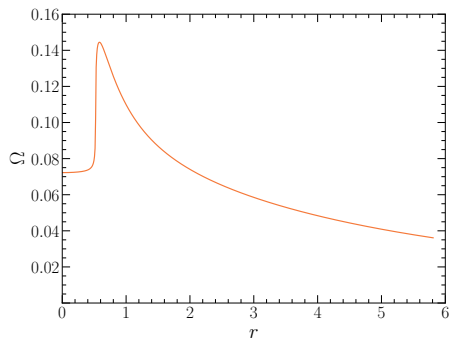
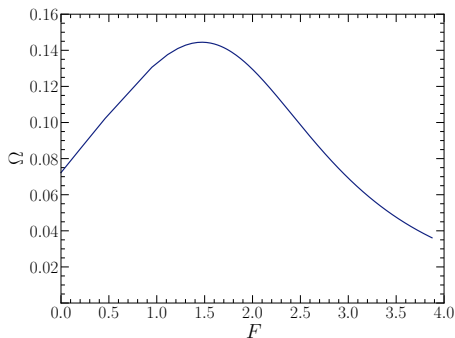
- Uryu et al presented only a few models in their work.
- This is the first systematic study of equilibrium models with the new rotation law.
- For details: *Iosif & Stergioulas (2020), arXiv:2011.10612*

Preparing comparison with KEH law

- We choose a cold, polytropic EOS with $N=1$ and $K=100$ (for direct comparison with models constructed with previous rotation law)
- We use dimensionless units of $c = G = M_{\odot} = 1$.
- We expanded the RNS code (that implements the KEH scheme with modifications from Cook+ 1992) for the case of the new Uryu+ law.
- RNS code: <https://gitlab.com/niksterg/rns1-1>

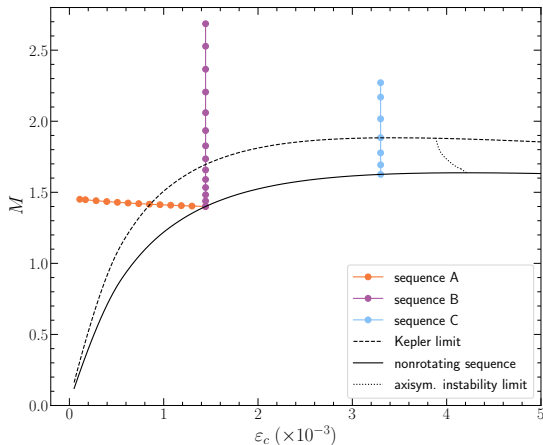
see: *Stergioulas et al (2004)*, *Iosif & Stergioulas (2014)*, *Stergioulas & Friedman (1995)*

What does this new law look like?



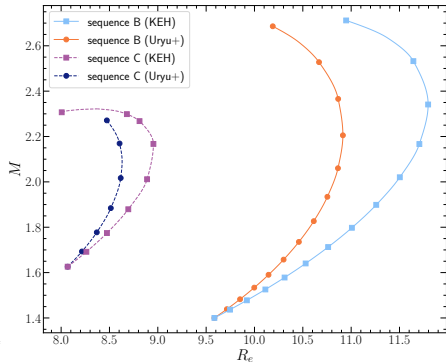
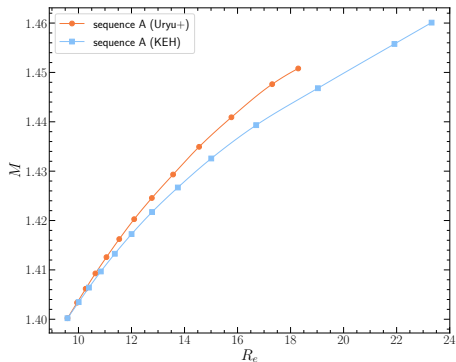
Angular velocity Ω profiles in the equatorial plane
(model C6, axis ratio $r_p/r_e = 0.43$)

Equilibrium sequences, polytropic EOS ($N=1$)



- Sequence A: constant rest mass $M_0 = 1.506$
- Sequence B: constant central energy density $\epsilon_c = 1.444 \times 10^{-3}$
- Sequence C: constant central energy density $\epsilon_c = 3.3 \times 10^{-3}$

Uryu+ vs KEH law: Mass vs circumferential radius R_e



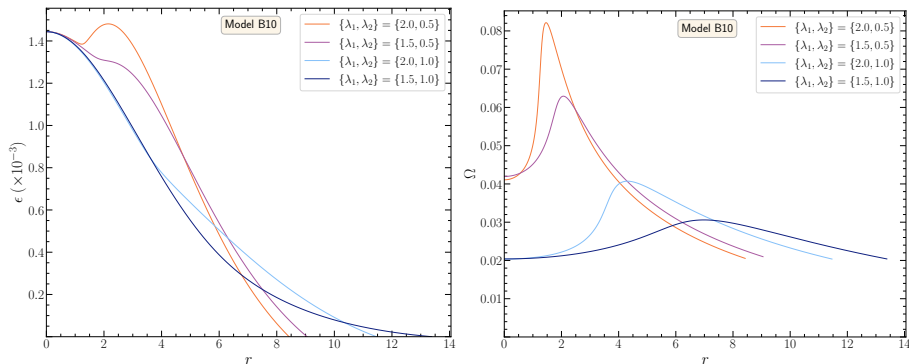
- Small influence of rotation law choice on the masses.
- Ω_e found to be smaller for Uryu+ models than for KEH models.
- This results in a weaker centrifugal force and thus in a smaller radius.

- We employed $p = 1$ and $q = 3$. Generalization to arbitrary non-integer $\{p, q\}$ values is not straightforward.
- Parameters A and B in the Uryu+ rotation law can be calculated indirectly by fixing the ratios:

$$\lambda_1 = \frac{\Omega_{\max}}{\Omega_c} \quad , \quad \lambda_2 = \frac{\Omega_e}{\Omega_c}$$

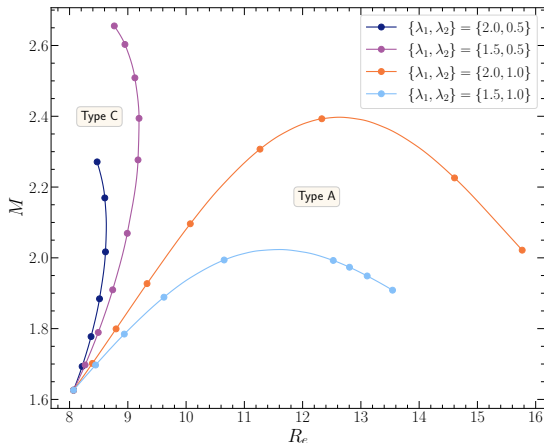
- Solve the system of equations for λ_1 and λ_2 to determine values for A and B in each iteration.

Effect of parameters λ_1, λ_2



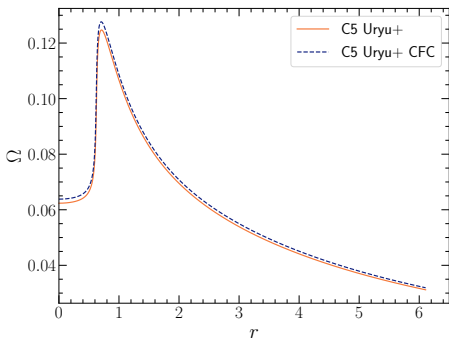
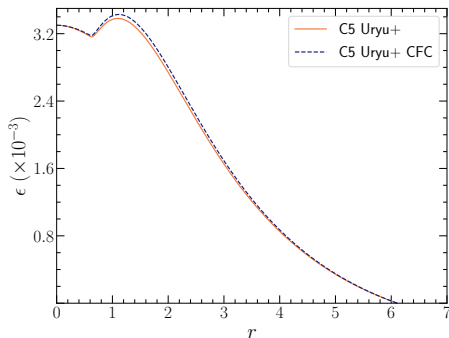
Energy density $\epsilon(r)$ and angular velocity $\Omega(r)$ profiles on the equatorial plane

Effect of parameters λ_1, λ_2



- Sequence C variations with different $\{\lambda_1, \lambda_2\}$ choices.
- Type A and type C solutions according to the classification of Ansorg+ 2009 are identified with the new Uryu+ law.

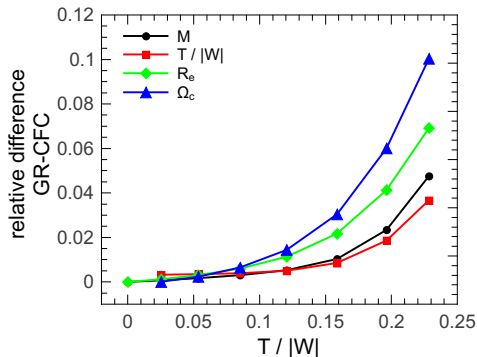
GR vs IWM-CFC configurations



- For models relevant to merger remnants the IWM-CFC approximation maintains an acceptable accuracy.
- Relative errors for local quantities are up to $\sim 2.5\%$ and $\sim 1\%$ for the masses and the ratio $T/|W|$.

see: *Isenberg (2008)*, *Wilson et al (1996)*, *Stergioulas et al (2004)*, *Iosif & Stergioulas (2014)*

Accuracy of the IWM-CFC approximation



- Relative errors between full GR and IWM-CFC approximation for sequence C, calculated with the KEH rotation law ($\hat{A} = 1$).
- Values at $T/|W| \sim 0.16$ correspond to an axis ratio $r_p/r_e = 0.5$.
- Uryu+ law case consistent with KEH law picture.

see: *Iosif & Stergioulas (2014)*

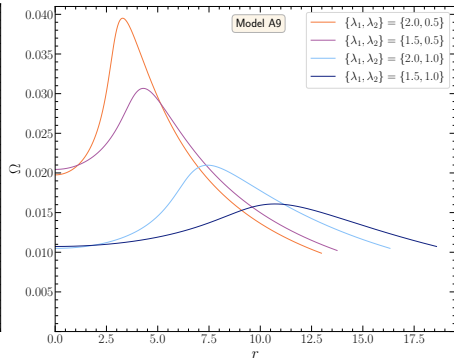
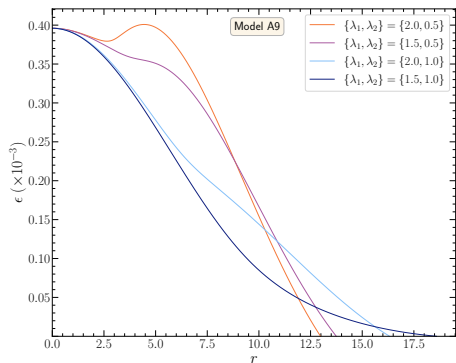
- There has been a shortage of reference models constructed with the newly proposed Uryu+ rotation laws. Our study partially fills this gap, for the case of the new 4-parameter rotation law.
- The versatility of the new law allows us to construct models that have a similar rotational profile and axis ratio as observed for merger remnants, while at the same time being quasi-spherical.
- Through the ratios λ_1 and λ_2 , one can easily take advantage of information provided from numerical simulations, in order to fine-tune the parameter values of the rotation law.

- tabulated EOS
- hot EOS
- T and Ω profiles taken directly from simulations
- perform time evolutions, study oscillations and dynamical collapse

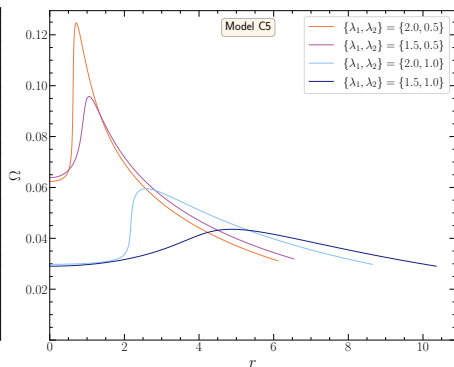
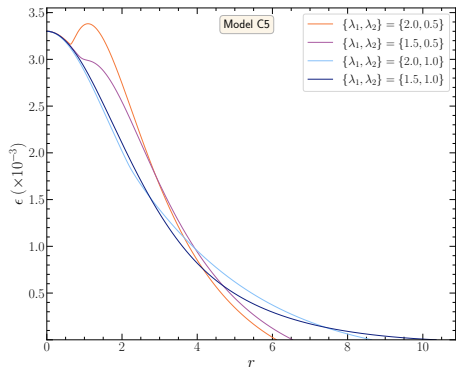
related recent works: Zhou et al (2019), Passamonti & Andersson (2020), Xie et al (2020), Camelio et al (2020)

extra information, definitions and brief technical details

Effect of parameters λ_1, λ_2 : low density



Effect of parameters λ_1, λ_2 : high density



- F denotes the gravitationally redshifted angular momentum per unit rest mass and enthalpy

$$F = u^t u_\phi = \frac{v^2}{(1 - v^2)(\Omega - \omega)} = \frac{(\Omega - \omega) r^2 \sin^2 \theta e^{-2\rho}}{\left[1 - (\Omega - \omega)^2 r^2 \sin^2 \theta e^{-2\rho}\right]}$$

- $j = hu_\phi$ is the specific angular momentum, i.e. the angular momentum per unit baryon mass, consistent with the integral expression of the total angular momentum $J = \int j dM_0$.
- j is conserved along an axisymmetric flow, i.e. its Lie derivative vanishes, $\mathcal{L}_u(hu_\phi) = 0$.

A comment on the integrability condition

- Consider the first integral of the hydrostationary equilibrium:

$$H - \ln u^t + \int_{\Omega_{\text{pole}}}^{\Omega} F(\Omega') d\Omega' = \text{constant}$$

- In the case of the transient compact remnant formed after a BNS merger, the numerically extracted $\Omega(r)$ profile in the equatorial plane, implies that $F(\Omega)$ is not a one-to-one function.
- This means: instead of $F = F(\Omega)$, we need to consider $\Omega = \Omega(F)$.

$$\int F d\Omega = \int F \frac{d\Omega}{dF} dF$$

A (very) brief look on the IWM-CFC approximation

- Assume conformal flatness for the spatial metric

$$\gamma_{ij} = \psi^4 \eta_{ij}$$

where ψ is a conformal factor and η_{ij} is the flat metric.

- The line element in the 3 + 1 formalism of GR

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

where α is the lapse function and β^i is the shift vector.

see: *Isenberg (2008), Wilson et al (1996)*

- Consider an stationary, axisymmetric star, in spherical-like coordinates.
- In the absence of meridional circulation, β^ϕ is the only non-zero component of the shift vector β^i .

- line element in IWM-CFC form

$$ds^2 = -\alpha^2 dt^2 + \psi^4(dr^2 + r^2 d\theta^2) + \psi^4 r^2 \sin^2 \theta (d\phi + \beta^\phi dt)^2$$

- line element in full GR

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{\gamma-\rho} r^2 \sin^2 \theta (d\phi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2)$$

- compare the above to get

$$\alpha = e^{(\gamma+\rho)/2}, \quad \psi = e^{\mu/2} = e^{(\gamma-\rho)/4}, \quad \beta^\phi = -\omega$$

- The line element in full GR can take the IWM-CFC form, if we set:

$$\mu = \frac{\gamma - \rho}{2}$$

- Get a numerical code (such as RNS), that already solves for the full GR metric.
- Convert it to CFC by imposing the above condition instead of solving for the metric potential μ .

see: *Stergioulas & Friedman (1995)*, *Stergioulas et al (2004)*, *Iosif & Stergioulas (2014)*