“Null String” Gas Cosmology: 1st steps.

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Introduction

This work is devoted to the study of the asymptotics of the gravitational field of primary particles with nonzero rest mass.

These particles are structurally composed of two closed “null strings” (thin closed tubes of a massless scalar field) in the shape of a circle, and they are formed in a gas of null strings as a result of gravitational interaction.

There is ample evidence that visible matter and detectable radiation make up only a small fraction of the mass of the universe, perhaps only a few percent.

Therefore, an urgent problem is to search for physical systems (models) for which the presence of dark matter is an intrinsic property.

One of such physical systems can be a “null string” gas (gas of thin tubes of a massless scalar field).

In this model, dark matter can be formed by extremely numerous and spatially diverse structures with nonzero rest mass, which are formed in a null string gas as a result of gravitational interaction.

It is important to note that the “lifetime” of such structures in the gas can be extremely short, but their number and permanent formation of new ones can make a considerable contribution to the overall mass.

An interesting problem is the study of the gravitational field of such structures, in particular, the study of the gravitational field of two gravitationally interacting null strings in the form of a circle (primary particles with nonzero rest mass), the trajectories of which are shown in Figure 1 and Figure 2.

These examples differ in the location in space of the regions within which oscillations of gravitationally interacting null strings occur.

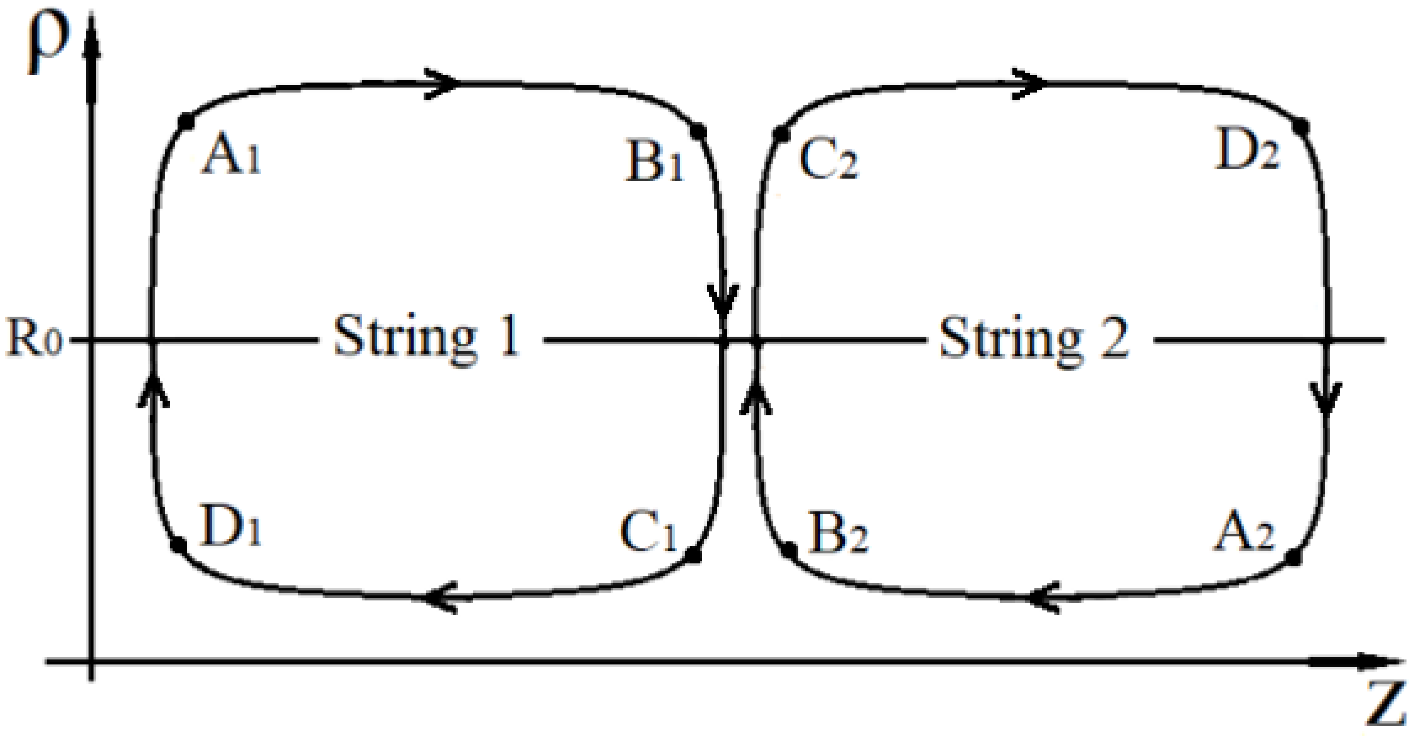

Figure 1. The figure shows, qualitatively, the trajectories of motion of two gravitationally interacting null strings, the meeting surface for which is orthogonal to the ρ axis ($\rho = R_0$).

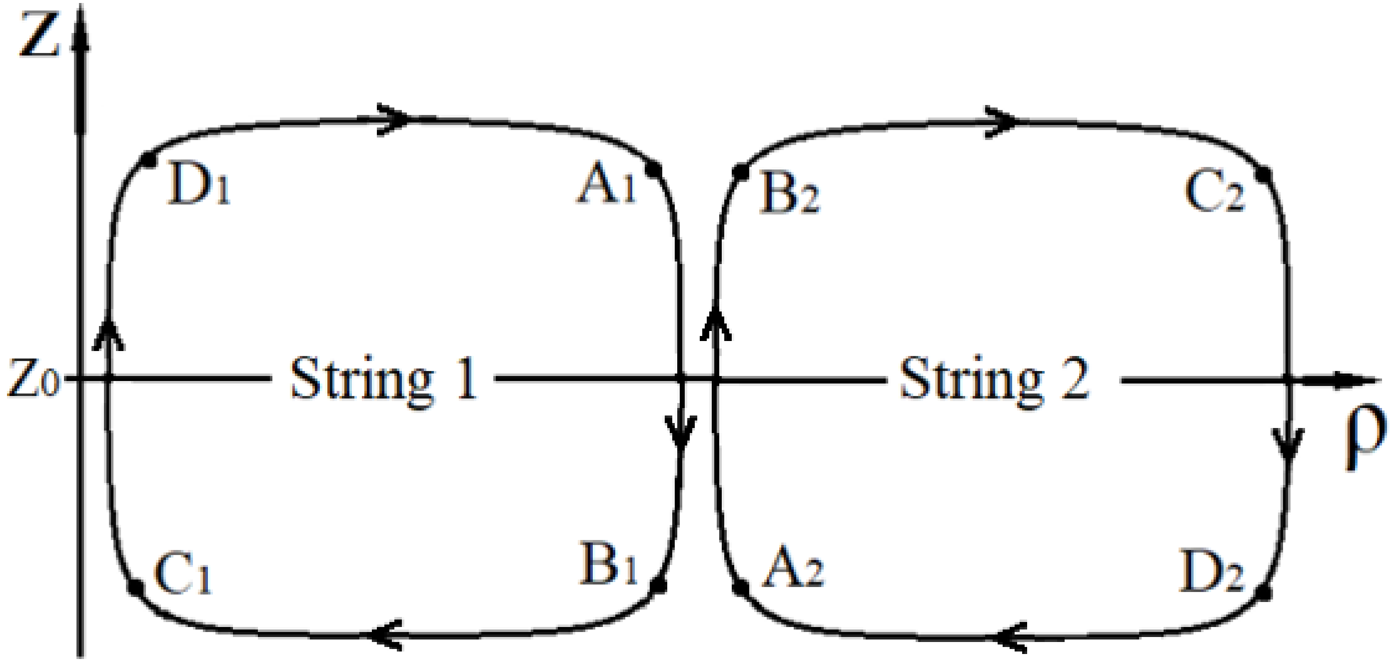

Figure 2. The figure shows, qualitatively, the trajectories of motion of two gravitationally interacting null strings, the meeting surface for which is orthogonal to the z axis ($z = z_0$).

For the cases shown in Fig. 1 and Fig. 2, two gravitationally interacting closed null strings have the shape of a circle, they are located in parallel planes and move “towards” each other.

The direction of movement for each null string is specified by a sequence of points: A_l; B_l; C_l; D_l; A_l; …, where index l denotes the number of the null string and takes the values: $1; 2$. The points on the trajectories, which are designated by the same letter (not dependent on indices) determine the position in space of each null string for the same value of the variable t.

Construction of the quadratic form

When constructing a quadratic form describing the field of two gravitationally interacting null strings, we can assume that the trajectory of motion of each of the interacting null strings consists of four segments (see Fig. 1, and Fig. 2).

Moreover, on the segments $A_l B_l$ and $C_l D_l$ the null string moves with a constant radius. On the segments $B_l C_l$ and $D_l A_l$, the null string radially decreases or increases its size (radius).

If a closed null string, on a certain segment, moves along the z axis without changing the size (radius), then its "trajectory" of motion (world surface) is determined by the equalities

$$t = t_0 + \tau,\ \rho = R_0,\ \theta = \sigma,\ z = z_0 \mp \tau,\ \tau \in (\tau_0; \tau_1),\ \sigma \in [0; 2\pi], \qquad (1)$$

where τ and σ are parameters on the world surface of the null string, τ_0 and τ_1 constants that determine the value of the τ parameter on the boundaries of the segment, R_0 null string radius, the constants t_0 and z_0, respectively, determine the initial time value and the initial position of the null string on the z axis. The case $z = z_0 - \tau$ describes the motion of a closed null string in the negative direction of the z axis, and the case $z = z_0 + \tau$ describes the motion in the positive direction of the z axis.

If, on some segment, a closed null string radially increases or decreases its size (radius) while being on the surface $z = z_0$, then the “trajectory” of motion is determined by the equalities

$$t = t_0' + \tau,\ \rho = R_0 \mp \tau,\ \theta = \sigma,\ z = z_0,\ \tau \in (\tau_0'; \tau_1'),\ \sigma \in [0; 2\pi], \tag{2}$$

where the constants t_0' and R_0, respectively, determine the initial time value and the initial radius of the null string. The case $\rho = R_0 - \tau$ describes radial compression, and the case $\rho = R_0 + \tau$ describes the radial expansion of a closed null string.

It can be noted that on time scales much larger than the time of one full cycle of oscillation of the null string (or at distances for which the geometric dimensions of the particles shown in Fig. 1 and Fig. 2 can be neglected), the functions of the required quadratic form must be invariant under the simultaneous inversion $t \to -t$, $z \to -z$, i.e.,

$$g_{mn}(t, \rho, z) = g_{mn}(-t, \rho, -z) . \tag{3}$$

Using (3), the quadratic form for the problem being solved can be represented as

$$dS^2 = e^{2\nu}(dt)^2 - A(d\rho)^2 - B(d\theta)^2 - e^{2\mu}(dz)^2 , \tag{4}$$

where ν, μ, A, B are functions of the variables t, ρ, z .

Null string motion

The null string motion in a pseudo-Riemannian space-time is determined by the set of equations

$$x^{\alpha}{}_{,\tau\tau} + \Gamma^{\alpha}_{pq} x^{p}{}_{,\tau} x^{q}{}_{,\tau} = 0\,, \qquad (5)$$

$$g_{\alpha\beta} x^{\alpha}{}_{,\tau} x^{\beta}{}_{,\tau} = 0, \quad g_{\alpha\beta} x^{\alpha}{}_{,\tau} x^{\beta}{}_{,\sigma} = 0\,, \qquad (6)$$

where $g_{\alpha\beta}$ and Γ^{α}_{pq} are respectively the metric tensor and the Christoffel symbols of the external space-time, $x^{p}{}_{,\tau} = \partial x^{p} / \partial \tau$, $x^{\beta}{}_{,\sigma} = \partial x^{\beta} / \partial \sigma$, the indices α, β, p, q take values 0, 1, 2, 3, and the functions $x^{\alpha}(\tau, \sigma)$ determine the motion trajectory (the world surface) of the null string.

Since the trajectories (1) and (2) simulate the motion of gravitationally interacting null strings, on the corresponding segments of the trajectories shown in Fig. 1, and Fig. 2, then the functions $x^{\alpha}(\tau,\sigma)$, $\alpha = 0,...,3$, must be particular solutions of the equations of motion of a null string.

In this case, the analysis of the equations of motion can give additional restrictions on the functions of the quadratic form (4).

For trajectories (1), equations (6) lead to the equality

$$e^{2\nu} \equiv e^{2\mu}, \qquad (7)$$

and for trajectories (2), to the equality

$$e^{2\nu} \equiv A. \qquad (8)$$

For (7), (8), the quadratic form (4) takes the form

$$dS^2 = e^{2\nu}\left((dt)^2 - (d\rho)^2 - (dz)^2\right) - B(d\theta)^2, \qquad (9)$$

where ν, B are functions of the variables t, ρ, z.

Note that on each of the segments: $A_l B_l$, $B_l C_l$, $C_l D_l$, $D_l A_l$, $l = 1, 2$ (see Fig. 1, Fig. 2), the null strings forming the primary particle move towards each other (in opposite directions). Then, the functions defining the trajectories of the interacting null strings, for each of the segments, must simultaneously satisfy the equations of motion (6).

For the case of motion of a closed null string of constant radius in the negative direction of the z axis, equations of motion (5), for (1), (9), lead to the only equation

$$\nu_{,t} - \nu_{,z} = 0\,. \tag{10}$$

For the case of motion of a closed null string of constant radius in the positive direction of the z axis, equations of motion (5), for (1), (9), lead to the equation

$$\nu_{,t} + \nu_{,z} = 0\,. \tag{11}$$

The joint solution of equations (10), (11) is

$$\nu = \nu(\rho)\,. \tag{12}$$

Equations of motion (5), taking into account (2), for the case of radial compression and radial expansion of a closed null string located in the plane lead to the equations

$$\nu_{,t} - \nu_{,\rho} = 0 \,, \tag{13}$$

$$\nu_{,t} + \nu_{,\rho} = 0 \,. \tag{14}$$

The joint solution of equations (13), (14) is

$$\nu = \nu(z) \,. \tag{15}$$

The consequence of the equalities (12), (15) is $\nu = \nu_0 = const$. Without loss of generality, we can fix the value of the constant $\nu_0 = 0$, then we finally get

$$e^{2\nu} = 1 \,. \tag{16}$$

Einstein's equations solution

The components of the energy-momentum tensor for a null string moving in a pseudo-Riemannian space-time are determined by the equalities

$$T^{mn}\sqrt{-g} = \gamma \int d\tau d\sigma x^m{}_{,\tau} x^n{}_{,\tau} \delta^4 \left(x^l - x^l(\tau,\sigma) \right), \qquad (17)$$

where indexes m, n, l take values $0,...,3$, functions $x^l(\tau,\sigma)$ define a trajectory of a null string motion (world surface), $g = |g_{mn}|$, g_{mn} is the metric tensor of external space-time, $\gamma = const.$

According to (17) outside the null string, i.e. in the region where $x^l \neq x^l(\tau,\sigma)$, all components of the null string energy-momentum tensor are identically zero. In this work, we investigate the asymptotics of the gravitational field of primary particles (i.e., we are looking for a solution in empty space surrounding the primary particle).

Then, without loss of generality, we can assume that all components of the energy-momentum tensor of gravitationally interacting null strings in the investigated region are equal to zero.

In the investigated region of space-time, the Einstein system of equations, for the quadratic form (9), (16), can be represented as

$$\left(\frac{B_{,t}}{B}\right)_{,t}+\frac{1}{2}\left(\frac{B_{,t}}{B}\right)^2=0,\ \left(\frac{B_{,z}}{B}\right)_{,z}+\frac{1}{2}\left(\frac{B_{,z}}{B}\right)^2=0,\ \left(\frac{B_{,\rho}}{B}\right)_{,\rho}+\frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2=0, \quad (18)$$

$$\left(\frac{B_{,z}}{B}\right)_{,t}+\frac{1}{2}\frac{B_{,t}}{B}\frac{B_{,z}}{B}=0,\ \left(\frac{B_{,\rho}}{B}\right)_{,t}+\frac{1}{2}\frac{B_{,t}}{B}\frac{B_{,\rho}}{B}=0,\ \left(\frac{B_{,\rho}}{B}\right)_{,z}+\frac{1}{2}\frac{B_{,\rho}}{B}\frac{B_{,z}}{B}=0. \quad (19)$$

The solution to equations (18), (19) can be represented as

$$B = (\beta_1 t + \beta_2 z + \beta_3 \rho)^2, \tag{20}$$

where β_1, β_2, and β_3 are constants.

Since the function B must be invariant under the simultaneous inversion of $t \to -t$, $z \to -z$, the consequence of (4) is

$$\beta_1 = \beta_2 = 0. \tag{21}$$

Also, without loss of generality, you can fix the value of the constant

$$\beta_3 = 1. \tag{22}$$

For constant values (21), (22), we find

$$B = \rho^2. \tag{23}$$

Applying (16), (23), for (9), we obtain the expression for the required quadratic form

$$dS^2 = dt^2 - d\rho^2 - \rho^2 d\theta^2 - dz^2. \tag{24}$$

Conclusions

It should be noted once again that the found solution (24) should be considered only as the asymptotics of the gravitational field of particles structurally consisting of gravitationally interacting null strings. It is incorrect to say that null strings forming particles, the trajectories of which are shown in Fig. 1, and Fig. 2, move in the Minkowski space-time, because when constructing the solution (24) we applied a number of approximations:

- The trajectory of motion of each of the interacting null strings (see Fig. 1, and Fig. 2) consists of four segments. On two of these segments, the null string moves with a constant radius, and on two more segments, the null string radially decreases or radially increases its size (radius). This approximation allowed us to obtain the equalities (7), (8), and (16). However, this approximation can be considered correct only at distances much larger than the size of the region inside which oscillations of interacting null strings occur;

- Symmetry of metric functions with respect to the simultaneous inversion $t \to -t$, $z \to -z$, the application of which made it possible to reduce the quadratic form to the form (5), can be considered correct only on time scales much larger than the time of one full cycle of oscillation of null strings forming a particle, or at distances much larger than the dimensions of the region inside which oscillations of interacting null strings occur.

It is important to note that in the case when the observation time scale is comparable to the time of one complete cycle of oscillation of the null strings that form the primary particle. Or in the case when the size of the study region is comparable to the size of the region inside which the oscillations of the null strings that form the primary particle occur. Then, in these cases, in a gas of null strings, significant deviations of the gravitational field from the flat Minkowski space-time should be observed.

It seems interesting to consider a structured massless scalar field (a gas of thin closed tubes) as the initial matter of the Universe. With such a choice, it may become possible to reduce all known types of interaction to various realizations of curvature (gravitational) interaction between elements (formed structures) of such a gas. In fact, it becomes possible to combine the concepts of "substance" and "field". The advantages of such a model include its four-dimensionality, as well as the obvious quantum properties of "macro" structures and "macro" processes that can be observed in such a gas. In essence, a future theory describing processes in a gas of thin tubes of a massless scalar field can be considered as a possible example of a hybrid theory. Since this theory should be partly quantum (for "macro" structures and "macro" processes) and partly classical (motion of the "null strings" forming a gas).

Thanks for your attention