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Reconstruction of models with variable cosmological parameter in $f(R,T)$ theory

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Abstract: The standard ΛCDM model is reasonably successful in describing the universe, and is the most widely acceptable model in cosmology. However, there are several theoretical issues, such as the initial singularity, the cosmological constant problem, the particle nature of dark matter, the existence of anomalies in the cosmic microwave background radiation and on small scales, the predictions and tests of the inflationary scenario, and whether general relativity is valid on the largest possible scales. Hence there is growing interest in looking at modified theories. In this presentation, a reconstruction is made of the Friedmann-Lemaitre-Robertson-Walker models with a dynamic cosmological parameter in f(R,T) modified gravity. This theory has a number of pleasing features, such as the avoidance of the initial big-bang singularity and a variable cosmological parameter. A dynamic cosmological parameter, which arises naturally in this theory, can solve the cosmological constant problem, and is also a candidate for dark energy. In addition, a variable cosmological parameter fits observations better than the standard ΛCDM model. The model exhibits a transition from deceleration to acceleration. The time evolution of the physical parameters such as energy density, pressure and equation of state are analyzed.

Keywords: Dark Energy; Variable cosmological parameter; f(R,T) gravity

1. Introduction

The most widely accepted theory to study the evolution of the universe is undoubtedly Einstein's general theory of relativity, which predicts that the universe was condensed into a very small, hot and dense state initially, and then expanded. In recent times, observations of type Ia supernova indicate that the current rate of expansion of the universe is accelerating. After this, many observations have supported the idea of an accelerated expansion [1–4]. In order to explain this acceleration, a new form of energy has been postulated that has a repulsive effect called dark energy (DE) [5–7]. According to the Planck mission team's best estimate [6], the universe is made up of three forms of matter/energy, viz., 68.5% DE, 26.6% dark matter (DM) and only 4.9% baryonic matter.

The most favoured explanation for DE is Einstein's cosmological constant which is obtained by adding a cosmological constant to the equations of Einstein's standard Friedmann-Lemaitre-Robertson-Walker (FLRW) equations, and leads to the ΛCDM model [6]. This has a solution that includes accelerated expansion. Despite the fine tuning and coincidence problems in the ΛCDM model, it is widely accepted as the best solution to the DE problem [8]. While this DE is constant, another consideration is a changing field, in which the equation of state (EOS) varies, viz., quintessence, phantom DE and scalar field models. For excellent reviews of DE, we refer to [8,9].

In another direction, the reason for the acceleration of the universe can be sought in modified theories of gravitation. One of these is $f(R,T)$ gravity [10], where R is the Ricci scalar and T is the trace of the energy momentum tensor. This theory allows for an explanation of accelerated expansion without DE, and for an avoidance of the initial singularity. A very interesting feature of the theory is that, in some sense, is may be thought of as general relativity with a modified matter part, thus allowing for a dynamic cosmological parameter [11]. This allows for the possibility of solving the cosmological constant problem. In this paper, we reconstruct the Friedman-Lemaitre-Robertson-Walker (FLRW) model in $f(R,T)$ theory, focussing on a variable cosmological parameter. This correspondence has not been fully studied. We find a model that exhibits a transition from deceleration in the past, to current acceleration. The behaviour of the physcial parameters is analysed, as are the energy conditions. The cosmological parameter varies from being large at early times, and decreases to a small value today.

The introduction should briefly place the study in a broad context and highlight why it is important. It should define the purpose of the work and its significance. The current state of the research field should be reviewed carefully and key publications cited. Please highlight controversial and diverging hypotheses when necessary. Finally, briefly mention the main aim of the work and highlight the principal conclusions. As far as possible, please keep the introduction comprehensible to scientists outside your particular field of research.

2. Basic Equations

In $f(R,T)$ gravity, the action is:

$$S = \int \left(\frac{1}{16\pi G} f(R,T) + S_m \right) \sqrt{-g} dx^4. \tag{1}$$

We choose [10]

$$f(R,T) = f_1(R) + f_2(T), \tag{2}$$

i.e., a sum of two independent functions of R and T, respectively. With this condition, equation (1) can be written as

$$f_1'(R)R_{ij} - \frac{1}{2}(f_1(R) + f_2(T))g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_1'(R) = 8\pi T_{ij} - f_2'(T)T_{ij} - f_2'(T)\Theta_{ij}, \tag{3}$$

where $\Box \equiv \nabla^i\nabla_j$ is the D'Alembertian operator. The tensor Θ_{ij} is defined as

$$\Theta_{ij} = g^{lm}\frac{\delta T_{lm}}{\delta g^{ij}}. \tag{4}$$

A prime denotes a derivative with respect to its argument.

In particular [12], we assume the forms

$$f_1(R) = \lambda_1 R, \;\; f_2(T) = \lambda_2 T \tag{5}$$

where λ_1 and λ_2 are arbitrary coupling constants of $f(R,T)$ gravity. We take the matter content in the universe to be a perfect fluid with energy momentum tensor:

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{6}$$

where ρ and p are energy density and pressure of the fluid, respectively, and u^i is four velocity vector satisfying the condition $u^i u_i = 1$. Then equation (4) yields:

$$\Theta_{ij} = -2T_{ij} - pg_{ij}, \tag{7}$$

Using equations (4)-(7), equation (3) becomes:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right) T_{ij} + \frac{\lambda_2}{\lambda_1}(p + \frac{1}{2}T)g_{ij}. \tag{8}$$

51 Note that this equation reduces to the general relativistic limit when we put $\lambda_1 = 1, \lambda_2 = 0$.

Now, in the general theory of relativity, the Einstein field equations with cosmological constant are (in units $G = c = 1$):

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + \Lambda g_{ij}, \tag{9}$$

We notice a similarity between equations (8) and (9). Hence, we may set:

$$\Lambda \equiv \Lambda(T) = \frac{\lambda_2}{\lambda_1}(p + \frac{1}{2}T), \tag{10}$$

which leads naturally to a varying cosmological parameter Λ as a function of T. Evaluating the trace T from equation (6), we obtain:

$$\Lambda = \frac{\lambda_2}{2\lambda_1}(\rho - p). \tag{11}$$

We consider the flat FLRW metric:

$$ds^2 = dt^2 - \Sigma_{i=1}^{3} a^2(t)(dx_i^2), \tag{12}$$

where $a(t)$ is the scale factor. The Hubble and deceleration parameters are defined by, respectively,:

$$H = \frac{\dot{a}}{a}, \qquad q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{13}$$

With the metric (12), the field equations (8) become, with the aid of equation (10):

$$3H^2 = \left[\left(\frac{8\pi + \lambda_2}{\lambda_1}\right) + \frac{\lambda_2}{2\lambda_1}\right]\rho - \frac{\lambda_2}{2\lambda_1}p, \tag{14}$$

$$2\dot{H} + 3H^2 = -\left[\left(\frac{8\pi + \lambda_2}{\lambda_1}\right) + \frac{\lambda_2}{2\lambda_1}\right]p + \frac{\lambda_2}{2\lambda_1}\rho, \tag{15}$$

We obtain the energy density and pressure from the above two equations as

$$\rho = \frac{\lambda_1}{(8\pi + 2\lambda_2)}\left[3 + \frac{\lambda_2}{(8\pi + \lambda_2)}(q+1)\right]H^2, \tag{16}$$

$$p = \frac{\lambda_1}{(8\pi + 2\lambda_2)}\left[-3 + \frac{(16\pi + 3\lambda_2)}{(8\pi + \lambda_2)}(q+1)\right]H^2, \tag{17}$$

and then, the cosmological parameter from equation (10) as:

$$\Lambda = \left[\frac{3\lambda_2}{(8\pi + 2\lambda_2)} - \frac{\lambda_2}{(8\pi + 2\lambda_2)}(q+1)\right]H^2. \tag{18}$$

The equation of state parameter ($\omega = p/\rho$) is given by

$$\omega = -1 + \frac{(16\pi + 4\lambda_2)(q+1)}{(24\pi + 4\lambda_2) + \lambda_2 q}. \tag{19}$$

3. Solution to field equations

To solve the field equations, we assume the following condition for q [13]:

$$q = -1 + \beta H, \tag{20}$$

where β is a constant. From the definitions of H and q, equation (13), we can write the solution as:

$$a = exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right], \quad H = \frac{1}{\sqrt{2\beta t + k}}, \quad q = -1 + \frac{\beta}{\sqrt{2\beta t + k}} \tag{21}$$

We note from equation (21) that q is positive at early times, corresponding to deceleration, and negative at late times, corresponding to acceleration.

We now use observations to estimate the values of the constant β, and the integration constant k. If we evaluate equation (20) at the present time, we have

$$q_0 = -1 + \beta H_0, \tag{22}$$

Observations tell us that $q_0 = -0.51$ [14] and that $H_0 = 75.35\ kms^{-1}Mpc^{-1}$ [15]. Substituting these values into equation (22), we get $\beta = 0.0065$. From equation (21), we then get the value of k as $k = 0.000175$. Now, how do we select appropriate values for λ_1 and λ_2? Since the general relativistic limit is given by $\lambda_1 \to 1, \lambda_2 \to 0$, we choose $\lambda_1 = 0.9$ and $\lambda_2 = 0.00016$ to plot all subsequent figures. These values of β, k, λ_1 and λ_2 are used in plotting all figures.

From the observational point of view, it is more useful to express the parameters in term of redshift z. The average scale factor a in terms of redshift is given by

$$a(z) = \frac{a_0}{1+z}. \tag{23}$$

From equation (21), we get

$$\frac{1}{\beta}\sqrt{2\beta t + k} = ln(a), \tag{24}$$

and from equation (23), we get

$$ln(a) = ln(a_0) - ln(1+z). \tag{25}$$

Hence, the Hubble and deceleration parameters in terms of redshift are as follows:

$$H(z) = \frac{1}{\beta[ln(a_0) - ln(1+z)]}, \tag{26}$$

$$q(z) = -1 + \frac{1}{[ln(a_0) - ln(1+z)]}. \tag{27}$$

We now plot the deceleration parameter q against the redshift z:

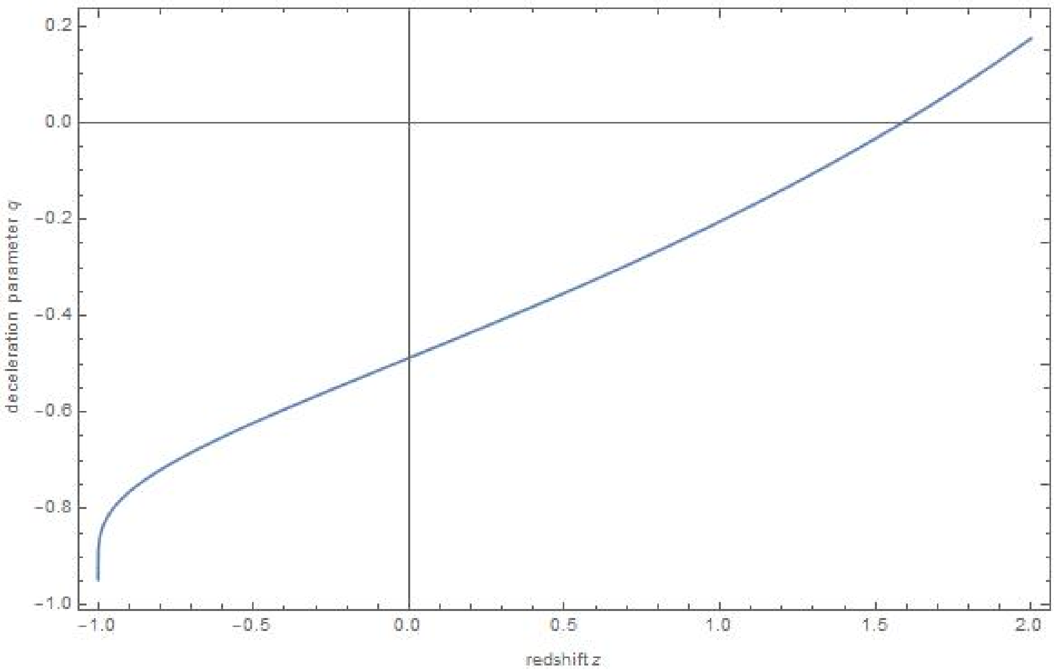


Figure 1: Deceleration parameter q against redshift z

Using the $t - z$ relationship, we get the energy density, pressure, cosmological parameter and equation of state parameters as follows:

$$\rho = \frac{\lambda_1}{(8\pi + 2\lambda_2)} \left[3 + \frac{\lambda_2}{(8\pi + \lambda_2)[ln(a_0) - ln(1+z)]}\right] \frac{1}{\beta^2[ln(a_0) - ln(1+z)]^2}, \tag{28}$$

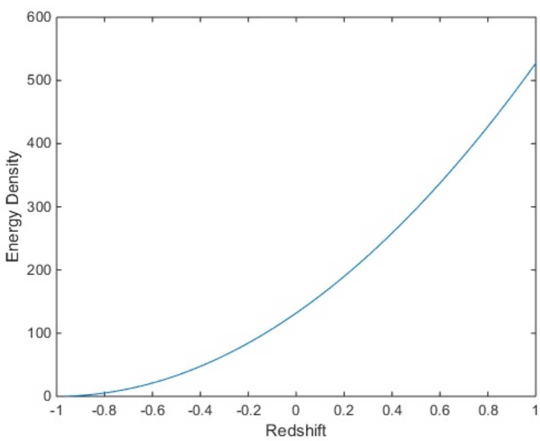
$$p = \frac{\lambda_1}{(8\pi + 2\lambda_2)} \left[-3 + \frac{(16\pi + 3\lambda_2)}{(8\pi + \lambda_2)[ln(a_0) - ln(1+z)]}\right] \frac{1}{\beta^2[ln(a_0) - ln(1+z)]^2}, \tag{29}$$

$$\Lambda = \left[\frac{3\lambda_2}{(8\pi + 2\lambda_2)} - \frac{\lambda_2}{(8\pi + 2\lambda_2)[ln(a_0) - ln(1+z)]}\right] \frac{1}{\beta^2[ln(a_0) - ln(1+z)]^2}, \tag{30}$$

and

$$\omega = -1 + \frac{(16\pi + 4\lambda_2)}{3(8\pi + \lambda_2)[ln(a_0) - ln(1+z)] + \lambda_2}. \tag{31}$$

We can plot these quantities as follows:



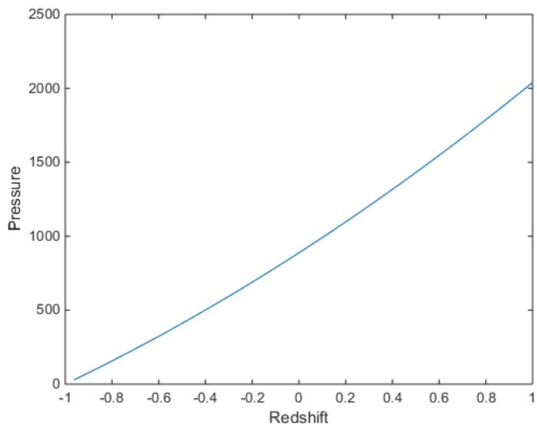
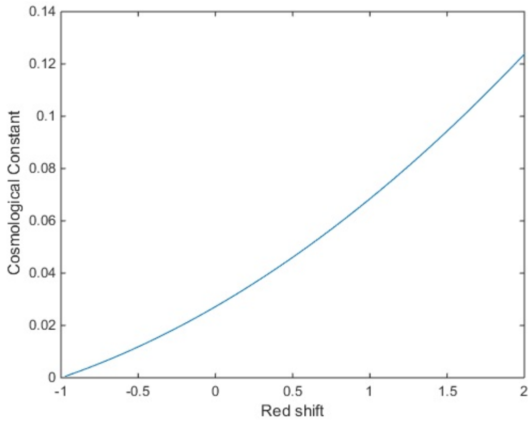


Fig 2: Energy density against redshift z. **Fig 3:** Pressure against redshift z.



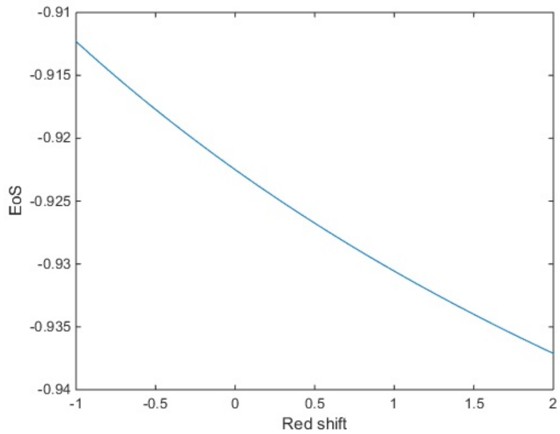


Fig 4: Cosmological parameter against redshift z.

Fig 5: EOS parameter against redshift z

4. Conclusions

In this paper, we have analysed a model with variable cosmological parameter in flat FLRW space-time in $f(R,T)$ theory. We graphed the various physical parameters against redshift. The vital characteristics of our model are the following. Our model was decelerating in the past, and is currently accelerating as can be seen from figure 1. The behaviour of the parameters q, ρ, p, ω and Λ are illustrated by means of figures 2-5. We have a variable cosmological parameter, which is large initially, and decreases with time. This could help in resolving the cosmological constant problem.

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