Dynamics of Anisotropic Cylindrical Collapse In Energy-Momentum Squared Gravity

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Stellar Structures

Dynamics of Stars

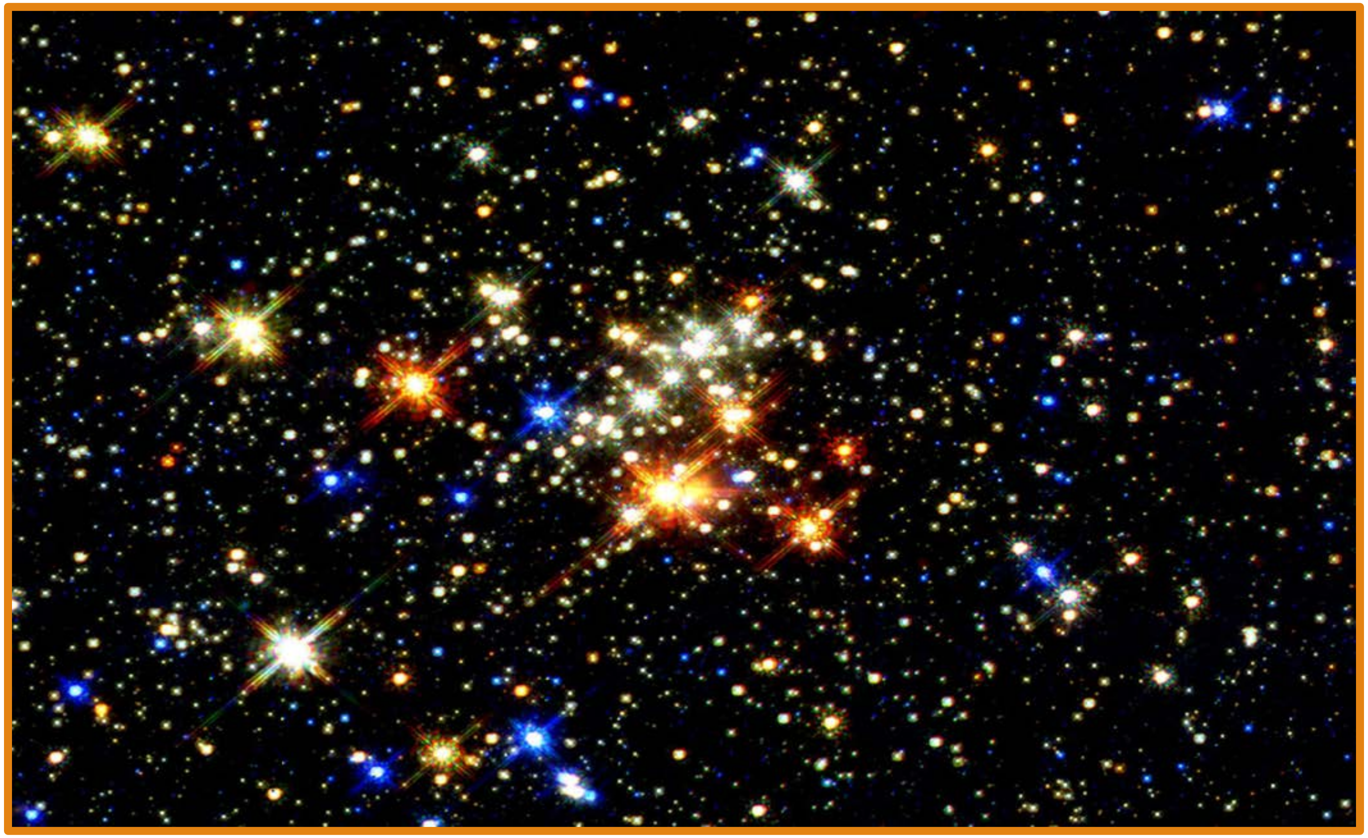
Theory of Gravitation

Modified Theories

Research Work

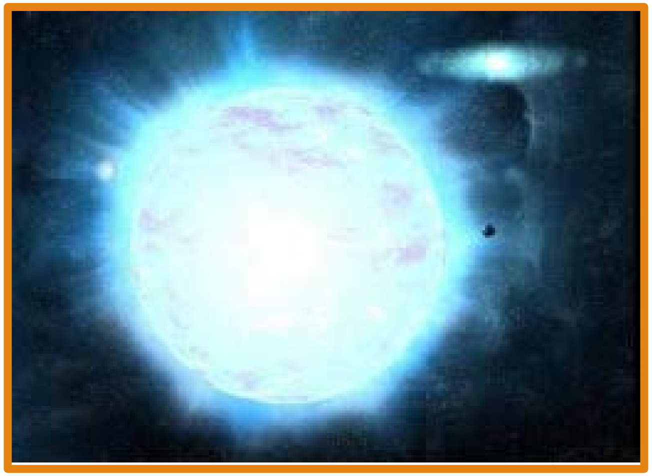
Concluding Remarks

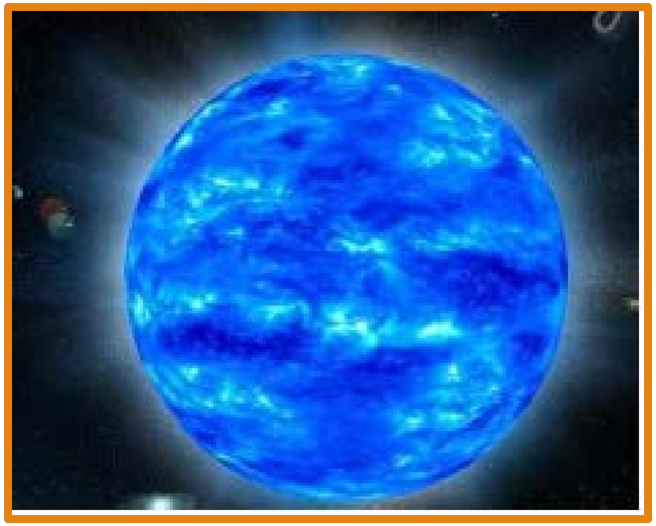
Stellar Structures



❖ Bounded together by their own gravity.

Temperature of Stars





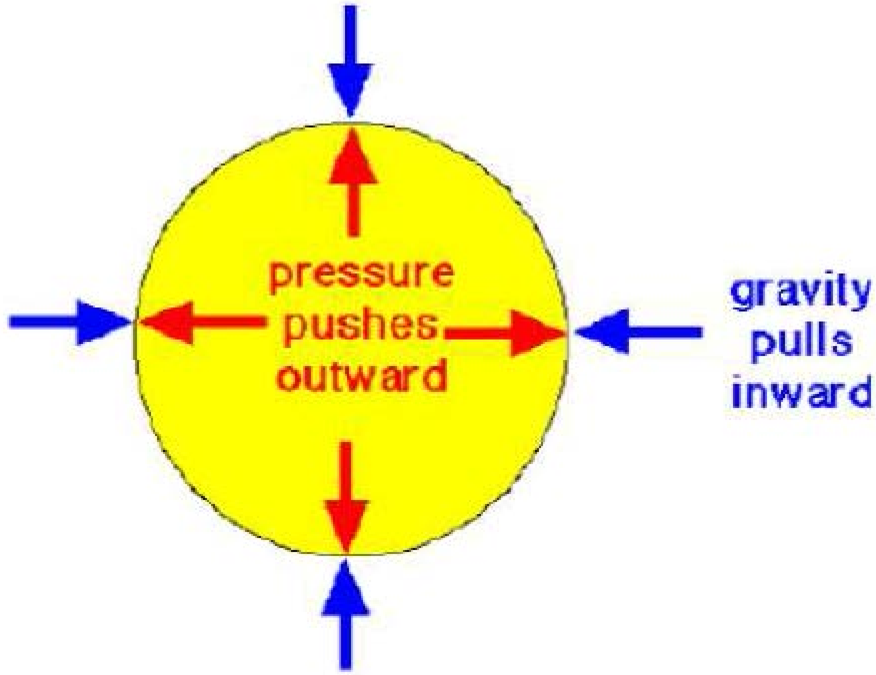
Surface temperatures in °C	Colour of the star	
30000 – 60000	Blue	
10000 – 30000	Blue white	
7500 – 10000	White	
6000 – 7500	Yellow white	
5000 – 6000	Yellow	
3500 – 5000	Orange	
< 3500	Red	

- Formation of Stars
- Stability of Stars
- Collapse of Stars
- Final Outcomes

Formation of Stars

❖ Stars are formed in clouds of dust and gasses known as nebulae.

Stability of Stars

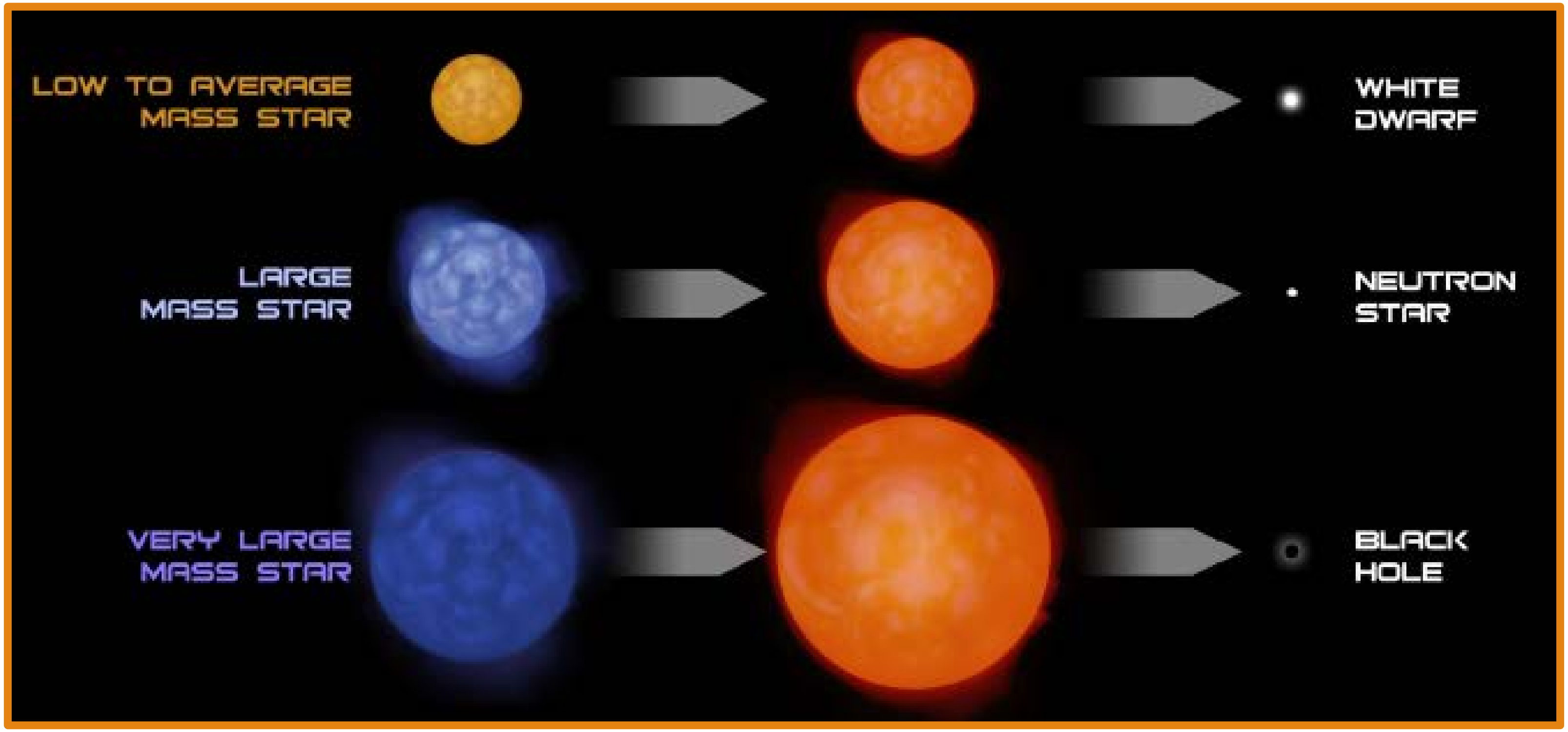


❖ A balance between pressure and gravity.

Gravitational Collapse

❖ Contraction of astronomical object due to its own gravity.

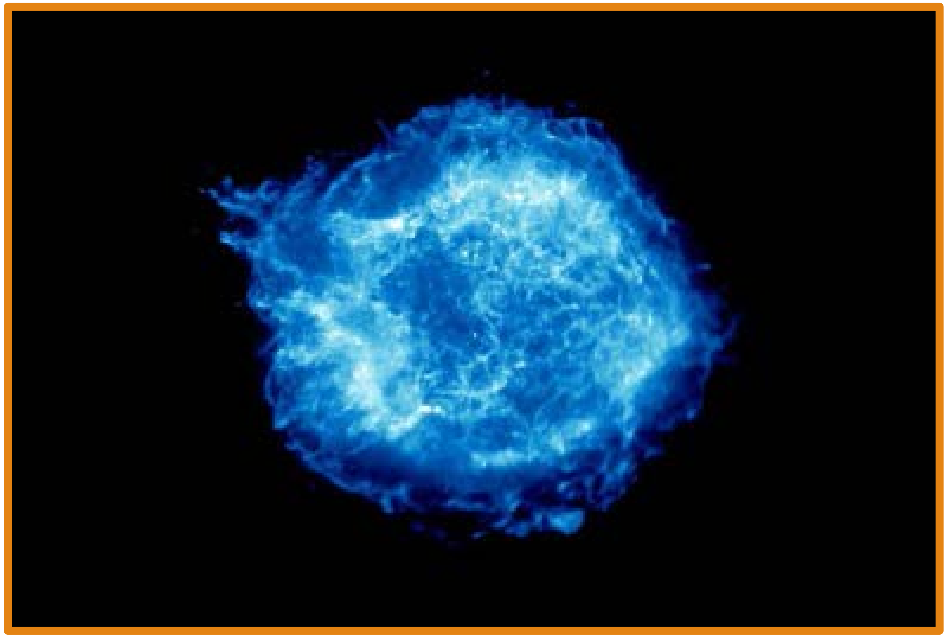
Final Outcomes



White Dwarfs

- White dwarfs are formed from the collapse of a star having mass less than or equal to 8 times mass of the sun.
- Compact stars of masses below 1.4 times mass of the sun.

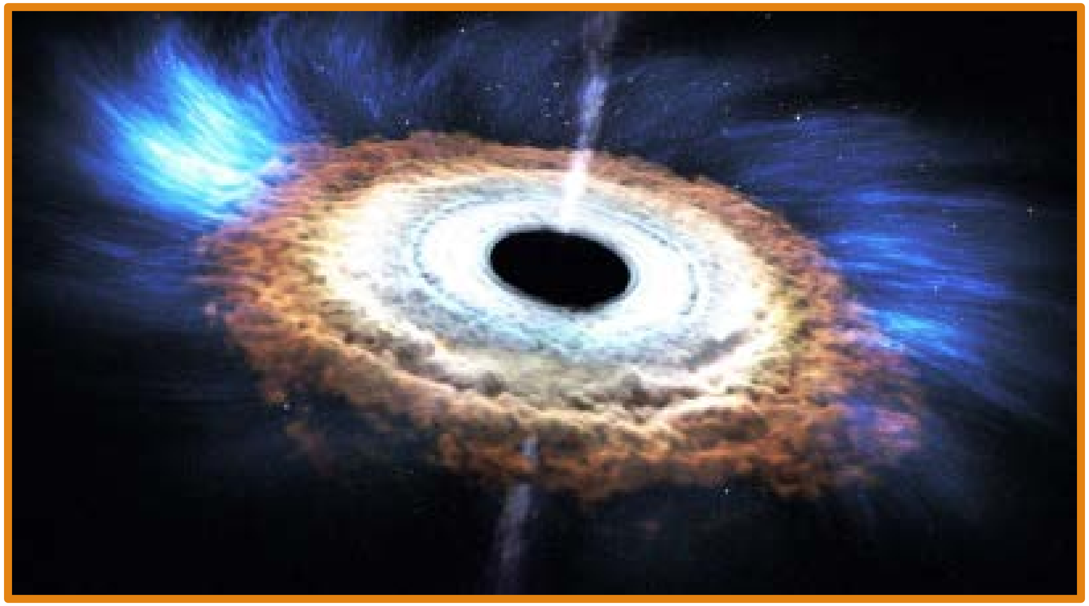
Neutron Stars



- Neutron stars are formed from the collapse of a star having mass between 8 to 20 times mass of the sun.

- Compact stars of masses between1.4 to 3 times mass of the sun.

Black Holes



- ❖ Black holes are formed from the collapse of a star having mass above 20 times mass of the sun.
- ❖ Strong gravitational effects.

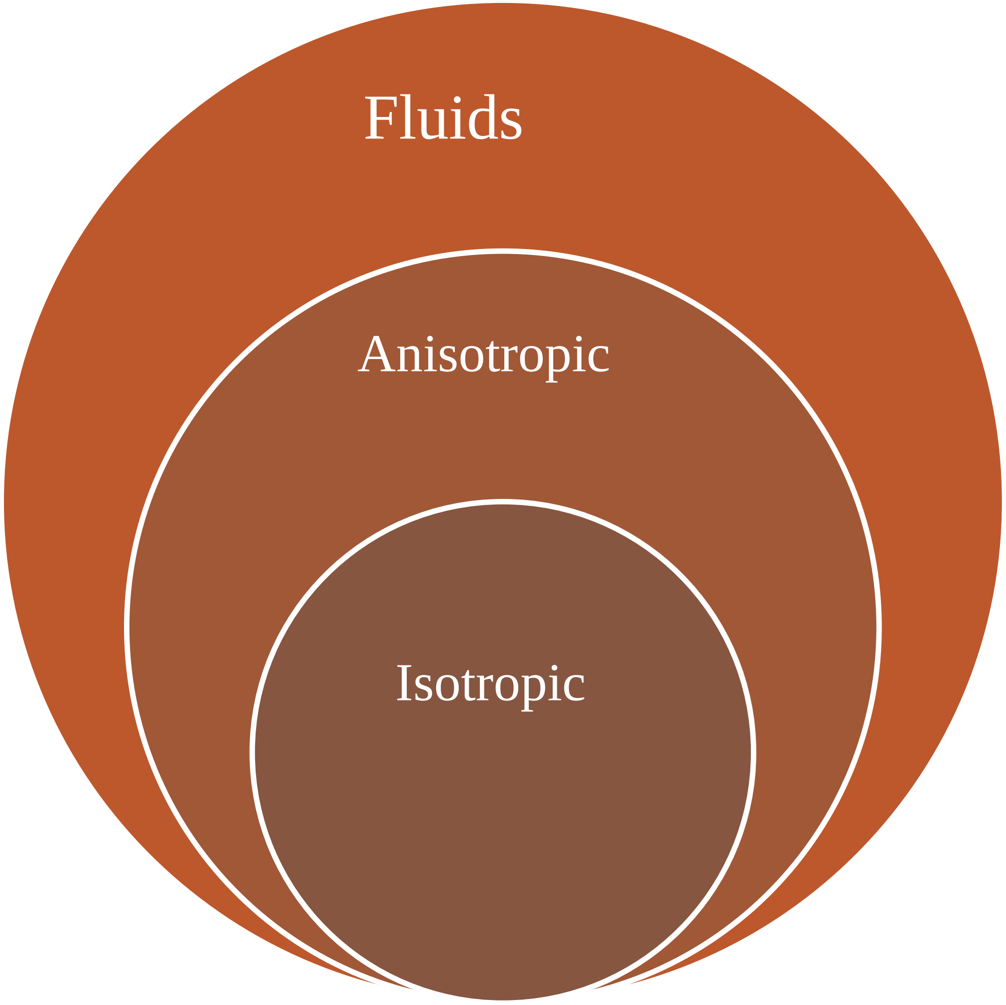
Energy-Momentum Tensor

- Symmetric second rank tensor.

- Describes the energy density, momentum and flow of energy at each point of the spacetime.

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}.$$

Classification of Fluids



Anisotropic Fluid

If the pressure of a fluid is dependent on the direction along which it is measured then it is named as anisotropic fluid.

$$\mathcal{T}^{(\mathcal{M})}_{\mu\nu} = (\rho + \mathfrak{p}_\perp)\,\vartheta_\mu\vartheta_\nu + g_{\mu\nu}\mathfrak{p}_\perp + (\mathfrak{p}_r - \mathfrak{p}_\perp)\mathcal{X}_\mu\mathcal{X}_\nu.$$

Isotropic Fluid

If the pressure of a fluid is independent of the direction along which it is measured then it is named as isotropic fluid.

Perfect Fluid

A fluid that can be completely characterized by its energy density and pressure.

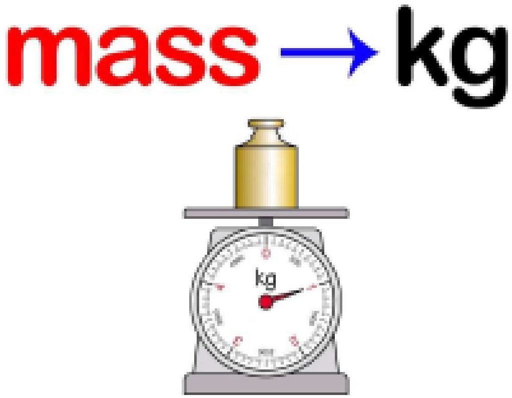
$$\mathcal{T}_{\mu\nu}^{(\mathcal{M})} = (\rho + \mathfrak{p})\,\vartheta_\mu \vartheta_\nu + g_{\mu\nu}\mathfrak{p}.$$

Dust Fluid

A dust fluid is a pressure less perfect fluid.

$$\mathcal{T}_{\mu\nu}^{(\mathcal{M})} = \rho \vartheta_\mu \vartheta_\nu.$$

Mass



- A scalar quantity describing total matter present in a body.

Classification of Mass

Inertial Mass

- It measures the resistance offered by an object in reaction to a change in its motion.

Gravitational Mass

- It is a property of matter that determines how matter deals with gravitational forces.

Inertia and Gravity

- A combination of inertia and gravity keep the Earth in orbit around the sun and the moon in orbit around the Earth.

- If there was no gravity, inertia would cause the moon to travel in a straight line.

- If only gravity existed, the Earth would be pulled into the sun.

Junction Conditions

Junction Conditions

- Hypersurface
- Normal Vector
- Intrinsic Curvature
- Extrinsic Curvature

Hypersurface

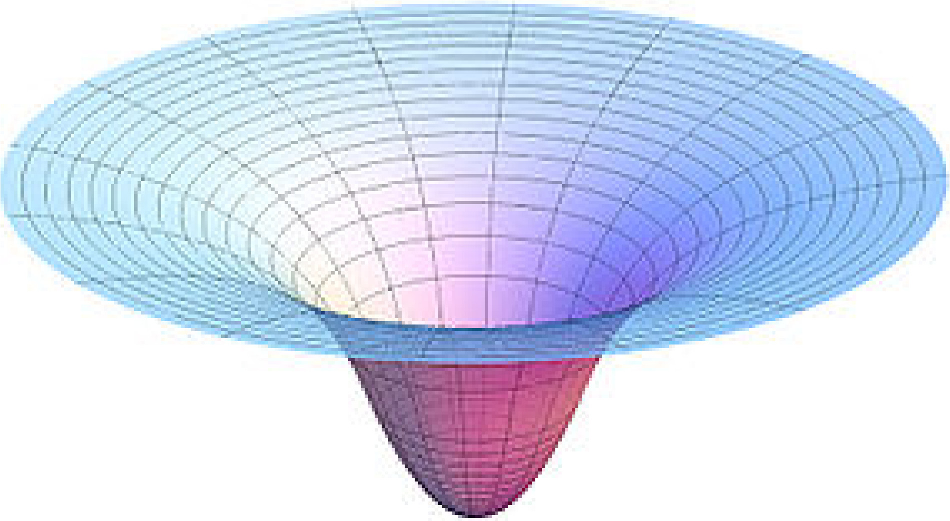
In n-dimensional manifold, a hypersurface is $(n - 1)$-dimensional submanifold.

Normal Vector

A vector changes its values in the orthogonal direction to hypersurface. A unit vector n_μ is defined as

$$n_\mu = \frac{l_{,\mu}}{g^{\mu\nu} l_{,\mu} l_{,\nu}}.$$

Curvature



- Characterizes the deviation of an object from being flat.
- Change in direction at some particular point.

Intrinsic Curvature

The deviation of a surface from the flat geometry within the whole surface.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu .$$

Extrinsic Curvature

The curvature of $(n\ \text{-}1)$-dimensional surface, which is embedded in a surface of n-dimension.

$$K_{ij} = -n_\mu^\pm \left(\frac{\partial^2 x_\pm^\mu}{\partial \psi_i \partial \psi_j} + \Gamma^\mu_{\alpha\beta} \frac{\partial^2 x_\pm^\alpha \partial^2 x_\pm^\beta}{\partial \psi_i \partial \psi_j} \right), \quad (i, j = 0, 2, 3).$$

Darmois Junction Conditions

First Fundamental Form

- Continuity of line elements.

Mathematical Form

$$(ds^2)_\Sigma = (ds^2_-)_\Sigma = (ds^2_+)_\Sigma .$$

Second Fundamental Form

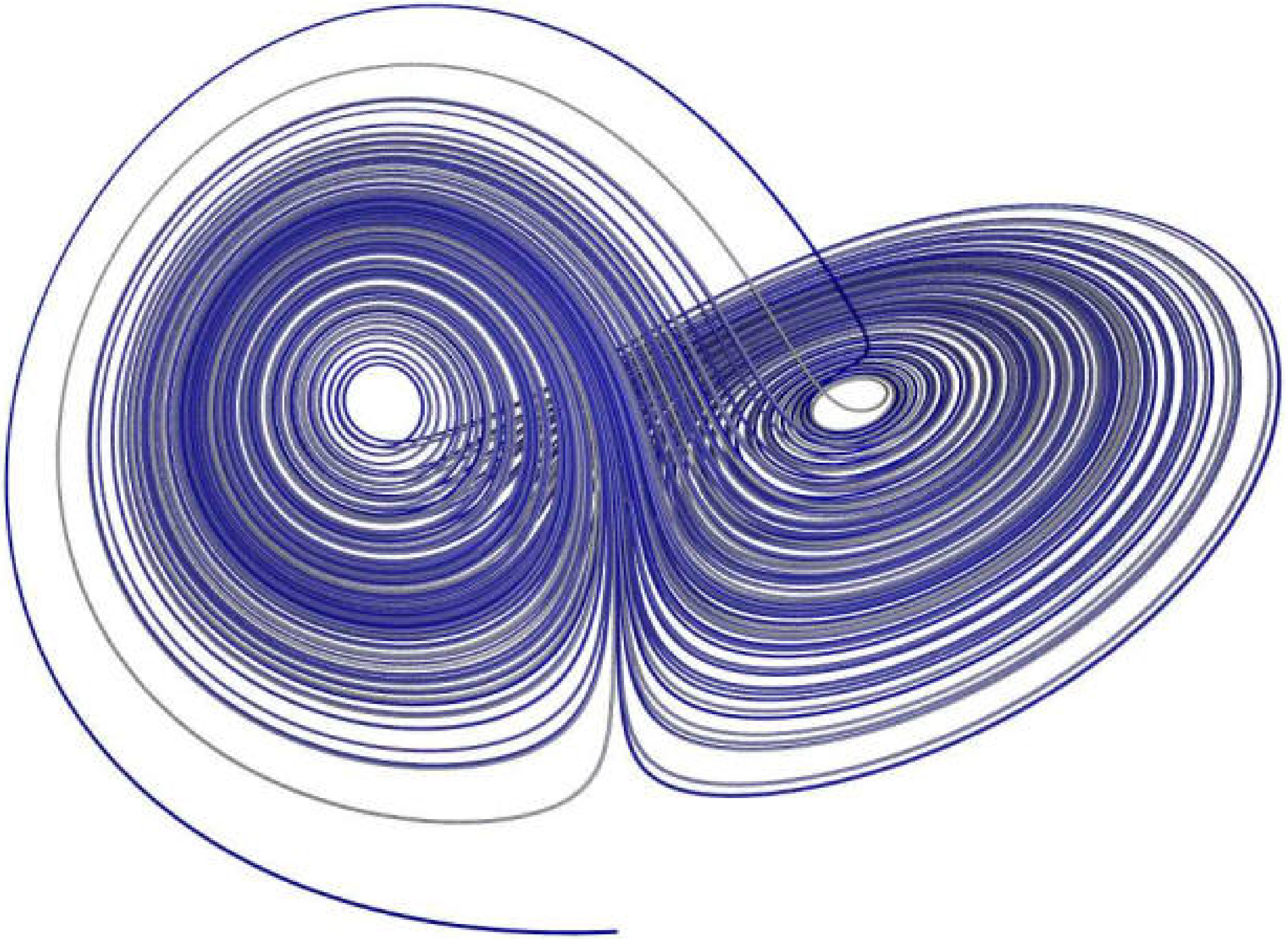
- Continuity of curvature.

Mathematical Form

$$[K_{ij}] = K_{ij}^{-} - K_{ij}^{+} = 0.$$

Darmois, G.: Memorial des Sciences Mathematiques (Gautheir-Villars, Paris, 1927).

Dynamical Equations



Weyl Tensor

- Fourth rank tensor.
- Determines distortion in the shape of body by tidal force.

Weyl Tensor

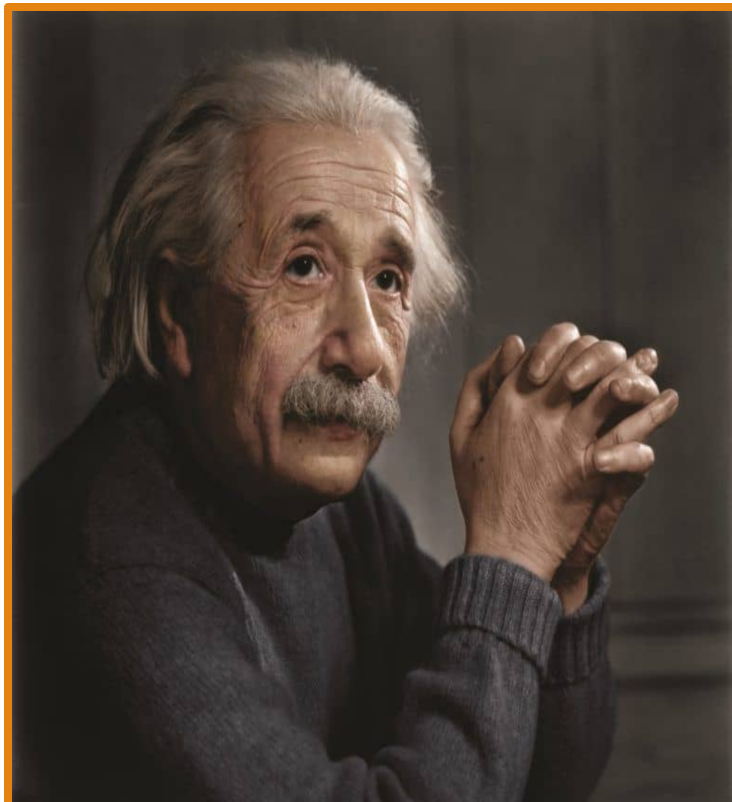
$$\mathbb{C}_{\mu\nu\alpha\beta} = \mathcal{R}_{\mu\nu\alpha\beta} + \frac{1}{n-2}(g_{\mu\beta}\mathcal{R}_{\alpha\nu} - g_{\mu\alpha}\mathcal{R}_{\beta\nu} + g_{\nu\alpha}\mathcal{R}_{\beta\mu} - g_{\nu\beta}\mathcal{R}_{\alpha\mu}) + \frac{1}{(n-1)(n-2)}(g_{\mu\alpha}g_{\beta\nu} - g_{\mu\beta}g_{\alpha\nu})\mathcal{R}, \quad n \geq 3.$$

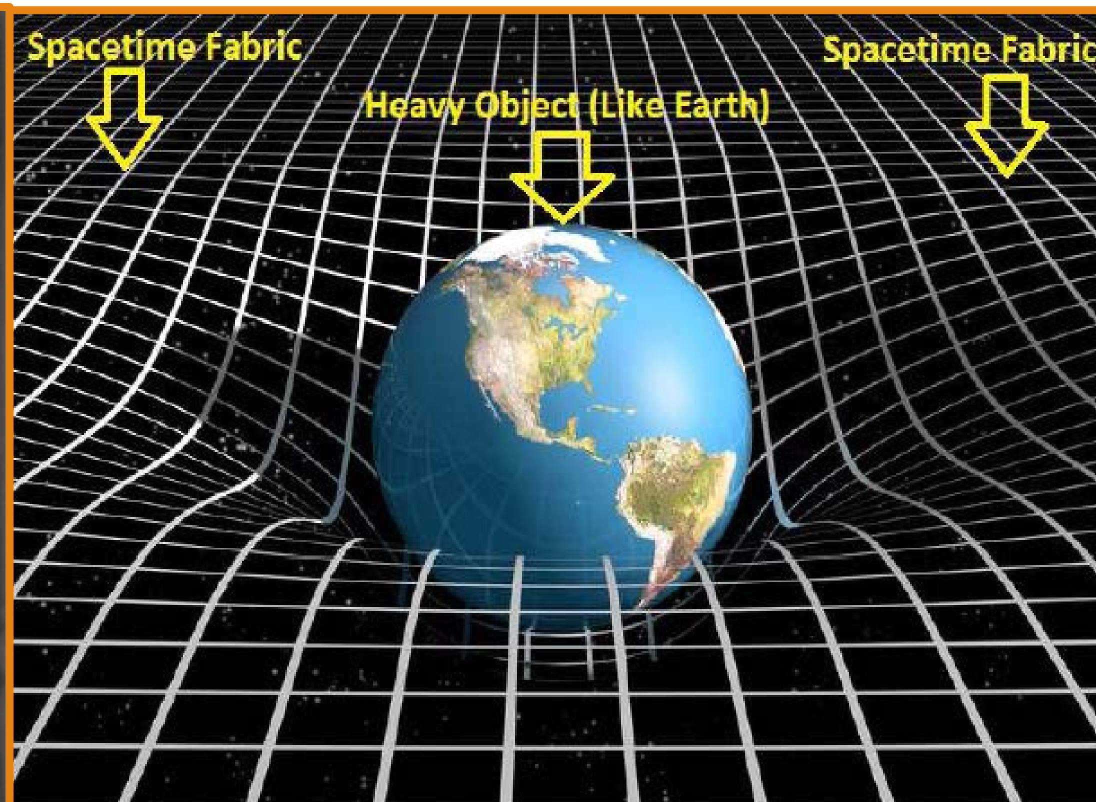
Weyl Scalar

$$\mathbb{C}^2 = \frac{1}{3}\mathcal{R}^2 + \mathbb{R} - 2\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu}.$$

$$\mathbb{R} = \mathcal{R}^{\mu\nu\xi\eta}\mathcal{R}_{\mu\nu\xi\eta}.$$

General Theory of Relativity





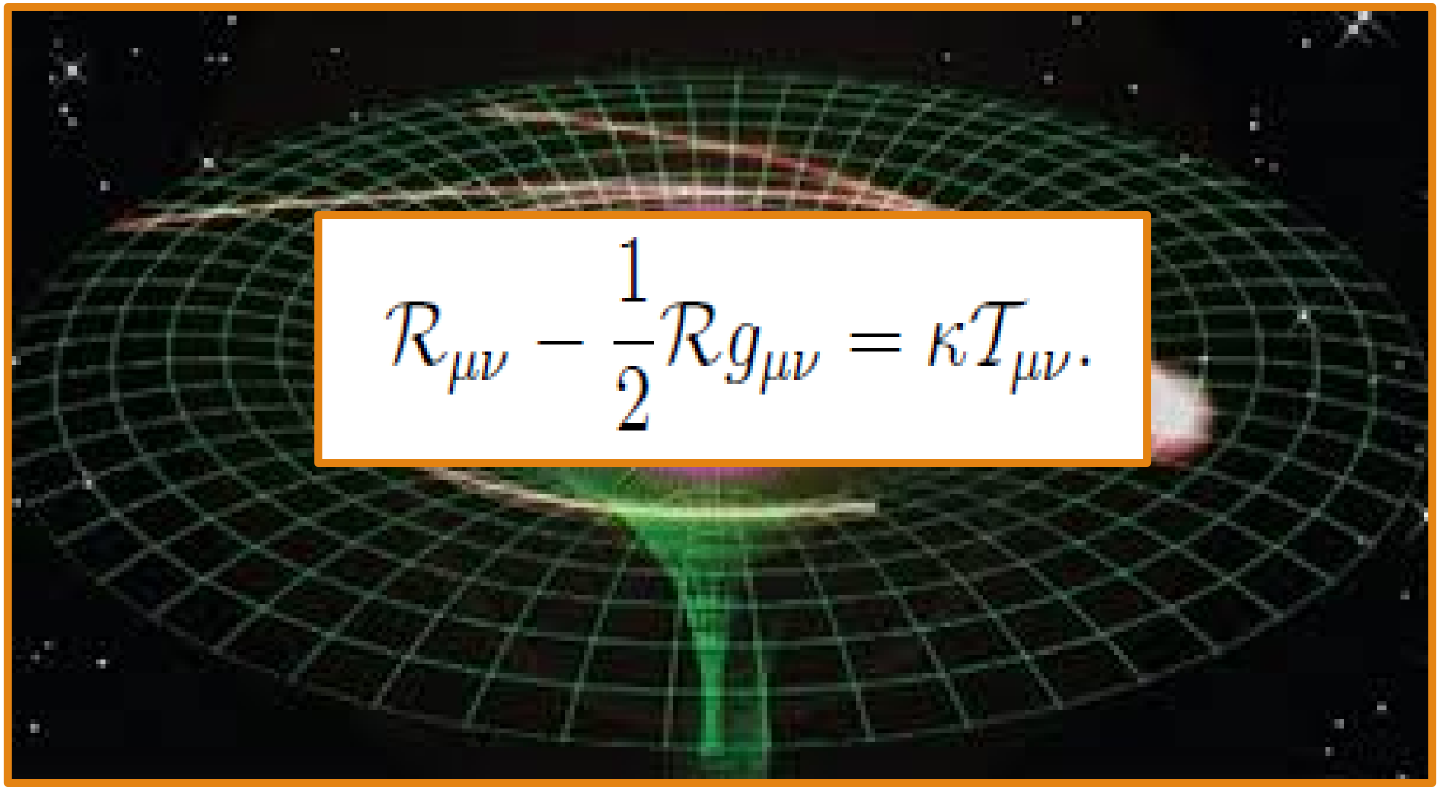
Principle of General Covariance

- All laws of physics remain same in any coordinate system.

Principle of Equivalence

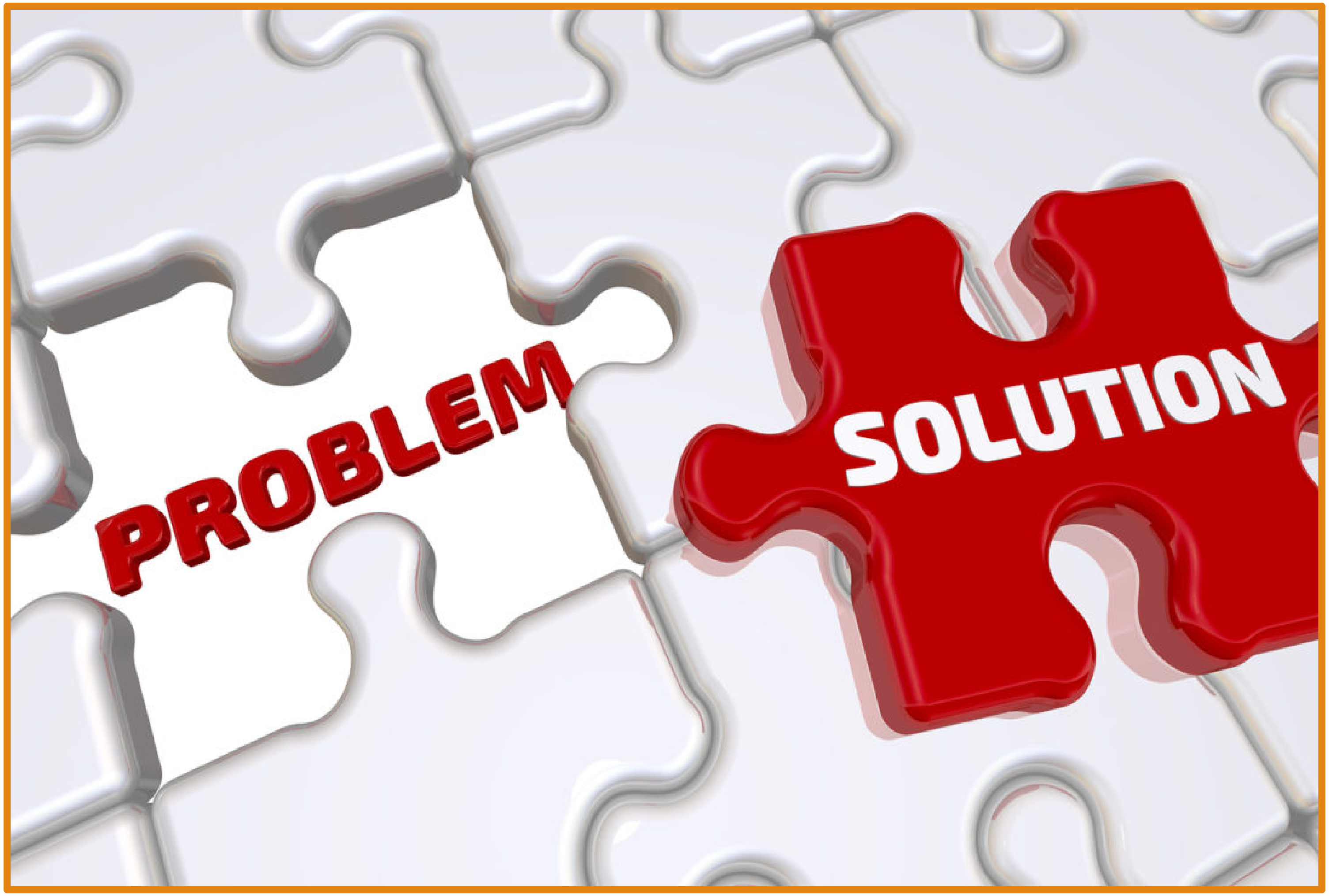
- There is no difference between the effects of gravitation and acceleration on spacetime.

Einstein Field Equations



$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = \kappa \mathcal{T}_{\mu\nu}.$$

Modified Theories



f(R) Theory

Buchdahl, H.A.: Mon, Not. R. Astron. Soc. **150**(1970)1.

Action

$$\mathcal{I}=\frac{1}{2\kappa^2}\int f\left(\mathcal{R}\right)\sqrt{-g}d^4x+\int\mathcal{L}_m\sqrt{-g}d^4x.$$

Field Equations

$$\mathcal{R}_{\mu\nu}f_{\mathcal{R}}+g_{\mu\nu}\Box f_{\mathcal{R}}-\nabla_\mu\nabla_\nu f_{\mathcal{R}}-\frac{1}{2}g_{\mu\nu}f=\kappa^2\mathcal{T}_{\mu\nu},$$

where

$$\Box=\nabla_\mu\nabla^\mu,\quad f\equiv f(\mathcal{R}),\quad f_{\mathcal{R}}=\frac{\partial f}{\partial\mathcal{R}}.$$

Curvature-Matter Coupling

Nojiri, S. and Odintsov, S.D.: Phys. Lett. B **599** (2004) 137 .

Cosmic Eras

Non-Conserved

Additional Force

Dark Components

Energy-Momentum Squared Gravity

Katirci, N. and Kavuk, M.: Eur. Phys. J. Plus **129**(2014)163.

Action

$$\mathcal{I} = \frac{1}{2\kappa^2}\int f\left(\mathcal{R}, \mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}\right)\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x.$$

Field Equations

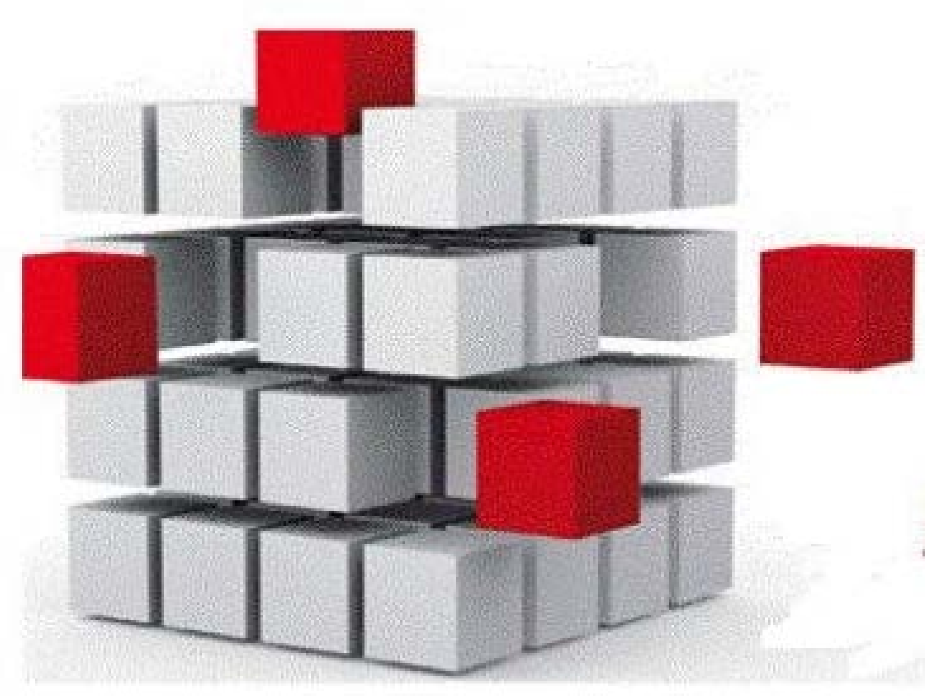
$$\mathcal{R}_{\mu\nu}f_{\mathcal{R}} + g_{\mu\nu}\Box f_{\mathcal{R}} - \nabla_\mu\nabla_\nu f_{\mathcal{R}} - \frac{1}{2}g_{\mu\nu}f = \mathcal{T}_{\mu\nu} - \Theta_{\mu\nu}f_{\mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}}.$$

where

$$\Box = \nabla_\xi\nabla^\xi,\ f \equiv f(\mathcal{R}, \mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}),\ f_{\mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}} = \frac{\partial f}{\partial\mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}},\ f_{\mathcal{R}} = \frac{\partial f}{\partial\mathcal{R}} \text{ and}$$

$$\Theta_{\mu\nu} = -2\mathcal{L}_m\left(\mathcal{T}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{T}\right) - 4\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\xi\eta}}\mathcal{T}^{\xi\eta} - \mathcal{T}\mathcal{T}_{\mu\nu} + 2\mathcal{T}_\mu^\xi\mathcal{T}_{\nu\xi}.$$

Salient Features



- ❖ Alternative for dark energy.
- ❖ Bounce at early-times.
- ❖ Non-conserved theory.


Research Work

Interior Metric

$$ds_-^2 = -\alpha^2(\mathfrak{t},\mathfrak{r})d\mathfrak{t}^2 + \beta^2(\mathfrak{t},\mathfrak{r})d\mathfrak{r}^2 + \gamma^2(\mathfrak{t},\mathfrak{r})d\phi^2 + \delta^2(\mathfrak{t},\mathfrak{r})dz^2.$$

Matter Distribution

$$\mathcal{T}_{\mu\nu}^{(\mathcal{M})} = (\rho + \mathfrak{p}_\perp)\,\vartheta_\mu\vartheta_\nu + g_{\mu\nu}\mathfrak{p}_\perp + (\mathfrak{p}_r - \mathfrak{p}_\perp)\mathcal{X}_\mu\mathcal{X}_\nu.$$

Exterior Metric

$$ds_+^2 = -\left(-\frac{2M(v)}{R}\right)dv^2 - 2dvdR + R^2\left(d\phi^2 + \psi^2 dz^2\right).$$

Mass Function

$$\mathbb{E} = \mathfrak{m}(\mathfrak{t},\mathfrak{r}) = \frac{1}{8}\left(1 - \mathbb{L}^{-2}\nabla^\nu r \nabla_\nu r\right).$$

$$\mathbb{E} = \frac{\mathbb{L}}{8} + \frac{1}{8\delta}\left\{\frac{1}{\alpha^2}\left(\gamma\dot{\delta} + \delta\dot{\gamma}\right)^2 - \frac{1}{\beta^2}\left(\gamma\delta' + \delta\gamma'\right)^2\right\}.$$

Throne, K.S.: Phys. Rev. D **138**(1965)B251.

Field Equations

$$
\begin{aligned}
\frac{1}{f_{\mathcal{R}}}\left(\rho+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}\right) &= \frac{1}{\beta^2}\left(\frac{\beta'\gamma'}{\beta\gamma}-\frac{\gamma'\delta'}{\gamma\delta}-\frac{\gamma''}{\gamma}+\frac{\beta'\delta'}{\beta\delta}-\frac{\delta''}{\delta}\right)\\
&+ \frac{1}{\alpha^2}\left(\frac{\dot{\gamma}\dot{\delta}}{\gamma\delta}+\frac{\dot{\beta}\dot{\delta}}{\beta\delta}+\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}\right),\\
\frac{1}{f_{\mathcal{R}}}\left(\mathfrak{p}_{\mathfrak{r}}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right) &= \frac{1}{\alpha^2}\left(-\frac{\dot{\gamma}\dot{\delta}}{\gamma\delta}+\frac{\dot{\alpha}\dot{\delta}}{\alpha\delta}-\frac{\ddot{\gamma}}{\gamma}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}-\frac{\ddot{\delta}}{\delta}\right)\\
&+ \frac{1}{\beta^2}\left(\frac{\alpha'\delta'}{\alpha\delta}+\frac{\alpha'\gamma'}{\alpha\gamma}+\frac{\gamma'\delta'}{\gamma\delta}\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{f_{\mathcal{R}}}\left(\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{22}^{(C)}}{\gamma^{2}}\right) &= \frac{1}{\beta^{2}}\left(\frac{\alpha'\delta'}{\alpha\delta}-\frac{\delta'\beta'}{\delta\beta}+\frac{\alpha''}{\alpha}-\frac{\alpha'\beta'}{\alpha\beta}+\frac{\delta''}{\delta}\right) \\
&+ \frac{1}{\alpha^{2}}\left(\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{\dot{\beta}\dot{\delta}}{\beta\delta}-\frac{\ddot{\beta}}{\beta}+\frac{\dot{\alpha}\dot{\delta}}{\alpha\delta}-\frac{\ddot{\delta}}{\delta}\right), \\
\frac{1}{f_{\mathcal{R}}}\left(\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{33}^{(C)}}{\delta^{2}}\right) &= \frac{1}{\beta^{2}}\left(\frac{\alpha'\gamma'}{\alpha\gamma}-\frac{\gamma'\beta'}{\gamma\beta}+\frac{\alpha''}{\alpha}-\frac{\alpha'\beta'}{\alpha\beta}+\frac{\gamma''}{\gamma}\right) \\
&+ \frac{1}{\alpha^{2}}\left(\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}-\frac{\ddot{\beta}}{\beta}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}-\frac{\ddot{\gamma}}{\gamma}\right), \\
\frac{1}{f_{\mathcal{R}}}\left(\mathcal{T}_{01}^{(C)}\right) &= \left(\frac{\gamma'\dot{\beta}}{\gamma\beta}+\frac{\alpha'\dot{\delta}}{\alpha\delta}-\frac{\dot{\gamma}'}{\gamma}+\frac{\alpha'\dot{\gamma}}{\alpha\gamma}-\frac{\dot{\delta}'}{\delta}+\frac{\delta'\dot{\beta}}{\delta\beta}\right).
\end{aligned}
$$

Darmois Junction conditions

$$\mathbb{E} - M = \frac{\mathbb{L}}{8}, \quad \psi = \frac{1}{2},$$

$$\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2} = -\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}.$$

Proper Time and Radial Derivatives

$$\mathfrak{D}_{\mathfrak{t}} = \frac{1}{\alpha}\frac{\partial}{\partial \mathfrak{t}}, \quad (1)$$

$$\mathfrak{D}_{\mathfrak{r}} = \frac{1}{\gamma'}\frac{\partial}{\partial \mathfrak{r}}. \quad (2)$$

Velocity of Fluid Particles

$$\mathcal{U} = \mathfrak{D}_{\mathfrak{t}}(\gamma) = \frac{\dot{\gamma}}{\alpha} < 0. \quad (3)$$

Misner, C.W. and Sharp, D.H.: Phys. Rev. B **136**(1964)571.

Dynamical Equations

$$\left(\mathcal{T}^{(\mathcal{M})\mu\nu}+\mathcal{T}^{(C)\mu\nu}\right)_{;\nu}\mathcal{X}_{\mu}=0,$$
$$\left(\mathcal{T}^{(\mathcal{M})\mu\nu}+\mathcal{T}^{(C)\mu\nu}\right)_{;\nu}\vartheta_{\mu}=0.$$

$$\frac{\alpha'}{\alpha}\left(\rho+\mathfrak{p}_{\mathfrak{r}}+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)+\left(\mathfrak{p}_{\mathfrak{r}}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)'-\frac{\beta}{\alpha}\left(\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\right)^{\cdot}$$
$$-\frac{\mathcal{T}_{01}^{(C)}}{\psi\alpha^2}\left(\frac{\dot{\beta}}{\beta}+\frac{\dot{\gamma}}{\gamma}\right)+\frac{\gamma'}{\psi\gamma}\left(\mathfrak{p}_{\mathfrak{r}}-\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}-\frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)=0, \quad (4)$$

$$\frac{\dot{\beta}}{\beta}\left(\rho+\mathfrak{p}_{\mathfrak{r}}+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)+\alpha^2\left(\frac{\mathcal{T}_{00}^{(C)}}{\alpha^4}\right)^{\cdot}+\frac{\dot{\alpha}}{\beta\alpha}\left(\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}\right)'$$
$$+\dot{\rho}-\frac{\alpha}{\beta}\left(\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\right)'+\frac{\dot{\gamma}}{\psi\gamma}\left(\rho+\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}+\frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)$$
$$-\frac{\mathcal{T}_{01}^{(C)}}{\psi\beta^2}\left(\frac{\alpha'}{\alpha}+\frac{\gamma'}{\gamma}\right)=0. \quad (5)$$

Proper Time Variation

$$\mathfrak{D}_{\mathfrak{t}}(\mathbb{E}) = -\frac{\psi\gamma^2}{2}\left\{\left(\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)\mathcal{U} - \frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\varphi\right\} + \frac{\psi\mathcal{U}}{4}. \quad (6)$$

Proper Radial Variation

$$\mathfrak{D}_{\mathfrak{r}}(\mathbb{E}) = \frac{\psi\gamma^2}{2}\left\{\left(\rho + \frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}\right) - \frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\frac{\mathcal{U}}{\varphi}\right\} + \frac{\psi}{4}. \quad (7)$$

Acceleration of the Fluid Particles

The acceleration of the collapsing system is

$$\mathfrak{D}_{\mathfrak{t}}(\mathcal{U}) = -\frac{\gamma}{2}\left(\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right) - \frac{1}{\psi\gamma^2}\left(\mathbb{E} - \frac{\mathbb{L}}{8}\right) + \frac{\varphi}{\beta}\frac{\alpha'}{\alpha}. \tag{8}$$

Using Eq.(4), we have

$$\begin{aligned}\frac{\alpha'}{\alpha} &= \left(\rho + \mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{00}^{(C)}}{\alpha^2} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)^{-1}\left\{\frac{\beta}{\alpha}\left(\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\right)^{\cdot} + \frac{\mathcal{T}_{01}^{(C)}}{\psi\alpha^2}\left(\frac{\dot{\beta}}{\beta} + \frac{\dot{\gamma}}{\gamma}\right)\right. \\ &- \left.\left(\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{Y^2}\right)' - \frac{\gamma'}{\psi\gamma}\left(\mathfrak{p}_{\mathfrak{r}} - \mathfrak{p}_{\perp} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2} - \frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)\right\}.\end{aligned}$$

Substituting this value in Eq.(8), we obtain

$$\mathcal{F}_{newtn} = -\mathcal{F}_{grav} + \mathcal{F}_{hyd} + \mathcal{F}_{ds},$$

where

$$\mathcal{F}_{newtn} = \left(\rho + \mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{00}^{(C)}}{\alpha^2} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)\mathfrak{D}_{\mathfrak{t}}(\mathcal{U}),$$

$$\mathcal{F}_{grav} = \left(\rho + \mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{00}^{(C)}}{\alpha^2} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right)\left\{\frac{\gamma}{2}\left(\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right) + \frac{1}{\psi\gamma^2}\left(\mathbb{E} - \frac{\mathbb{L}}{8}\right)\right\},$$

$$\mathcal{F}_{hyd} = -\varphi^2\left\{\mathfrak{D}_{\mathfrak{r}}\left(\mathfrak{p}_{\mathfrak{r}} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2}\right) + \frac{1}{\psi\gamma}\left(\mathfrak{p}_{\mathfrak{r}} - \mathfrak{p}_{\perp} + \frac{\mathcal{T}_{11}^{(C)}}{\beta^2} - \frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)\right\},$$

$$\mathcal{F}_{ds} = \varphi\left\{\mathfrak{D}_{\mathfrak{t}}\left(\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\right) + \frac{\mathcal{T}_{01}^{(C)}}{\psi\alpha\beta}\left(\frac{\dot{\beta}}{\alpha\beta} + \frac{\mathcal{U}}{\gamma}\right)\right\}.$$

Relation between Weyl Scalar and Matter Variables

Manipulating the Kretschmann scalar, we have

$$\begin{aligned}
\mathbb{R} &= \frac{48}{\psi^2\gamma^6}\left(\mathbb{E}-\frac{\mathbb{L}}{8}\right)^2-\frac{16}{\psi\gamma^3}\left(\mathbb{E}-\frac{\mathbb{L}}{8}\right)\left(\frac{\mathcal{G}_{00}}{\alpha^2}-\frac{\mathcal{G}_{11}}{\beta^2}+\frac{\mathcal{G}_{22}}{\gamma^2}\right)\\
&- \frac{2\mathcal{G}_{00}\mathcal{G}_{11}}{\beta^2\alpha^2}+\frac{4\mathcal{G}_{22}}{\gamma^2}\left(\frac{\mathcal{G}_{00}}{\alpha^2}-\frac{\mathcal{G}_{11}}{\beta^2}\right)+4\left(\frac{\mathcal{G}_{22}}{\gamma^2}\right)^2-4\left(\frac{\mathcal{G}_{01}}{\alpha\beta}\right)^2\\
&+ 3\left(\frac{\mathcal{G}_{00}}{\alpha^2}\right)^2+3\left(\frac{\mathcal{G}_{11}}{\beta^2}\right)^2.
\end{aligned}$$

Substituting the value of Kretschmann scalar, Ricci scalar and Ricci tensor, we have

$$\frac{\mathcal{C}\gamma^3\psi}{\sqrt{48}}=\mathbb{E}-\frac{\mathbb{L}}{8}-\frac{\psi\gamma^3}{6}\left(\rho-\mathfrak{p}_{\mathfrak{r}}+\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}-\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}+\frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right).$$

The proper radial derivative of Weyl scalar becomes

$$\mathfrak{D}_{\mathfrak{r}}\left(\frac{\mathcal{C}\gamma^3\psi}{\sqrt{48}}\right)+\frac{\psi\gamma^3}{6}\mathfrak{D}_{\mathfrak{r}}\left\{\left(\rho-\mathfrak{p}_{\mathfrak{r}}+\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{00}^{(C)}}{\alpha^2}-\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}+\frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)\right\}$$
$$-\frac{\gamma^2\psi}{2}\left\{\left(\mathfrak{p}_{\mathfrak{r}}-\mathfrak{p}_{\perp}+\frac{\mathcal{T}_{11}^{(C)}}{\beta^2}-\frac{\mathcal{T}_{22}^{(C)}}{\gamma^2}\right)-\frac{\mathcal{U}}{\varphi}\frac{\mathcal{T}_{01}^{(C)}}{\alpha\beta}\right\}=0.$$

Minimal Coupling Model

$$f(\mathcal{R}, \mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}) = f_1(\mathcal{R}) + f_2(\mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu}).$$

$$\mathfrak{D}_{\mathrm{r}}\left(\frac{\mathcal{C}\gamma^3\psi}{\sqrt{48}}\right) - \frac{\gamma^2\psi}{2}\left(\mathfrak{p}_{\mathfrak{r}} - \mathfrak{p}_{\perp}\right) + \frac{\gamma^3\psi}{6f_{\mathcal{R}}}\mathfrak{D}_{\mathfrak{r}}(\rho - \mathfrak{p}_{\mathfrak{r}} + \mathfrak{p}_{\perp} + f_0) = 0.$$

where

$$f_0 = \frac{\mathcal{R}_0 f_{\mathcal{R}} - f(\mathcal{R}_0) - f_2(\mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu})}{2}.$$

Bahamonde, S., Marciu, M. and Rudra, P.: Phys. Rev. D **100**(2019)083511.

We assume isotropic matter distribution, above equation becomes

$$\mathfrak{D}_{\mathfrak{t}}\left(\frac{\mathcal{C}\gamma^3\psi}{\sqrt{48}}\right)+\frac{\gamma^3\psi}{6f_{\mathcal{R}}}\mathfrak{D}_{\mathfrak{t}}(\rho+f_0)=0.$$

This implies that

$$\mathcal{C}=0 \Longleftrightarrow \mathfrak{D}_{\mathfrak{t}}\rho=0.$$


CONCLUSION

- We have analyzed the dynamics of anisotropic cylindrical collapse in the background of energy-momentum squared gravity.

- The collapsing phenomenon is reduced by the positive values of correction terms and anisotropy whereas negative values enhance the collapse rate.

- The total energy in the spherical star and adjacent surfaces increases due to outward effective radial pressure and energy density.

- The spacetime is conformally flat if and only if energy density is homogeneous through out the system. It is concluded that the rate of collapse is slower as compared to general relativity.

Thankyou