Article

Second Order Glauber Correlation of Gravitational Waves using the LIGO observatories as Hanbury Brown and Twiss detectors

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Abstract: The second order Glauber correlation of a simplified gravitational wave is investigated, using parameters from the first signal detected by LIGO. This simplified model spans the inspiral, merger, and ringdown phases of a black hole merger and was created to have a continuous amplitude, so there is no discontinuity between the phases. This allows for a trivial extraction of the intensity, which is necessary to determine the correlation between detectors. The two LIGO observatories can be used as detectors in a Hanbury Brown and Twiss interferometer for gravitational waves, these observatories measure the amplitude of the wave, so these measurements were used as the basis of the simplified model. The signal detected by the observatories is transient and is not consistent with steady waves and thus the second order Glauber correlation function was calculated to produce physically meaningful results. To find correlations that are comparable to applications to electromagnetic waves, different weighting functions were studied in the time average integrals in the Glauber correlation functions. The relationship between transient and steady signals and their respective correlation functions was also examined. The second order Glauber correlation functions are a measure of intensity interference between independent detectors and has proven to be useful in both optics and particle physics. It has also been used in theoretical studies of primordial gravitational waves. The correlations can be used for example to identify the degrees of coherence of a field, characterize multi-particle processes, and assist in image enhancement.

Keywords: Gravity; Gravitational Waves; Hanburry Brown and Twiss; LIGO; Interferometer; Glauber Correlation

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1. Introduction

The goal of this research is to develop techniques to find the 2nd order Glauber correlation functions for gravitational waves. We used Glauber correlation functions to characterize the fundamental structure of gravitational wave signals similar to the methodology and characterizations used for electromagnetic waves and particle physics, which has not been done for gravitational waves produced by binary inspirals. The analysis of a signal detected by LIGO introduces a problem, the signal received from a binary merger is short lived, where meaningful detection is only observed for ~ 0.2 seconds. This differs from signals analyzed by the quantum optics [1,2] and particle physics [3] communities which are typically not transient. A method of calculating the second order Glauber correlation functions utilizing intensity weighting is proposed, as the method normally prescribed averages over a characteristic time period, which results in a measure of the correlation that is dependent on an arbitrary time interval in the time average. This is a consequence of using a constant weighting function in the time average. We instead apply an intensity weighting function in the time averages, which produces Glauber correlation functions that are independent of time integration limits. Using the alternative weighting, correlations are produced that are similar to previosly analyzed electromagnetic waves, and comparing the two can provide insight into e.g. the coherence properties of gravitational waves.

2. Methods

Classically, interferometers are used to study the characteristics of waves by interfering the wave with itself. The LIGO observatories consist of two separate gravitational wave interferometers that measure the strain amplitude of the observed gravitational wave. Each of the two observatories can be thought of as the two detectors of a Hanbury Brown and Twiss interferometer [1–3], allowing the intensity of the wave to be extracted and analyzed with the second order Glauber correlation function.

These functions are dependent on the time between detections and describes the correlations of observed intensities [1]. The calculation of the Glauber correlation functions is defined as,

$$g^2(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}. \tag{1}$$

Where τ is the time between detections and $\langle ...\rangle$ signifies a time averaging over a characteristic time period [4]. Typically the function is expanded using a characteristic time interval, T, in the time averages,

$$g^2(\tau) = \frac{\frac{1}{T}\int_{\frac{T}{2}}^{\frac{T}{2}} I(t)I(t+\tau)dt}{\frac{1}{T^2}\int_{\frac{T}{2}}^{\frac{T}{2}} I(t)dt \int_{\frac{T}{2}}^{\frac{T}{2}} I(t+\tau)dt}. \tag{2}$$

This does not create any problems with the typical signals being analyzed, but does introduce a dependence of the time interval, T, in the time averages for transient signals. To remedy this an intensity weighted average was used in the time averages for calculating the intensity correlations,

$$g^2(\tau) = \frac{G^{(2)}(t,t+\tau)}{G^{(1)}(t)G^{(1)}(t+\tau)}, \tag{3}$$

where the weighting term is the intensity when $\tau = 0$,

$$\begin{aligned} G^{(2)}(t,t+\tau) &= \frac{\int_{-\infty}^{\infty} I(t)I(t+\tau)I(t+0)dt}{\int_{-\infty}^{\infty} I(t+0)dt}, \\ G^{(1)}(t+\tau) &= \frac{\int_{-\infty}^{\infty} I(t+\tau)I(t+0)dt}{\int_{-\infty}^{\infty} I(t+0)dt}. \end{aligned} \tag{4}$$

This method of time averaging with an intensity weighting has the advantage of being independent of an arbitrary time interval in averaging the correlations.

Later, a simplified model for the strain amplitude of the gravitational wave signal for a binary inspiral will be described, this model of the strain amplitude is based off of the first gravitational wave detection at LIGO. The extraction of the intensity from the signal is not trivial [4]. The relationship between the strain amplitude and intensity is,

$$I(t) \propto h(t)^2, \tag{5}$$

with the exact conversion being [5],

$$I(t) = A\omega^2\, h(t)^2, \tag{6}$$

where,

$$A = \frac{c^2}{16\pi G}. \tag{7}$$

Substituting $I(t)$ as described in (6) into equation (4) will yield a Glauber correlation function that properly extracts the intensity of the signal,

$$g^2(\tau) = \frac{A^2\omega^4\, G^{(2)}(t, t+\tau)}{(A\omega^2\, G^{(1)}(t))(A\omega^2\, G^{(1)}(t+\tau))}, \tag{8}$$

and simplifies to,

$$g^2(\tau) = \frac{G^{(2)}(t, t+\tau)}{G^{(1)}(t)G^{(1)}(t+\tau)}. \tag{9}$$

We find that the $A\omega$ terms cancel completely meaning the square of the strain amplitude model may be used as the intensity for our purposes with no negative repercussions. Simply substituting $h(t)^2$ for $I(t)$, we get equation (4) back.

Now that both techniques have been established a known correlation was analyzed to validate this new technique. An oscillatory signal from Fox[1] was analyzed with both forms of the second order Glauber correlation function. The oscillatory intensity function has the form,

$$I(t) = I_0(1 + A_0 \sin(\omega t)). \tag{10}$$

The constant weighting time-average correlation equation (2), gives a function that agrees with the amplitude provided by Fox at $\tau = 0$. The closed form solution obtained is,

$$g^2(\tau) = \frac{1}{2}(2 + A_0^2 \cos(\omega\tau)), \tag{11}$$

see Figure 1. The correlation at $\tau = 0$,

$$g^2(0) = 1 + \frac{A_0^2}{2}, \tag{12}$$

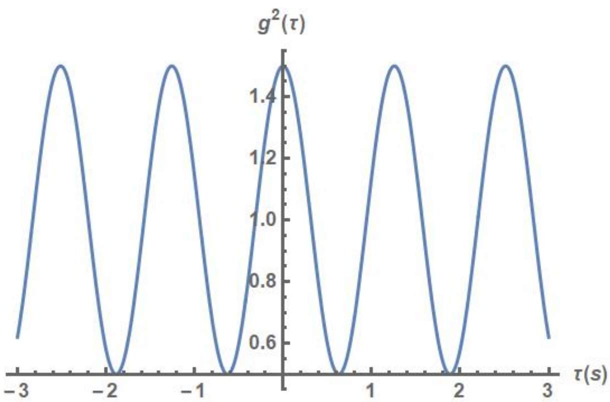

Figure 1. Glauber correlation function (11).

which agrees with Fox [1].

Using the intensity weighted correlation from equation (4) we obtained the following closed form solution,

$$g^2(\tau) = \frac{4(1 + \frac{1}{2}A_0^4 \cos(\omega\tau))}{(2 + A_0^2)(2 + A_0^2 \cos(\omega\tau))}, \tag{13}$$

see Figure 2. The correlation at $\tau = 0$ is,

$$g^2(0) = \frac{4(1 + \frac{1}{2}A_0^4)}{(2 + A_0^2)^2}. \tag{14}$$

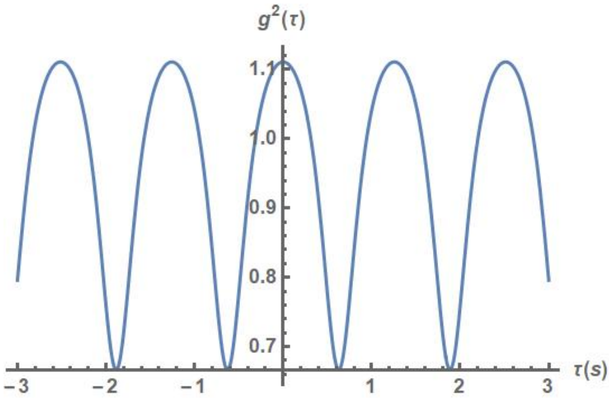

Figure 2. Glauber correlation (13).

Clearly Figure 1 and Figure 2 exhibit similar characteristics, with the former having a larger amplitude, though it must be said that both methods when applied to steady periodic functions produce correlations not dependent on a time period T.

3. Results

The gravitational wave signals detected by LIGO will be modeled using a sine-Gaussian. The correlations of this transient model were calculated with both constant and intensity weightings in the time averages. This model was fit by eye to describe the strain amplitude observed at LIGO using three parameters, ω, b, t_m,

$$h(t) = h_{max}\, e^{-\left(\frac{t+t_m}{b}\right)^2} \cos(2\pi\omega t), \tag{15}$$

where t_m is the time of the black hole merger, b is a dampening parameter used to fit the function, and ω is the frequency of the wave, which is assumed to be constant for this simplified model, see Figure 3.

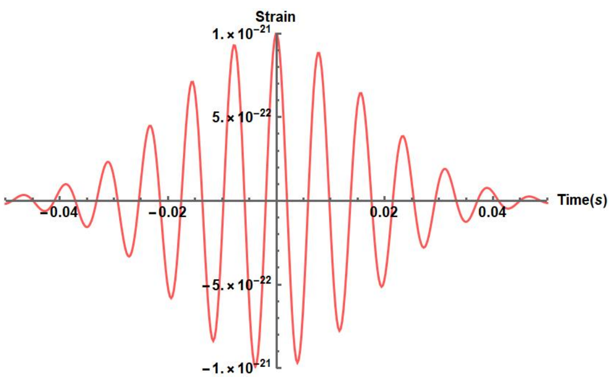

Figure 3. LIGO waveform from equation (15).

The relationship between the strain amplitude (15) and the intensity is shown in (6). Substituting (15) into (2) we obtain a more complicated correlation function than before. The constant weighting in the time average (2) yields the following closed form solution,

$$g^2(\tau) = \frac{2e^{-\frac{\tau^2}{b^2}} T(\text{erf}\,(\frac{T-2t_m-\tau}{b}) + \text{erf}\,(\frac{T+2t_m+\tau}{b}))}{b\sqrt{\pi}((\text{erf}\,(\frac{T-2t_m}{\sqrt{2}b}) + \text{erf}\,(\frac{T+2t_m}{\sqrt{2}b}))(\text{erf}\,(\frac{T-2(t_m+\tau)}{\sqrt{2}b}) + \text{erf}\,(\frac{T+2(t_m+\tau)}{\sqrt{2}b})))}, \tag{16}$$

see Figure 4. The correlation at $\tau = 0$ is,

$$g^2(0) = \frac{2T(\text{erf}\,(\frac{T-2t_m}{b}) + \text{erf}\,(\frac{T+2t_m}{b}))}{b\sqrt{\pi}(\text{erf}\,(\frac{T-2t_m}{\sqrt{2}b}) + \text{erf}\,(\frac{T+2t_m}{\sqrt{2}b}))^2}. \tag{17}$$

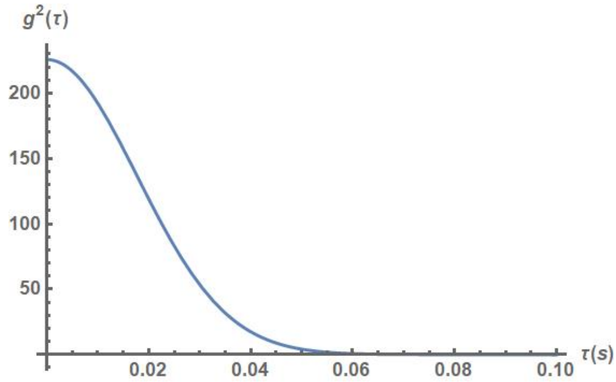

Figure 4. Glauber correlation (16).

There is a dependence on the time period being averaged over T when looking at the correlation at $\tau = 0$, which was not true for the steady signal (10). Using the intensity weighting yields a correlation with no dependence on the period T being averaged over. Substituting (15) into (4) yields a more concise correlation,

$$g^2(\tau) = \frac{2e^{-\frac{\tau^2}{3b^2}}}{\sqrt{3}}, \tag{18}$$

see Figure 5, with the magnitude at $\tau = 0$ being,

$$g^2(0) = \frac{2}{\sqrt{3}}. \tag{19}$$

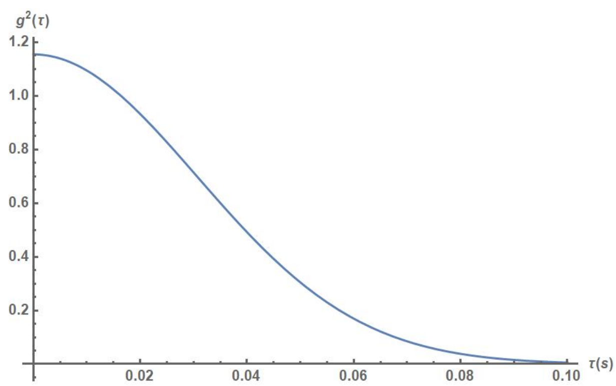

Figure 5. Glauber correlation of (18).

Both Figure 4 and Figure 5 appear to exhibit similar behavior, as $\tau \to \infty$, both correlation functions go to 0. They also have similar behavior around $\tau = 0$, with the major difference being the steepness of the correlation.

There are distinct characteristics that are exhibited when looking at the second order Glauber correlation functions of electromagnetic waves, for example, the value of the correlation at $\tau = 0$ and the value as $\tau \rightarrow \infty$ can be used to differentiate between steady and chaotic signals [1,2,6]. We compare these characteristics to those of our correlations for gravitational waves. The signal in our model is transient and undergoing amplitude modulations, although if you look at just the 2nd order Glauber correlations, the correlation functions produced are ambiguous [6]. A chaotic signal or a steady signal with amplitude modulations could produce Glauber correlation functions with similar characteristics to the functions we calculated for the transient signal. Properties of these correlations are listed below to highlight this ambiguity;

- Coherent light of a single frequency [1,2] is defined as $g^2(\tau) = 1$.
- For a laser $g^2(\tau = 0) = 2$ for chaotic light [6].
- $g^2(\tau = 0) > 1$ for most signals except e.g. steady and bunched [1,2].
- For chaotic light $g^2(\tau) = 0$ as $\tau \rightarrow \infty$, [7].

4. Conclusions

A technique for calculating 2nd order Glauber correlation functions using an intensity weighting was investigated and tested, which produced comparable results to the constant weighting in the time-averaging technique for a steady signal and better characterization of a transient signal. A sine-Gaussian was introduced to be used as a simplified model for the first signals detected by LIGO. This model's 2nd order Glauber correlation function was then calculated with the constant weighting time-average correlation and the intensity weighted correlation.

Conflicts of Interest: "The authors declare no conflict of interest."

References

1. Mark Fox, *Quantum optics - an introduction* (Oxford University Press, 2006).
2. R. Louden, *The Quantum Theory of Light* (Oxford University Press, third edition, 2000).
3. Gordon Baym, Acta Phys. Polo. B **29**, 1839-1884 (1998).
4. M. Facao, Ana Silva, Alexandre Lopes, Pedro Silva,"Computer simulation for calculating the second-order correlation function of classical and quantum light" European Journal Physics (2011)
5. Bernard Schutz,B.S. Sathyaprakash,"Physics, Astrophysics, and Cosmology, with Gravitational Waves" Living Reviews in Relativity (2009)
6. A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip, and A. Beveratos, Phys. Rev. A **88**, 013801 (2013).
7. Wikipedia contributors, "Degree of coherence," Wikipedia, The Free Encyclopedia, "Degree of coherence" (accessed January 30, 2021).