Second Order Glauber Correlation of Gravitational Waves using the LIGO observatories as Hanbury Brown and Twiss detectors

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Classical interferometers

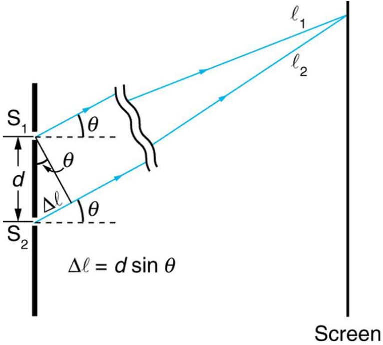

Figure 1: Classical amplitude interferometer [1].

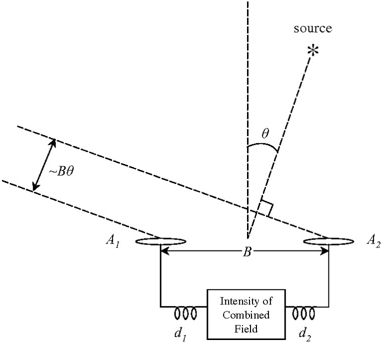

Figure 2: Classically connected intensity interferometer [2].

The LIGO-Virgo HBT interferometer

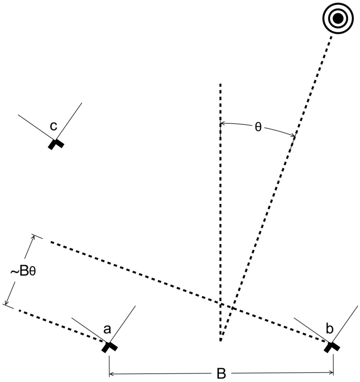

Figure 3: An HBT interferometer is not classically connected. The interference is in the non-classical wave functions [3].

2nd order correlation function

$$g^{(2)}(\tau) = \frac{\left\langle h_a^2(t)\, h_b^2(t+\tau) \right\rangle}{\langle h_a^2(t)\rangle \left\langle h_b^2(t+\tau)\right\rangle}$$

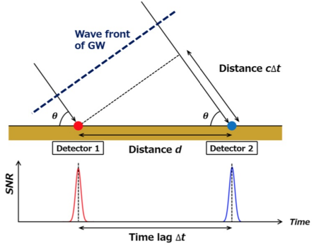

Figure 4: Time lag [4] $\Delta t \rightarrow \tau$ for LIGO-Virgo network.

2nd order correlations

The section on Glauber correlations is a demonstration that signals detected by LIGO-Virgo are not coherent.

Coherent or "steady" signal
$\langle I(t) \rangle = \langle I(t+\tau) \rangle$

Second order Glauber correlation function

$$g^{(2)}(0) = 1 + \frac{\langle [\Delta h^2(t)]^2 \rangle}{\langle h^2 \rangle^2}$$

$g^{(2)}$ for AM signals are similar to chaotic signals [5].

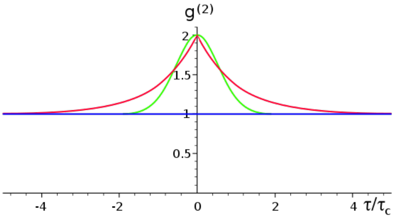

Figure 5: Two chaotic and one coherent signal's second order Glauber correlation [6]

Time Weighted Average

Using the conventional method the correlation function takes the form: $g^2(\tau) = \dfrac{\frac{1}{2T}\int_{-T}^{T} I(t)I(t+\tau)dt}{\frac{1}{4T^2}\int_{-T}^{T} I(t)dt \int_{-T}^{T} I(t+\tau)dt}$ An already known correlation was investigated, which was generated from the oscillatory intensity described by:

$I(t) = I_0(1 + A_0 \sin(\omega t))$

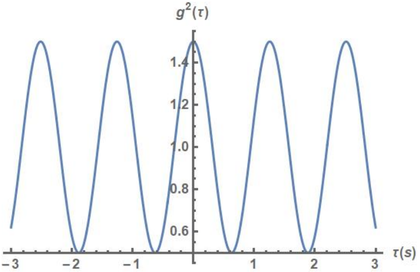

Figure 6: Glauber correlation of the oscillatory intensity using a time weighted average [7]

Intensity Weighted Average

$$
\begin{aligned}
g^2(\tau) &= \frac{G^{(2)}(t, t+\tau)}{G^{(1)}(t)G^{(1)}(t+\tau)} \\
G^{(2)}(t, t+\tau) &= \frac{\int_{-\infty}^{\infty} I(t)I(t+\tau)I(t+0)dt}{\int_{-\infty}^{\infty} I(t+0)dt} \\
G^{(1)}(t+\tau) &= \frac{\int_{-\infty}^{\infty} I(t+\tau)I(t+0)dt}{\int_{-\infty}^{\infty} I(t+0)dt}
\end{aligned}
\tag{1}
$$

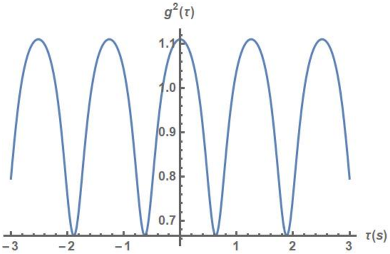

Figure 7: Glauber correlation of the oscillatory intensity using an intensity weighted average [7]

Comparison of Glauber Correlation Functions

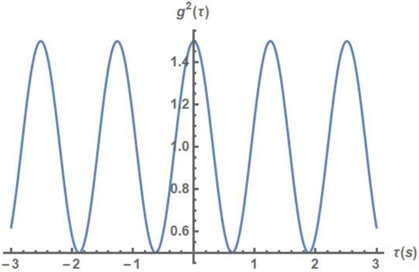

Figure 8: Standard time average [7]

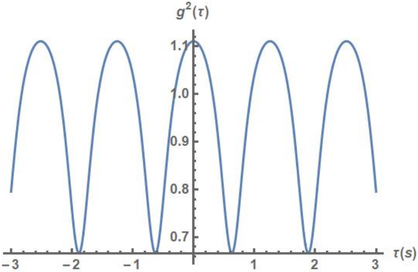

Figure 9: Intensity weighting [7]

$$g^2(\tau) = \frac{1}{2}(2 + A_0^2\cos(\omega\tau)) \quad (2)$$

$$g^2(\tau) = \frac{4(1 + \frac{1}{2}A_0^4\cos(\omega\tau)}{(2 + A_0^2)(2 + A_0^2\cos(\omega\tau)} \quad (3)$$

Sine-Gaussian Approximation of a Black Hole Merger

Using a function of the form: $h(t) = \frac{A_r}{d} e^{-(\frac{t+t_m}{b})^2} \cos(2\pi\omega t)$

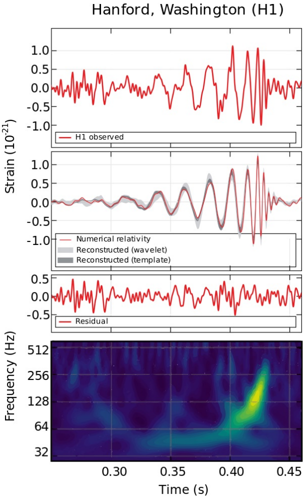

Figure 10: Discovery response [8]

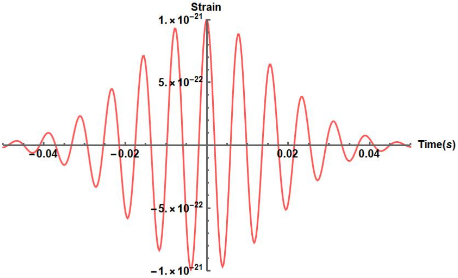

Figure 11: Sine-Gaussian Model [7]

Sine-Gaussian Correlation With a Time Weighted Average

The correlation was calculated using the time weighted average:

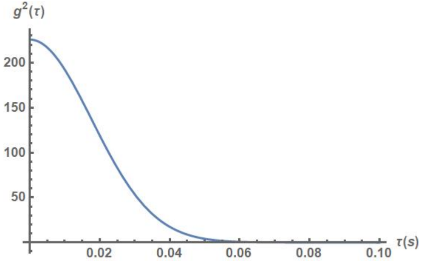

Figure 12: Sine-Gaussian correlation with a time weighted average [7]

$$g^2(\tau) = \frac{2e^{-\frac{\tau^2}{b^2}} T((\frac{T-2t_m-\tau}{b}) + (\frac{T+2t_m+\tau}{b}))}{b\sqrt{\pi}(((\frac{T-2t_m}{\sqrt{2}b}) + (\frac{T+2t_m}{\sqrt{2}b}))((\frac{T-2(t_m+\tau)}{\sqrt{2}b}) + (\frac{T+2(t_m+\tau)}{\sqrt{2}b})))} \quad (4)$$

$$\lim_{\tau\to\infty} g^2(\tau) = 0$$

Sine-Gaussian Correlation With an Intensity Weighted Average

The correlation was calculated using an intensity weighted average:

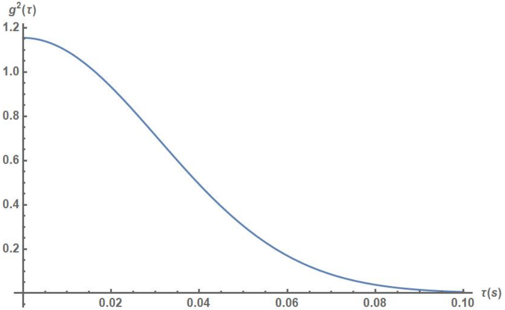

Figure 13: Calculated Correlation [7]

$$g^2(\tau) = \frac{2}{\sqrt{3}}\, e^{-\dfrac{\tau^2}{3b^2}} \qquad (5)$$

$$\lim_{\tau \to \infty} g^2(\tau) = 0$$

Some Characteristics of 2nd Order Glauber Correlation Functions

- Coherent light of a single frequency [9, 10] is defined as $g^2(\tau) = 1$.
- For a laser $g^2(\tau = 0) = 2$ for chaotic light [11].
- $g^2(\tau = 0) > 1$ for most signals except e.g. steady and bunched [9, 10].
- For chaotic light $g^2(\tau) = 0$ as $\tau \to \infty$, [6].

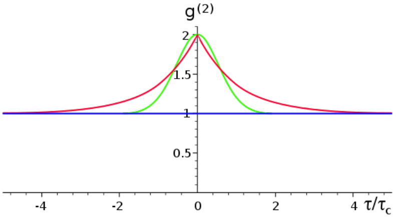

Figure 14: Two chaotic and one coherent signal's second order Glauber correlation [6]

Questions

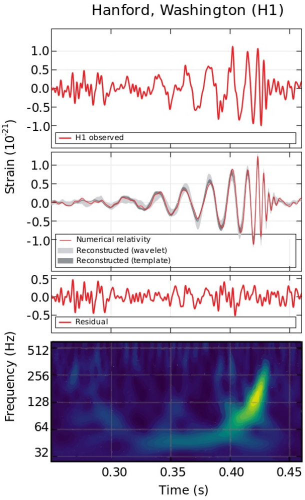

Figure 15: Discovery response [8]

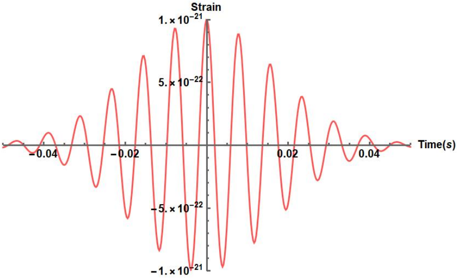

Figure 16: Sine-Gaussian Model [7]

References I

[1] Lumen Learning, "Wave Optics, Young's Double Slit Experiment".

[2] Frank Sebastian Rotondo, "Copy Imaging with Amplitude and Intensity Interferometers." (2004).

[3] Preston Jones, Andri Gretarsson, Ellie Gretarsson, Brennan Hughey, Darrel Smith, Michele Zanolin, and Douglas Singleton, (2019), arXiv:1907.00100 [gr-qc].

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[5] A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip, and A. Beveratos, Phys. Rev. A **88**, 013801 (2013).

References II

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[7] Alexander Barrett, Preston Jones, "Second Order Glauber Correlation of Gravitational Waves using the LIGO observatories as Hanbury Brown and Twiss detectors"

[8] B.P. Abbott, et al. "Observation of gravitational waves from a binary black hole merger." Physical Review Letters, 116(6), Feb 2016

[9] Mark Fox, *Quantum optics - an introduction* (Oxford University Press, 2006).

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References III

[11] A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip, and A. Beveratos, Phys. Rev. A **88**, 013801 (2013).