

Maximum Entropy Reasoning

- ▶ Beliefs are represented by **probabilistic conditionals**

$$(B|A)[p]$$

"If proposition *A* holds, then *B* follows with probability *p*."

- ▶ Probabilities reflect reasoner's **degrees of belief** in possible worlds $\omega \in \Omega$ ($\hat{=}$ interpretations)
- ▶ **Belief states** ($\hat{=}$ probability distributions) are inferred from belief bases \mathcal{R} ($\hat{=}$ sets of conditionals) following the **principle of maximum entropy**:

$$ME(\mathcal{R}) = \arg \max_{\mathcal{P} \models \mathcal{R}} - \sum_{\omega \in \Omega} \mathcal{P}(\omega) \cdot \log(\mathcal{P}(\omega))$$

Syllogisms

- ▶ Syllogisms are **quantified inference schemas**:

If $\{ \text{all, no, some, some not} \} M$'s are P 's
and $\{ \text{all, no, some, some not} \} S$'s are M 's,
then $\{ \text{all, no, some, some not} \} S$'s are P 's.

- ▶ Varying the quantifiers and the order of *M*, *P*, *S* leads to **256 syllogisms**, of which 24 are logically valid
- ▶ Acceptance of syllogisms is investigated to better understand **human understanding of quantification** (and of uncertainty in general)

PreCORE Challenge

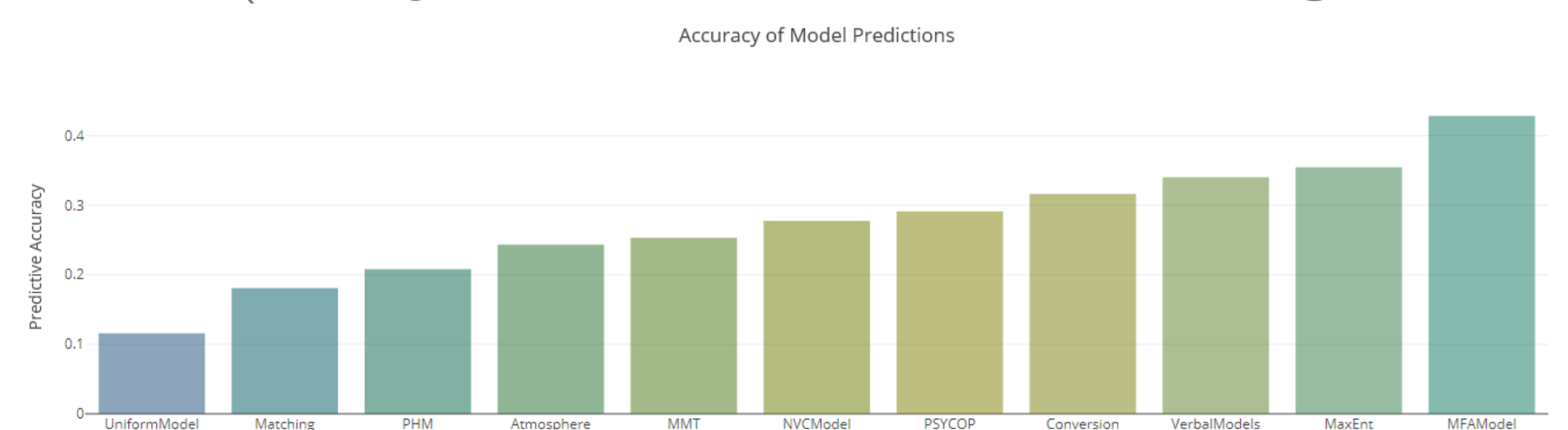
- ▶ **Initial situation**: The **premises of syllogisms** are shown to study participants and the participants are asked to draw the conclusion among

$\{ \text{all, no, some, some not} \} \{ S \text{'s are } P \text{'s}, P \text{'s are } S \text{'s} \}$

they do accept most ("no valid conclusion" (**nvc**) is also allowed)

- ▶ **Challenge**: Submit a model which **predicts the choices** of the participants (training data and answers of individuals to previous syllogism tasks are provided)
- ▶ Models compete on unknown data against **cognitive models** (cf. diagram) (similar data available on web to test models beforehand)

- ▶ **Results**¹ (training data: **Veser2018.csv**, test data: **Ragni2016.csv**)



Prediction accuracy of MaxEnt: **35.45 %**

¹wrt. data available on web (cf. refs) showing the same tendency as in the challenge

MaxEnt Model – Basic Version

1. Translate **premises of syllogism** into probabilistic conditionals ($\rightarrow \mathcal{R}$):
 $\{ \text{all, no, some, some not} \} M$'s are P 's $\rightarrow (M|P)[p]$
2. Calc. **MaxEnt probabilities** $q = ME(\mathcal{R})(P|S)$ for possible conclusions
"α *S*'s are *P*'s" with $\alpha \in \{ \text{all, no, some, some not} \}$
3. Return conclusion for which q **fits best** to α (if none fits, return **nvc**)

- ▶ **Parameter selection**:

- Translate $\alpha \in \{ \text{all, no, some, some not} \}$ to $p \in \{ 1, 0, 0.65, 0.15 \}$
- Tolerance of ± 0.1 when saying that q fits to $\alpha \in \{ \text{some, some not} \}$
- If several conclusions fit, return the first one

- ▶ **Variant submitted to challenge**:

Portfolio Model \rightarrow Test if participant is a **MaxEnt reasoner** beforehand

References

- ▶ **Maximum Entropy Reasoning**:
J.B. Paris (2006): **The Uncertain Reasoner's Companion: A Mathematical Perspective**. Cambridge University Press
- ▶ **Syllogisms**:
S. Khemlani, P.N. Johnson-Laird, P.N. (2012): **Theories of the Syllogism: A Meta-Analysis**. Psychological Bulletin, 138, 427-457
- ▶ **PreCORE Challenge**:
M. Ragni, N. Riesterer, S. Khemlani (2019): **Predicting Individual Human Reasoning: The PRECORE-Challenge**. Proceedings of CogSci2019 (Workshop)
<https://www.cognitive-computation.uni-freiburg.de/modelingchallenge> (**PreCORE Challenge**)
<https://orca.informatik.uni-freiburg.de/ccobra> (**CCobra Framework**)

Acknowledgments

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- ▶ <https://www.hybrid-reasoning.org/>