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Abstract

Information Geometry of Estimating Functions in Parametric Statistical Models †

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In information geometry, a parametric statistical model (a family of probability density functions) is treated as a differentiable manifold, where the Riemannian metric called Fisher metric and the pair of two torsion-free dual affine connections called the exponential and mixture connections play essential roles for statistical inference. For example, the maximum likelihood estimation in an exponential family can be interpreted as the orthogonal projection of the geodesic defined by the mixture connection. This comes from the fact that an exponential family is a dually flat space, where the curvature and the torsion tensors of the two dual affine connections are all equal to zero. Recently, it has been found by the authors that a general estimating function naturally induces a similar geometric structure on a statistical model, that is, a Riemannian metric and a pair of dual affine connections, through the concept called pre-contrast function. In this case, however, one of the affine connections is not necessarily torsion-free, especially when the estimating function is not integrable with respect to the parameter of the statistical model. In this presentation, we explain the construction and some properties of this geometric structure with related concepts in information geometry. In addition, some of its statistical implications are discussed using an example of non-integrable estimating functions which induces a partially flat space, where only one of the induced affine connections is flat (curvature-free and torsion-free).