Gauge Freedom of Entropies on q-Gaussian Distributions [†]

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A q-Gaussian distribution is a generalization of an ordinary Gaussian distribution. The set of all q-Gaussian distributions admits information geometric structures such as an entropy, a divergence and a Fisher metric via escort expectations. The ordinary expectation of a random variable is the integral of the random variable with respect to its probability distribution. Escort expectations admit us to replace the law to any other distributions. A choice of escort expectations on the set of all q-Gaussian distributions determines an entropy and a divergence. The q-escort expectation is one of most important expectations since this determines the Tsallis entropy and the alpha-divergence.

The phenomenon gauge freedom of entropies is that different escort expectations determine the same entropy, but different divergences.

In this talk, we first introduce a refinement of the q-logarithmic function. Then we demonstrate the phenomenon on an open set of all q-Gaussian distributions by using the refined q-logarithmic functions. We write down the corresponding Riemannian metric.