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A KINETIC MODEL FOR PEDESTRIAN EVACUATION IN A CORRIDOR WITH AN AGGRESSIVE SPARSE COUNTERCURRENT.

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OUTLINE

- A kinetic model is proposed to simulate a pedestrian flow in a 1D, two-way evacuation situation where a dense outward passive flow encounters a scarse, yet aggressive, countercurrent.
- Pedestrians walking in opposite directions are considered as two distinct species, resulting in a set of two kinetic (Boltzmann-like) coupled equations.
- Frontal interactions with no passing, in which one individual changes direction are considered reactions.
- Two scenarios are addressed within a two-moment approach, depending on the relative aggressiveness of the species and the density of the passive crowd.
- A three moment model is in progress, where a viscous-type effect enters the stabilization process.

KINETIC MODEL

Two kinetic equations are established, featuring a force term arising from the aggressiveness (desire to attain a higher speed $w_i(x,v_i,t)$ in a characteristic time τ_i):

$$Df_i \equiv \frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial x} + \frac{\partial}{\partial v_i}\left(\frac{w_i(x,v_i,t)-v_i}{\tau_i} f_i\right) = \sum_{j=1}^{2} I_{ij}, \quad \text{for} \quad i=1,2.$$

Here $i=1$ species moves to the right and $i=2$ to the left.
The interaction model is based on the following hypotheses:

1. Only binary interactions are considered in two modalities: direct (same species "catch up") and cross, or frontal, interactions (different species "face to face").
2. Overtaking (direct) or passing (cross) is allowed with a probability p, which depends on the local density.
3. In overtaking or passing, both pedestrians continue with their velocities prior to the encounter.
4. When overtaking is not possible, the individual reaching from behind adapts to the slower speed.
5. In a frontal encounter we define a probability r_{ij} that dictates which individual changes direction, and thus acquires the other's speed.

The proposed general structure for the interaction terms is:

$$I_{i1} + I_{i2} = (1-p)\sum_{j=1}^{2}\left(\Gamma_{ij}^{(+)} - \Gamma_{ij}^{(-)}\right) \quad \text{for} \quad i=1,2, \tag{1.1}$$

such that $(1-p)\Gamma_{ij}^{(+/-)} dx dv_i dt$ stands for the total number of pedestrians added/subtracted from the set $[x, x+dx] \times [v_i, v_i+dv_i]$ as a consequence of the interactions between both type of walkers in a time no longer that dt.

INTERACTION MODEL

Direct interactions

- Gain term Γ_{ii}^{+}:
  - Select one specific non-primed walker with velocity v_i.
  - The individuals approaching it from behind with $|v_i'| > |v_i|$, will reach the one in the front in a given time interval dt. - The primed (faster) pedestrians will slow down, which leads to a gain in the phase space cell corresponding to v_i.
  - The number of these interactions in a time interval dt is the product of the number of points in the given cell, $f_i dv_i dx$, and the one in an 1D cylinder of height $|v_i' - v_i|$, that is

$$\Gamma_{ii}^{(+)} dx dv_i dt = dx dv_i dt f_i \int_{|v_i'|>|v_i|} dv_i' f_i' |v_i' - v_i|. \tag{2.1}$$

- Loss term Γ_{ii}^{-}:
  - Focus on the slower (primed) individuals in the front, with velocities $|v_i'| < |v_i|$.
  - The speed reduction of the unprimed walker leads to a loss in the cell v_i.

$$\Gamma_{ii}^{(-)} dx dv_i dt = dx dv_i dt f_i \int_{|v_i'|<|v_i|} dv_i' f_i' |v_i - v_i'|. \tag{2.2}$$

Cross interactions

Define r_{ij} as the probability that the individual of class i changes direction and acquires speed v_j.

- Gain term $\Gamma_{ij}^{(+)}$ (probability r_{ji}):

$$\Gamma_{ij}^{(+)} dx dv_i dt = (1-r_{ij}) dx dv_i dt f_i \int_{D_j} dv_j f_j |v_i - v_j|, \tag{2.3}$$

- Loss term $\Gamma_{ij}^{(-)}$ (probability r_{ij}):

$$\Gamma_{ij}^{(-)} dx dv_i dt = r_{ij} dx dv_i dt f_i \int_{D_j} dv_j f_j |v_i - v_j|. \tag{2.4}$$

where D_j is the domain of f_j ($D_1 = [0,\infty)$, $D_2 = (-\infty, 0]$).

KINETIC EQUATIONS

$$Df_1 = (1-p)\left\{ f_1 \int_0^\infty dv'_1 f_1' (v_1' - v_1) + (r_{21} - r_{12}) f_1 \int_{-\infty}^0 dv_2 f_2 (v_1 - v_2)\right\}, \tag{3.1}$$

$$Df_2 = (1-p)\left\{ f_2 \int_{-\infty}^0 dv_2' f_2' (v_2 - v_2') + (r_{12} - r_{21}) f_2 \int_0^\infty dv_1 f_1 (v_1 - v_2)\right\}, \tag{3.2}$$

TWO MOMENT MODEL FOR HASTY WALKERS

- The two moment model considers the evolution of densities (ρ_i) and flow velocities (u_i):

$$\rho_i = \int_{D_i} f_i dv_i, \quad \rho_i u_i = \int_{D_i} v_i f_i dv_i, \tag{4.1}$$

- In the hasty walkers model one assumes $w_i = \omega_i v_i$ and defines an aggressivness parameter a_i:

$$a_i = \frac{(\omega_i - 1)}{\tau_i}$$

From Eqs. (3.1-3.2) one obtains (here $\Delta_u = u_1 - u_2 > 0$ and $R = r_{21} - r_{12}$):

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial(\rho_i u_i)}{\partial x} = \pm R \rho_i \rho_j (1-p)\Delta u, \quad i,j = 1,2 \quad i \neq j \tag{4.2}$$

$$\frac{\partial(\rho_i u_i)}{\partial t} + \frac{\partial\left(\rho_i \theta_i + \rho_i u_i^2\right)}{\partial x} - a_i \rho_i u_i = (1-p)\left\{\mp \rho_i^2 \theta_i \pm R \rho_j \rho_i (u_i \Delta u \pm \theta_i)\right\}, \quad i,j = 1,2, \quad i \neq j \tag{4.3}$$

The velocity variance $\rho_i \theta_i = \int_{D_i} (v_i - u_i)^2 f_i dv_i$ will close the system within some adequate assumption (see below).

ADDITIONAL ASSUMPTIONS

- The passing probability is here defined as $p = 1 - \frac{\rho}{\rho_M}$, where $\rho = \rho_1 + \rho_2$ and ρ_M is the maximum density for the system.
- For r_{ij} we propose

$$r_{ij} = 1 - \frac{a_i \rho_i}{a_1 \rho_1 + a_2 \rho_2}. \tag{4.4}$$

  such that the relevant quantity for deciding which individual turns around is local density times agressiveness.
- For the closure of the system we propose θ_i to be given by the solution for the distribution functions f_i corresponding to an homogeneous and stationary situation, where it is assumed that cross interactions are scarce. Under these assumptions, one obtains (see Ref.[1]) $\theta_i = u_i^2/\alpha_i$ where

$$\alpha_i = \pm(1-p)\frac{\rho_i^s u_i^s}{a}. \tag{4.5}$$

  and s stands for the stationary and homogeneous state.

RESULTS FOR AN EVACUATION SCENARIO

- In order to examine the response of the system to small perturbations, a stability analysis is carried out. We write the perturbed variables as $X_i = \bar{X}_i + \delta X_i$ where δX_i is a small fluctuation
- The 0th order system has solutions: $\bar{\rho}_i = \bar{u}_i = 0$, $\bar{u}_1 = \bar{u}_2 = 0$, and $\frac{\bar{\rho}_2}{\bar{\rho}_1} = \frac{a_1}{a_2}$.
- If the evacuation route is to the right we have $\bar{\rho}_2 \ll \bar{\rho}_1$ and $a_1 \ll a_2$ (see the first bullet of the OUTLINE). Thus, in this scenario we consider is $\frac{\bar{\rho}_2}{\bar{\rho}_1} = \frac{a_1}{a_2}$.
- For simplicity, we describe the flow from the comoving frame of the exiting crowd ($\bar{u}_1 = 0$).

Considering $\frac{a_1}{a_2} = \mu \ll 1$

At least one root of the dispersion relation (both in zeroth and first order in *mu*) for the linearized system lies on the right hand side of the complex plane, resulting in an instability.This translates in an unavoidable congestion in this scenario.

Considering $\bar{\rho}_2 = \nu \ll 1$

In this case, to lowest order in ν one obtains oscillating (stable) modes. To first order in *nu* one can observe three different scenarios depending the ratio of aggressiveness and the magnitude of $\bar{\rho}_1^2 \bar{u}_2$ (relative to u_1):
1. If $a_1 < a_2 \leq \sqrt{3} a_1$ the system is unstable. However we require $a_2 \gg a_1$ and thus this scenario is irrelevant. 2. If $a_2 > 3a_1$, the system is stable only if

$$\bar{\rho}_1^2 \bar{u}_2 < 4 a_2 \nu^4 \frac{3\nu^2 - 1}{(4\nu^2 + \nu - 1)^2} \tag{6.1}$$

CONCLUSIONS AND FINAL REMARKS

- A model for bidirectional pedestrian flow in a corridor has been established based on kinetic theory. This model could be useful for evacuation planning.
- The system has been linearly analyzed within a two moment approach considering a closure obtained form the stationary homogeneous state with negligible front encounters.
- Two scenarios have been explored. A congestion was found to be unavoidable for a small ratio of agressiveness. In the case of such ratio being only small when compared with the density of the outgoing flow, a stable flow is attainable as long as the relative velocity of both velocities is small enough.
- A viscous term in the transport equations, which appears with a more realistic closure for θ_i, will lead to stabilization depending on the wavenumber of the perturbations. This is work in progress and will be reported elsewhere.

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