Dissipative extension of electrodynamics

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Abstract

A derivation of the coupled system of electromechanical continua is presented in the framework of nonequilibrium thermodynamics. Nonlinear polarisation, Gauss's law, Maxwell stress, entropy current are consequences of a general dissipative treatment, based on the ideas in [1].

Electrodynamics and dissipation

Dissipative effects are represented in electrodynamics through electric resistance and nonclassical polarisation and magnetisation. Sometimes radiation itself is considered as a dissipative effect. However, the field equations of electrodynamics are accepted as ideal and nondissipative.

There are some well known problematic aspects of ideal electrodynamics. Those are the singular energies of point charges, point dipoles, and the radiation reaction force, leading to the self accelerated motion when mechanics and electrodynamics are to be treated together. We know well how charges are moving in an external field, and we also can calculate the electromagnetic fields produced by charges moving on fixed paths. However, moving charges self-interacting with their self-fields is problematic both from a mathematical and a physical point of view.

The study of electromagnetic singularities leads to the concept of mass renormalisation [2]. According to several experts, renormalisation has a dissipative flavour but without a conceptual framework of dissipation.

Gradient theories of electrodynamics and mechanics

A weakly nonlocal, that is, gradient extension of the Maxwell equations is a promising approach to remove the mentioned singularities directly from the field theory [3, 4]. Gradient extension is also a long-standing program of continuum theories in general, particularly in continuum mechanics [5]. There are different methods [6, 7] that considering the second law or not. Also, the representation of the frame indifference may be different, which is a key aspect when second law compatibility is investigated.

Nonequilibrium thermodynamics and ideal evolution

Nonequilibrium thermodynamics is dealing with the consequences of the second law. That kind of consequences independent of the material structure and only the fundamental balances are introduced as conditions. Modern approaches can construct evolution equations without the help of variational principles, only with the help of the second law. This is well known for a long time ago for homogeneous systems. For internal variables, the second law results in relaxation type ordinary differential equations. Another known example is extended heat conduction[8, 9]. In the general case, for weakly nonlocal state spaces, it is not straightforward, and the various methods differ in complexity and in the number of assumptions.

One of the essential benchmarks is to reproduce the ideal evolution, construct Euler-Lagrange equations without variational principles. It is generally and systematically possible if thermostatics and extensivity are suitably generalised for gradient dependent thermodynamic state spaces. Remarkably, Newtonian gravity emerges from thermodynamics if a weakly nonlocal scalar field is considered as a thermodynamic state variable and balances of mass, momentum and energy are considered as constraints [10].

Coupled hydrodynamics and electrostatics

In this case, the state variable is the electric potential φ. The Gibbs relation for fluids with specific quantities is written as

$$T\mathrm{d}s - p\mathrm{d}v - \frac{\mathbf{P}}{\rho}\mathrm{d}\mathbf{E} = \mathrm{d}u - \mathrm{d}(e\varphi) - \mathrm{d}\left(\epsilon_0 \frac{\mathbf{E}^2}{2\rho}\right).$$

Here u, e, s, v are the specific internal energy, charge density, entropy and volume, T, p are the temperature and the pressure, $\mathbf{E} = -\nabla\varphi$ is the electric field and $\mathbf{P}$ is the polarisation field, and ϵ_0 is the permeability of the vacuum. Therefore the specific entropy and the internal energy are both depend on the electric potential and its gradient:

$$\hat{s}(u, e, \rho, \varphi) = s\left(u - e\varphi + \epsilon_0 \frac{\nabla\varphi\cdot\nabla\varphi}{2\rho}, \rho, \nabla\varphi\right)$$

The balances of mass, momentum, energy and charge are considered as constraints, characterising the extensive quantities. In a substantial form these balaces are the following

$$\begin{aligned}\dot{\rho} + \rho\nabla\cdot\mathbf{v} &= 0,\\ \rho\dot{\mathbf{v}} - \nabla\cdot\mathbf{T} &= \mathbf{0},\\ \rho\dot{u} + \nabla\cdot\mathbf{q} &= \mathbf{T} : \nabla\mathbf{v},\\ \rho\dot{e} + \rho\nabla\cdot\mathbf{v} &= 0.\end{aligned}$$

Here $\mathbf{v}, \mathbf{q}, \mathbf{T}, \mathbf{j}$ are the velocity field, the heat flux, the second-order stress tensor and the electric current density, respectively. The overdots are substantial time derivatives; the central dots denote contractions.

The calculation of the entropy production is straightforward, the simplest method is the separation of divergences. The entropy balance is obtained in the following form:

$$\rho\dot{s} + \nabla\cdot\left[\frac{1}{T}(\mathbf{q} - \varphi\mathbf{j} + \dot{\varphi}\mathbf{D})\right] =$$

$$(\mathbf{q} - \varphi\mathbf{j} + \dot{\varphi}\mathbf{D})\cdot\nabla\left(\frac{1}{T}\right) + \frac{\dot{\varphi}}{T}(\nabla\cdot\mathbf{D} - \rho e) + \left(\mathbf{T} + p\mathbf{I} - \mathbf{DE} + \frac{\mathbf{E}^2}{2}\mathbf{I}\right) : \frac{\nabla\mathbf{v}}{T} \geq 0.$$

Here the electric displacement $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ and $\mathbf{I}$ denotes the second-order unit tensor.. The thermodynamic forces and fluxes for the thermal, mechanical, electric matter and electric fields can be identified conveniently. Remarkable is the coupling of the scalar electric interaction with the scalar part of the mechanical interaction. It is also remarkable that the usual relation Maxwell stress and electrostatic force density, $\rho\mathbf{f}_{estat}$, that is

$$\nabla\cdot\left(\mathbf{DE} - \frac{\mathbf{E}^2}{2}\mathbf{I}\right) = -\rho e\mathbf{E} - \nabla\mathbf{E}\cdot\mathbf{P} = \rho\mathbf{f}_{estat}$$

appears only in the ideal limit, without dissipative effects.

Concluding remarks

- Dissipative evolution equations of electrodynamics coupled to continuum thermomechanics can be derived from the second law in the *quasistationary approximation*. Radiation is the basic challenge.
- Ideal, nondissipative evolution emerges when the corresponding thermodynamic force is zero. Stress-force relations are natural consequences.
- A simplified example is given in this poster. A similar calculation is straightforward for the complete system of Maxwell equations without radiation.
- For a relativistic extension, radiation reaction must be considered. In this respect, distribution theory and absolute treatment of spacetime can be instructive, see [11, 12].

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Acknowledgements

The work was supported by the grants National Research, Development and Innovation Office NKFIH 124366(124508) and NKFIH 123815. The support of TKP is acknowledged.