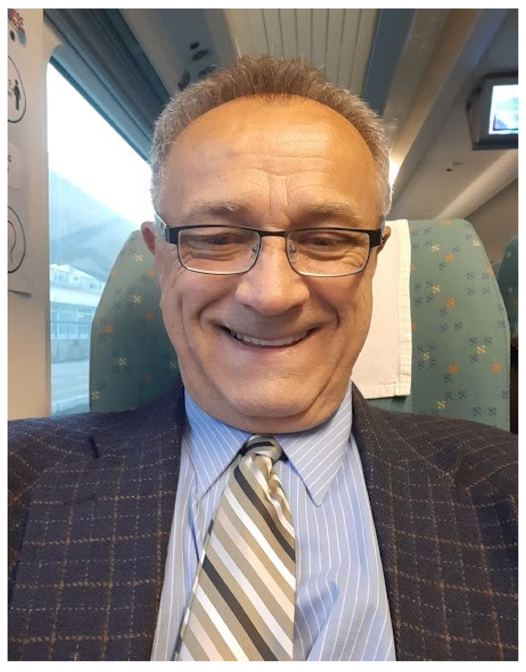


Entropy of Vostok Ice Core Data and Kalman Filter Harmonic Bank Climate Prediction Engine

[1]Migdat Hodzic and [2]Ivan Kennedy

[1]American University in Bosnia and Herzegovina, Sarajevo

[2]Sydney Institute of Agriculture, University of Sydney, NSW Australia

migdathodzic@gmail.com

Introduction. In our current work we are looking to use Spectral Entropy (based on normalized FFT harmonics squared amplitude) of Vostok CO_2 data prediction via KFHB [1] as a measure of data complexity and stochastic distance from the maximum entropy of White Noise signal. In particular we define a novel idea of Spectral Entropy Ratio (Figure 5) as a ratio between corresponding white noise entropy and the Vostok signal entropy for various number of harmonics used. This ratio is compared to KL Distance and Dissequilibrium Distance, and shown to be superior measure of KFHB complexity as it exhibits much large changes over number of harmonics used, compared to other two measures. Arrows (Figures 5 and 6) indicate effectiveness of KFHB with 7 harmonics with respect to the Entropy Ratio. The Ratio change in Figure 5 indicates "slowing down" for higher N hence the contribution of higher harmonics is getting smaller and smaller. The other two distance measure are very much flat over the whole range of number of harmonics.

Figure 1. Vostok Ice Core CO2 and Temperature Data

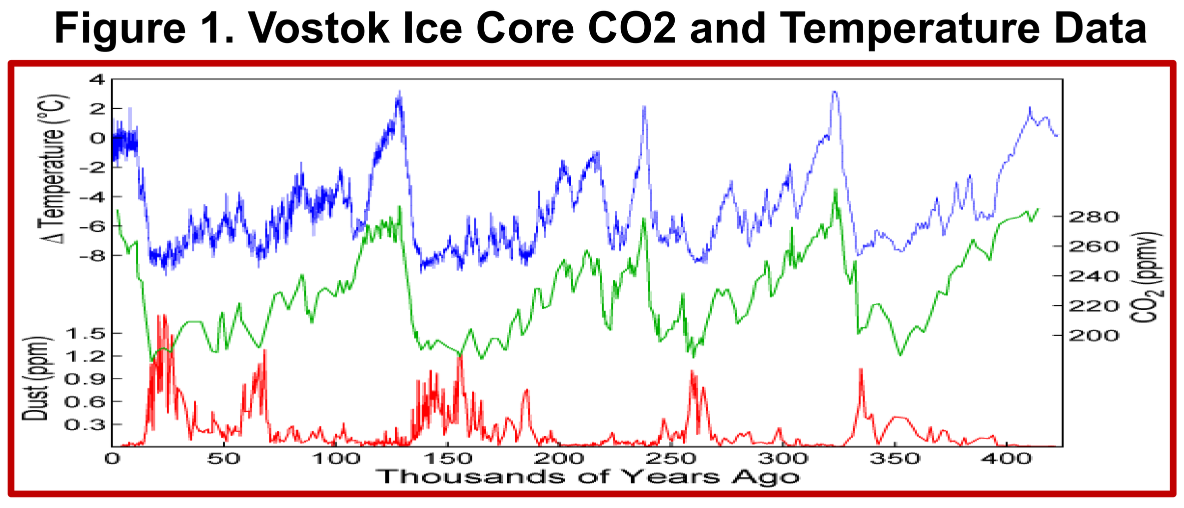


Figure 2. Climate Prediction Engine (CPE)

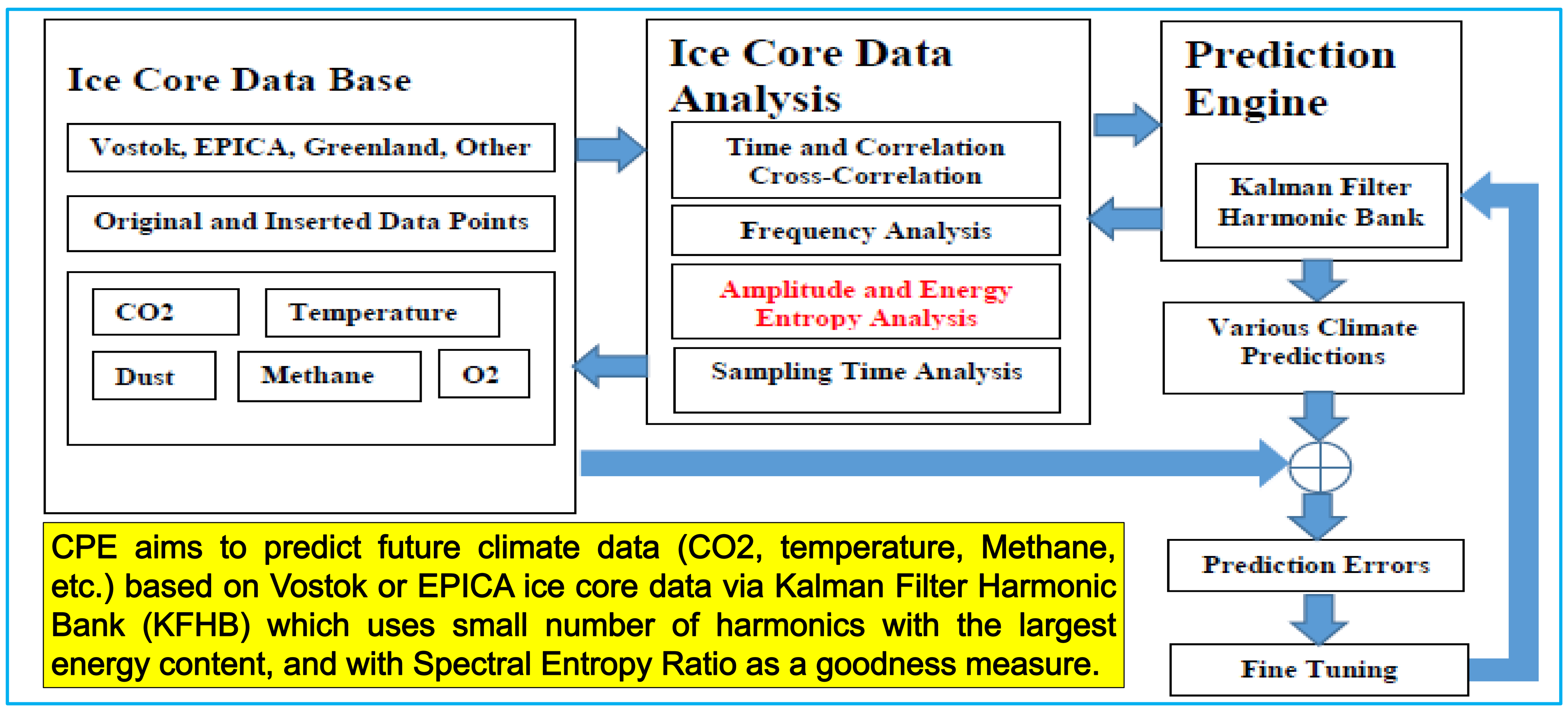
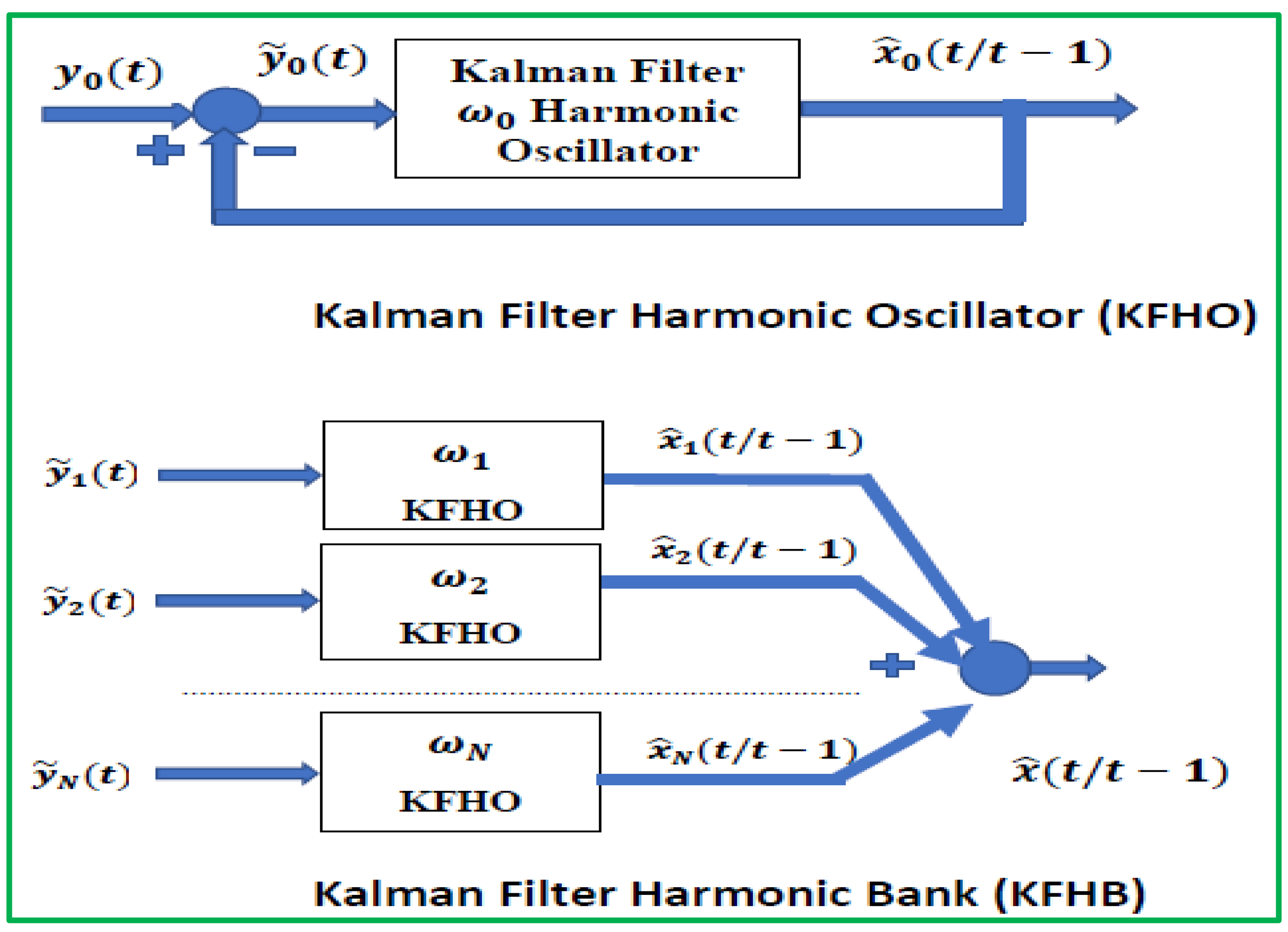
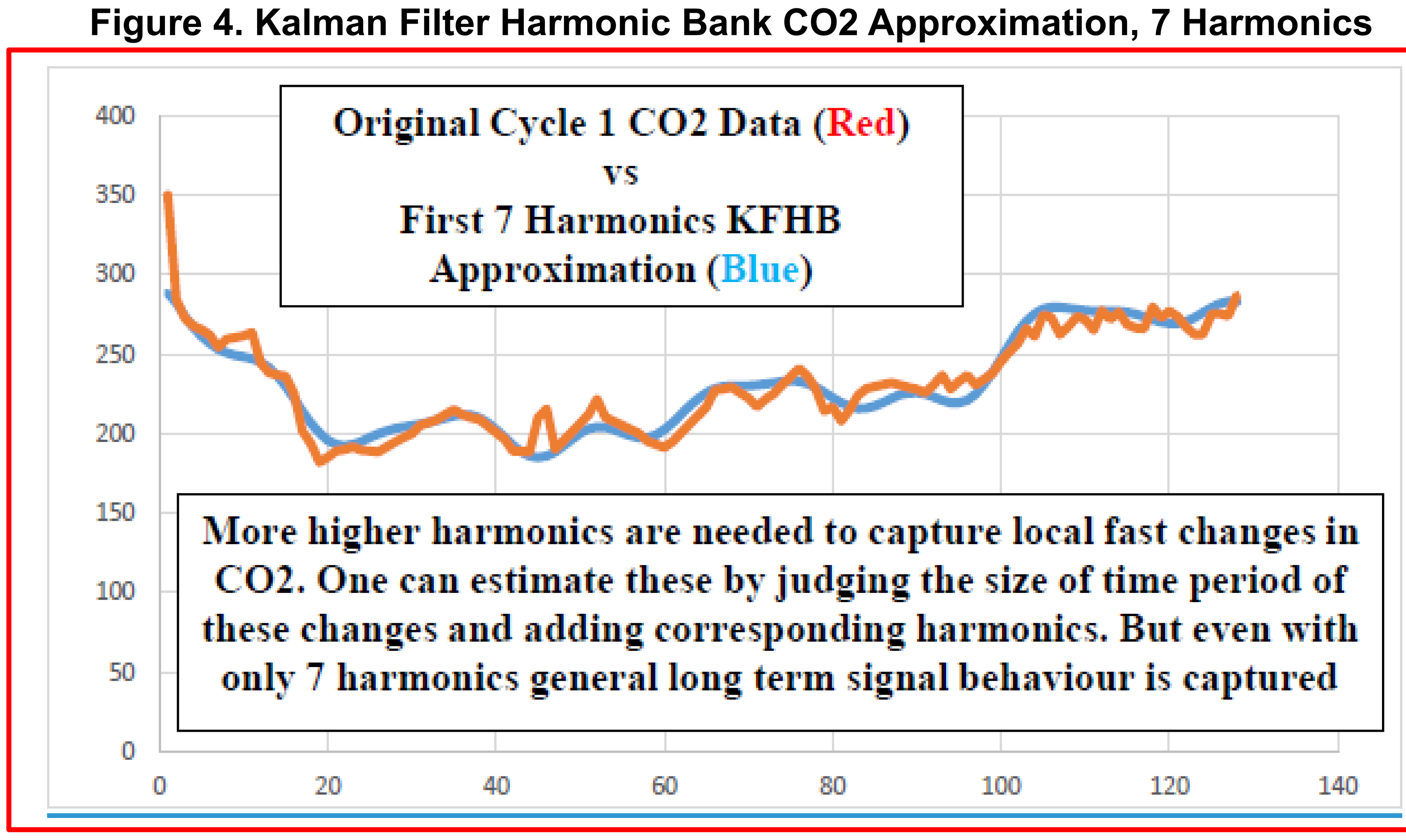


Figure 3. Kalman Filter Harmonic Oscillator (single Harmonic) and Harmonic Bank for N Vostok FFT Data Harmonics



KFHB consists of N harmonics with frequencies ω1 … ωN

Figure 4. Kalman Filter Harmonic Bank CO2 Approximation, 7 Harmonics



Similar results can be obtained for other Ice Core data such as temperature and Methane

Figure 5. KFHB Spectral Entropy Ratio and white noise distance measures vs number of harmonics

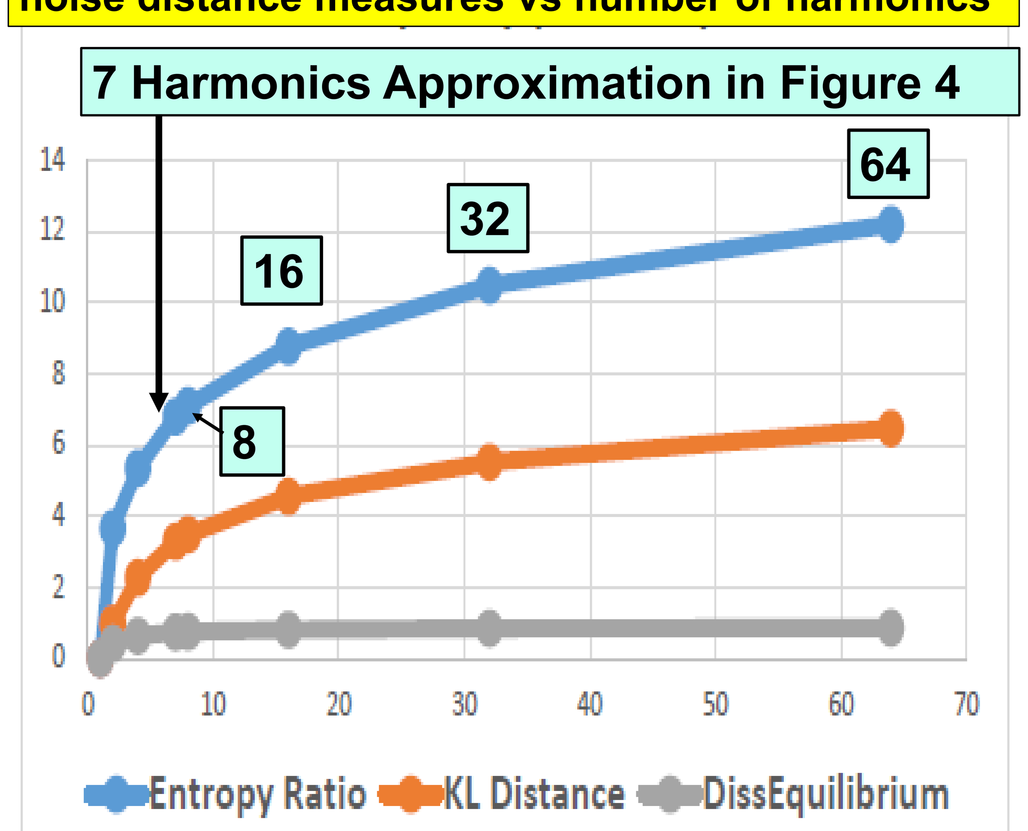


Figure 6. White noise and Vostok data Spectral Entropies vs number of harmonics

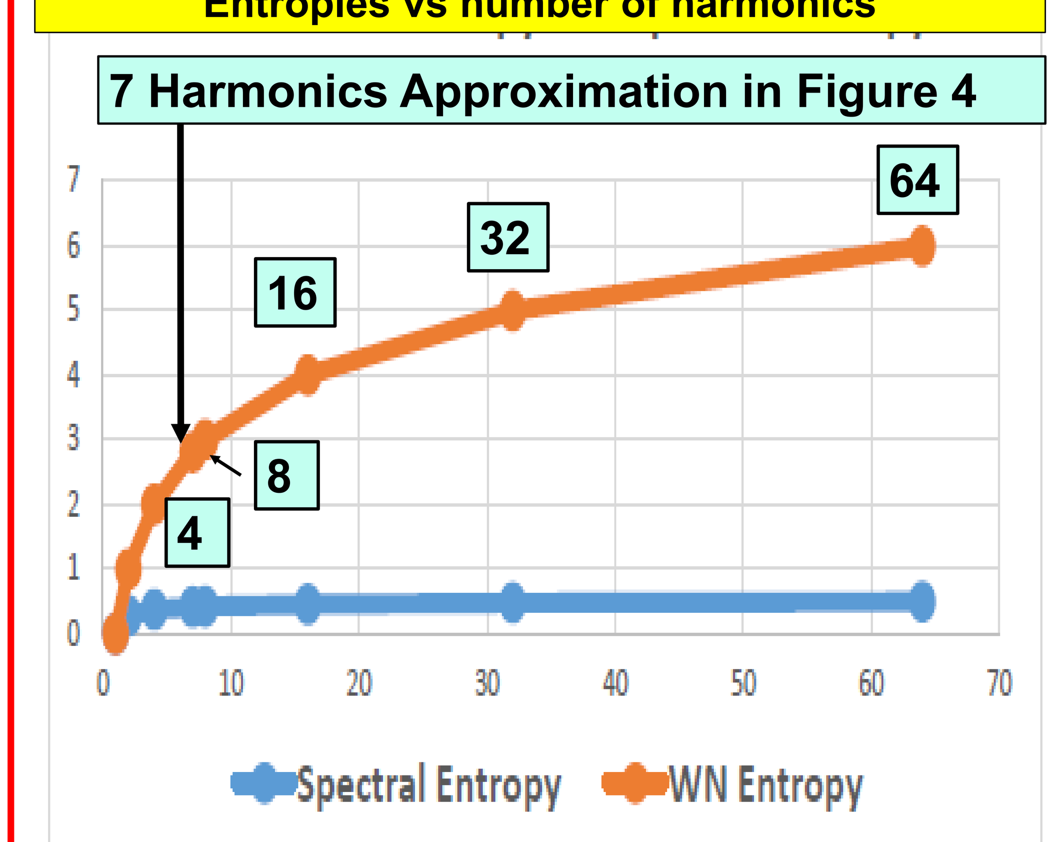


Table 1. Numerical values for Figure 5.

No of Harmonics	Entropy Ratio	KL Distance	Diss-Equilibrium
1	0	0	0
2	3.655731	0.983341	0.41040579
4	5.346831	2.271951	0.63875172
7	6.796865	3.283705	0.73944005
8	7.103256	3.483447	0.75584176
16	8.763917	4.55109	0.81346438
32	10.48351	5.521762	0.84222255
64	12.2002	6.465139	0.85620451

KL Distance = (1/2)*Σ{[p(x)*log[p(x)/(1/N)]+[(1/N)*log[(1/N)/p(x)]]}

p(x) = Normalized FFT coefficients, squared

DissEquilibrium from White Noise: Σ[p(x) – 1/N] **2

Table 2. Numerical values for Figure 6.

No of H's	Spectral Entropy in BITS	White Noise Entropy in BITS	Entropy Ratio
1	0	0	0
2	0.273543	1	3.655731
4	0.374053	2	5.346831
7	0.413037	2.807355	6.796865
8	0.422342	3	7.103256
16	0.456417	4	8.763917
32	0.476939	5	10.48351
64	0.491795	6	12.2002

White Noise Entropy: H = LOG (N,2), N – Number of harmonics

Description and Conclusions

- Vostok historical climate data is shown in Figure 1.
- Figure 2 is a block diagram of our Climate Prediction Engine (CPE) [1].
- In Figure 3 we have block diagram of single harmonic KF Oscillator and a bank of KF's for N harmonics
- Figure 4 shows KF bank for N = 7, first 7 harmonics with the highest signal energy [1].
- Effectiveness of KFHB can be measured in time domain using MAPE or other time based error measures [1].
- In frequency domain there are also various ways for assess effectiveness of KFHB based on number of harmonics used
- Figure 5 indicates Spectral Entropy Ratio together with two additional "distance" measures. The idea is to find a measure which captures the best complexity of the Vostok Data set compared to an underlying white noise process with the same number of spectral harmonics. Larger the range of values of these measures, the better it is a measure to determine goodness of KFHB estimates
- Figure 6 shows White Noise as well as Vostok data spectral entropies for reference
- Tables 1 and 2 are numerical values for Figures 5 & 6
- A full description of the paper is in preparation. We are planning to fully analyse other ways to employ Spectral Entropy as a measure of goodness of KFHB data estimates [2].
- We will also look into other types of relational entropies, such as, for example, Spectral Cross Entropies and the corresponding coefficients to capture informational relationship between CO2 and temperature (or other ice core data) in one or more common data cycles, but also between different cycles [1],[2].

References

1. Hodzic M and Kennedy IR, Chapter in Glaciers and Polar Environment, IntechOpen 1/2021.
2. Hodzic M and Kennedy IR, In preparation, 2021