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Entropy from thermodynamics to signal processing

Jordão N. O. Júnior[1] Marcelo Pinheiro[2] Ivonete Ávila[3] Carlos D. Maciel[1]

[1]Signal Processing Laboratory [2]Versatus HPC [3]Unesp Guaratinguetá

Introduction

Entropy as a concept is misused [1] and misunderstood [2] even by academics. It is often described as "disorder" which trivialise the term or it is mistakenly used as a measure of complexity. This is due the many different backgrounds from researchers using the same concept. However, it can be show that all different kinds of entropy are in fact the same and not equivalent in the formal sense as the concept of complexity.

Method

- Show mathematical **equivalence between entropies** from from Clausius $dS = \delta Q/T$, Boltzmann $S = k\log W$ and Gibbs $S = -k\sum_i p_i \log p_i$.
- Show **equivalence with Shannon's Entropy**.
- Bibliography about entropy in thermodynamics being **dimensionless**.
- **Physical links**: Maxwell's Demon, Landauer's principle.
- (Kolmogorov) Complexity vs Entropy

Results

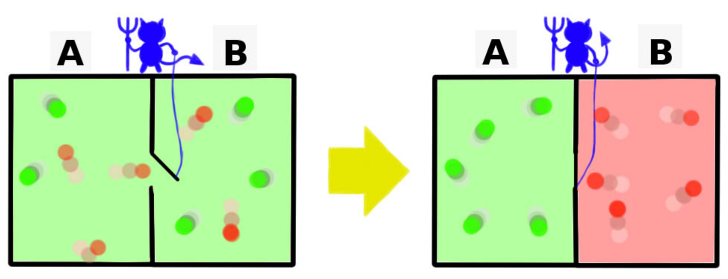

Figure 1: The Maxwell's Demon: a being who knows the velocity of every particle in the box, and can select their passages using a opening in the wall that divides it could separate those with high energy from those with low energy without realising work, **violating the second law of thermodynamics**. Actually, the demon has to forget past states of the system, **and by the Landauer's Principle, this process generates heat** (at least $kT\log 2$ J per bit erased) and entropy.

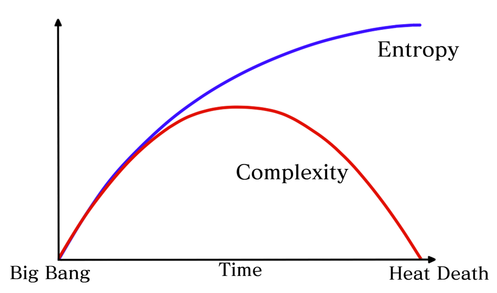

Figure 2: Complex structures need to **exchange energy with the environment to reduce their entropy** (this way, increasing the entropy of the environment). Therefore, they can only form in times in which entropy can increase. Entropy, however, is always increasing by the second law of thermodynamics. The early universe and the far future universe are both simple, uniform hot and dense state in the beginning and empty space in the end, but the entropy was low in the past and will be at its maximum at the end by heat death. Source of figure: adapted from [3].

Results

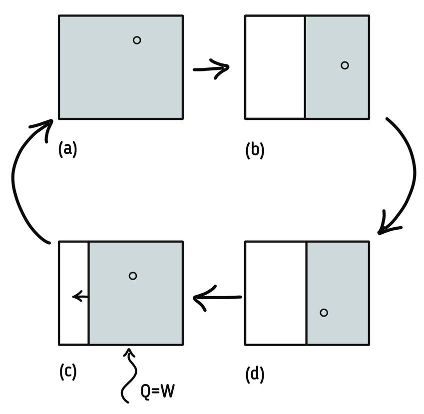

Figure 3: The process to extract work from a system thought by Szilard: in (a) there is a single molecule of a fluid inside a box with energy Q. If one knows in which half of the box the molecule is (i.e., a single bit of information about is position), it can insert a piston halving the box (b) and from the expansion of the fluid, extract work ((c) and (d)) $W = Q$ from the system while it returns to its initial state. Figure adapted from [4].

- $$S(W_1) + S(W_2) = S(W_1 W_2) \implies S = k\log W \qquad (1)$$
- $$\text{Thermal equilibrium} \implies \text{Maximum entropy} \xrightarrow[\text{partition function } Z]{\text{Lagrangian}} S_{\text{Clausius}} \equiv H_{\text{Shannon}} \qquad (2)$$
- $$\text{KL-divergence} \implies \text{Thermal system} \xrightarrow[\text{Markov chain}]{\text{Time evolution}} \text{Second Law of Thermodynamics} \qquad (3)$$

Conclusion

- Conceptual and mathematical background for equivalence between entropies;
- Relating entropy and complexity in thermodynamics and information theory;

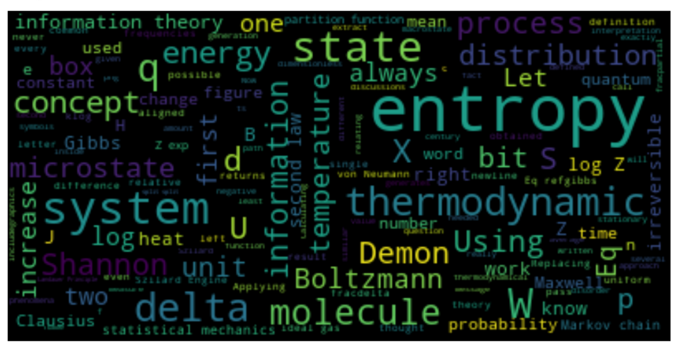

Figure 4: Word-cloud of a future work compressing these themes.

References

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[4] Leff, H.S.; Rex, A.F.Maxwell's demon: entropy, information, computing; Princeton University Press, 2014.

LPS

Contact: jordao.oliveira@usp.br