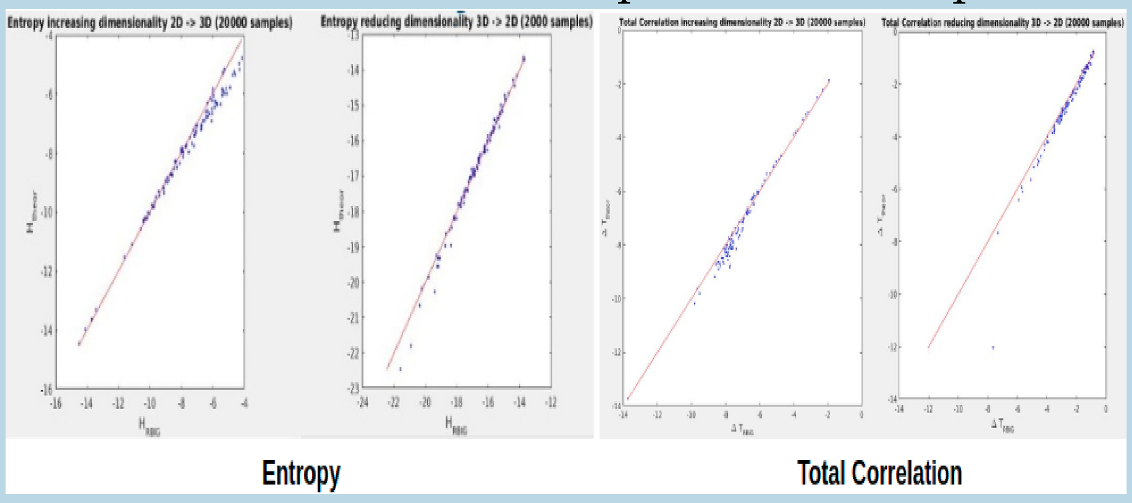
Redundancy under change of dimensionality: quantifying the efficiency of V1 cortex

Qiang Li, Emman Johnson, Jose Juan Esteve, Valero Laparra, Jesús Malo
qiang.li@uv.es, jesus.malo@uv.es

Problem and Aim

A way to confirm the *Efficient Coding Hypothesis* in theoretical neuroscience requiresthe analysis of the statistical performance of biological systems that have not been statistically optimized. However, when quantifying the information-theoretic performance of visual systems, cortical magnification poses a theoretical problem. Cortical magnification stands for the increasein signal dimensionality between the retina and the V1 cortex.

In this work, we address the information theoretic analysis of nonlinear mappings that do not preserve dimension using *no* approximation. On the one hand we derive the theory to compute variations of entropy and total correlation under such transforms, which involves the knowledge of the Jacobian of the output wrt the input.



Basic Concepts

Cortical magnification The number of neurons in the visual cortex responsible for processing a given size of visual stimulus changes as the stimulus moves across the visual field. **Total correlation** is one of several generalizations of the mutual information. It is also known as the multivariate constraint. ***RBIG*** is a Gaussianization-based methodologies to estimate total correlation, entropy, mutual information and Kullback-Leibler divergence.

Models

Model1 Computational visual model with dimensional preserved,

$$\mathrm{IM} \xrightarrow{WF} \mathrm{V1} \xrightarrow{S} \mathrm{V1S} \xrightarrow{n1} \mathrm{V1S+N}$$

Model2 Computational visual model with dimensional changed,

$$\mathrm{IM} \xrightarrow{DCT} \mathrm{LGN} \xrightarrow{n1} \mathrm{LGN+n1} \xrightarrow{WF} \mathrm{V1} \xrightarrow{n2} \mathrm{V1+n2} \xrightarrow{S} \mathrm{V1S} \xrightarrow{n3} \mathrm{V1S+n3}$$

Model3 A standard spatio-chromatic psychophysical pathway,

$$x^{(0)} \xrightarrow{\text{LMS}} r^{(1)} \xrightarrow{\text{Adapt.}} x^{(1)} \xrightarrow{\text{ATD}} r^{(2)} \xrightarrow{\text{Satur.}} x^{(2)} \xrightarrow{\text{DCT-CSF}} r^{(3)} \xrightarrow{\text{Div. Norm.}} x^{(3)}$$

References

[1] Duncan, R.; Boynton, G: *Cortical Magnification within Human Primary Visual Cortex Correlates withAcuity Thresholds*, Neuron 2003,38,659–71.

[2] Simoncelli, E.P.; Olshausen, B.A *Natural Image Statistics and Neural Representation*, Annual Review ofNeuroscience 2001,24,1193–1216.

[3] Malo, J.: *Spatio-chromatic information available from different neural layers via Gaussianization.* J. Math. Neurosc 2020,10,18.

Acknowledgements

This work was partially funded by the Spanish Government through the grant MINECO DPI2017-89867 and by the Generalitat Valenciana through the grant GrisoliaP/2019/035.

Total Correlation estimate with Theoretical and RBIG method

Here we show the theoretical result and how the RBIG estimations confirm it. The below figure shows the total correlation measured between the retina and the LGN representation using the theoretical estimation through the Jacobian of the model and the empirical estimation using RBIG in the luminance/contrast plane of natural images.

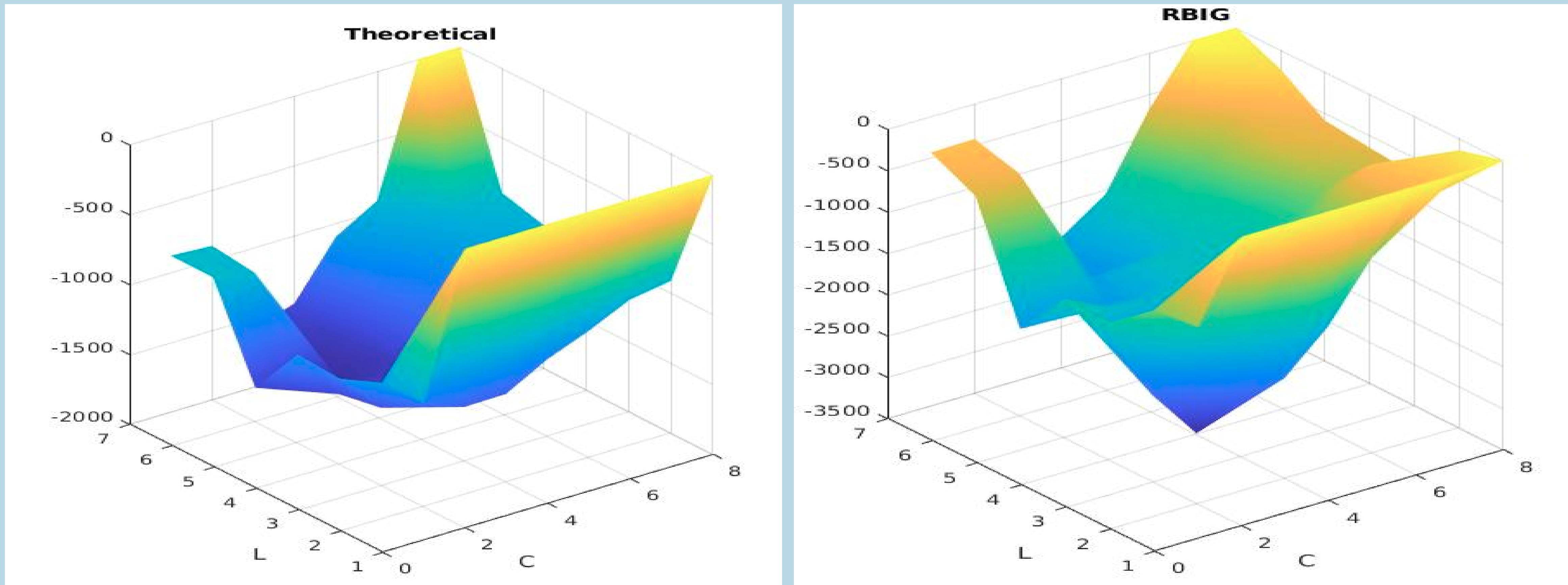


Fig.1 Left shown theoretical estimated total correlation between layer2 and layer1 in the model2. The right has shown RBIG estimated total correlation between layer2 and layer1 in the model2.

Information Transmission via Model1, Model2 and Model3

Here we show the theoretical result of model1 and model2. Fig.2 and Fig.3 show the total correlation measured between different layers using the theoretical estimation through the Jacobian of the model in the luminance/contrast plane of natural images. Fig.4 shows redundancy reduced at different layers of the network considering samples over the whole image manifold. Redundancy is measured in total correlation (unit bits).

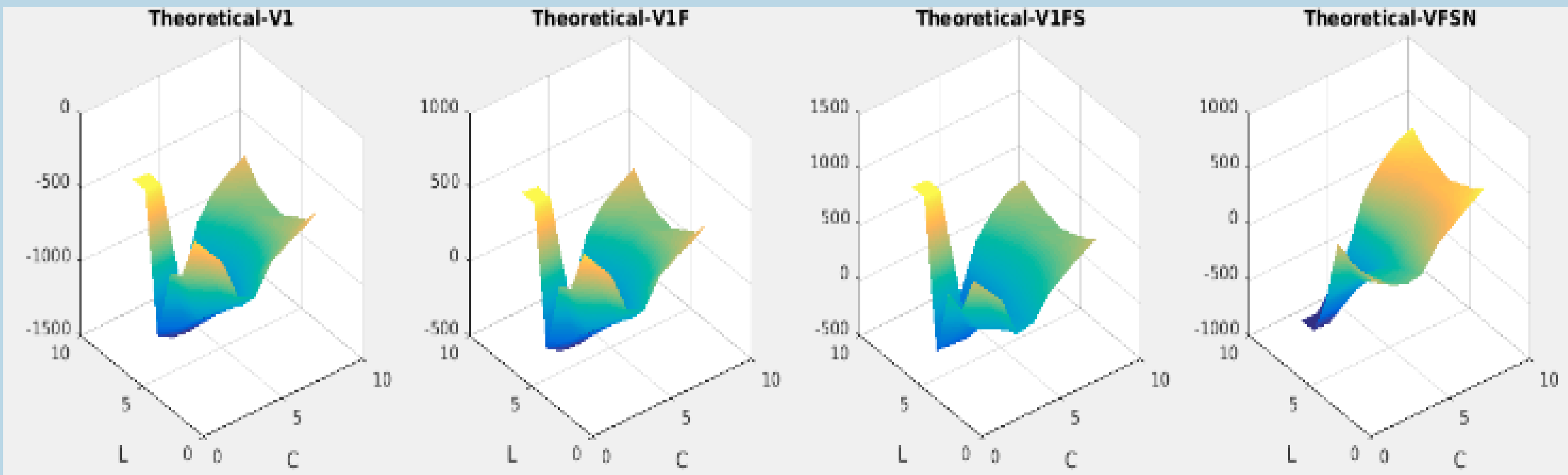


Fig.2 Theoretical information transmission at different layer via model1

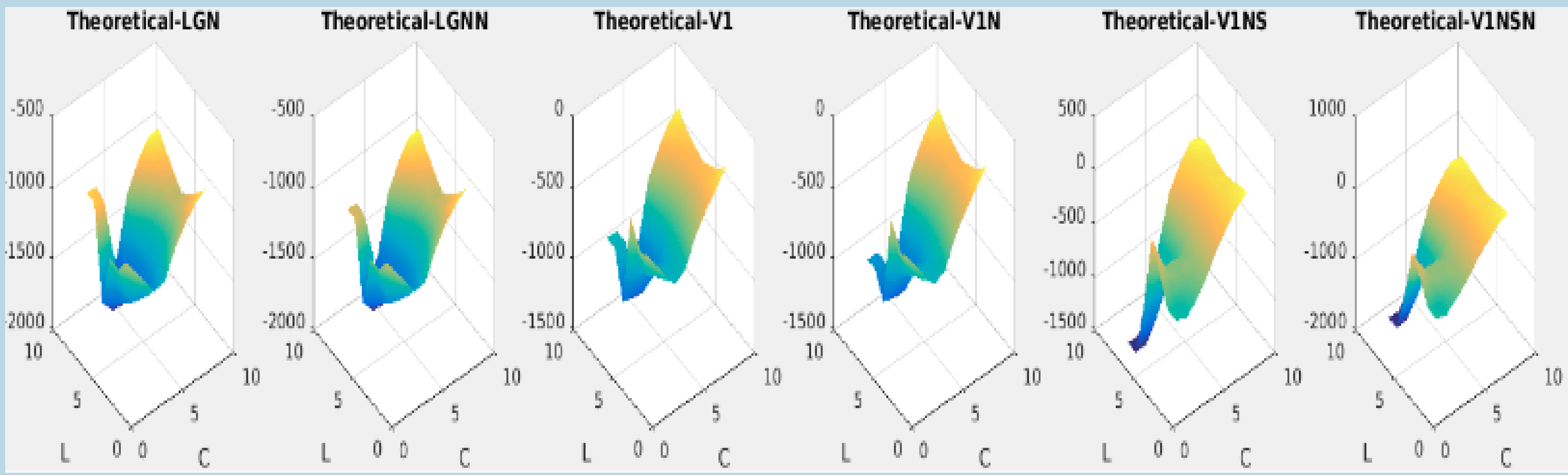


Fig.3 Theoretical information transmission at different layer via model2

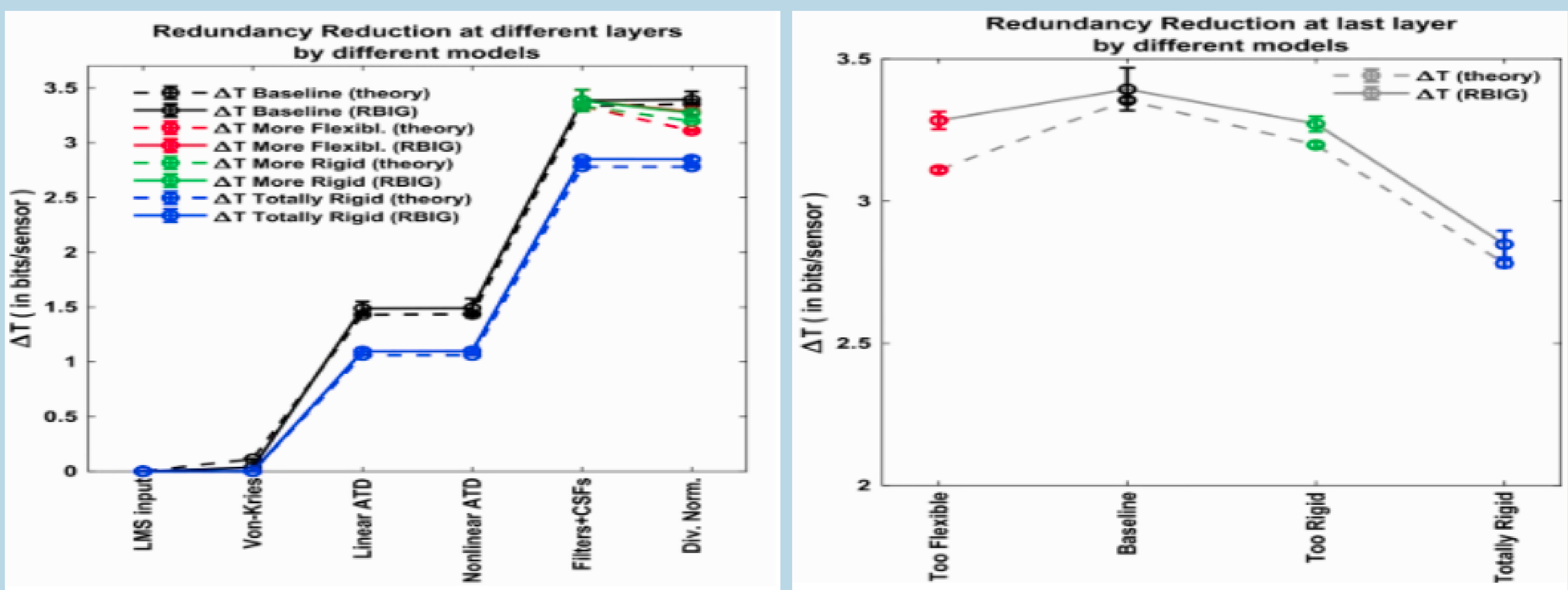


Fig.4 Information available at different layers of model3. The figure reproduced from Malo,J. 2020.