The touted defiance of Bell's inequality by quantum probabilities derives from a mathematical error

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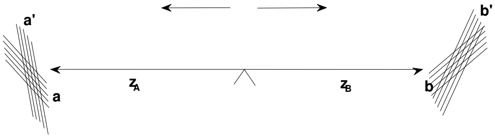
The optical setup for Bell in CHSH form ala Aspect

The Journeys of a pair of prepared photons



and their detection via angled polarizers

Observation Notation:

$$A(\mathsf{a}^*, \lambda) = +1 \quad \text{when parallel detection}$$

$$A(\mathsf{a}^*, \lambda) = -1 \quad \text{when perpendicular detection, and}$$

and similarly for $B(\cdot, \cdot) = \pm 1$

Undisputed Quantum Probabilities

QM-motivated probabilities

$$P[(A(a^*) = +1)(B(b^*) = +1)]$$

$$= \; P[(A(a^*) = -1)(B(b^*) = -1)] = \tfrac{1}{2}\, cos^2(a^*, b^*),$$

and using **relative angle** notation (a^*, b^*)

$$P[(A(a^*) = +1)(B(b^*) = -1)]$$

$$= \; P[(A(a^*) = -1)(B(b^*) = +1)] = \tfrac{1}{2}\, sin^2(a^*, b^*).$$

N.B. These imply $E[A(a^*)B(b^*)] \; = \; cos\, 2(a^*, b^*)$,

and btw

$$P[A(a^*) = +1] = P[B(b^*) = +1] \; = \; 1/2\,.$$

and the Entanglement Equations

Notice then the conditional probabilities

$$P[(A(\mathrm{a}^*) = +1)|(B(\mathrm{b}^*) = +1)] \; = \; cos^2(\mathrm{a}^*, \mathrm{b}^*)$$

and

$$P[(A(\mathrm{a}^*) = +1)|(B(\mathrm{b}^*) = -1)] \; = \; sin^2(\mathrm{a}^*, \mathrm{b}^*)$$

$$\neq \; P(A(\mathrm{a}^*) = +1) \; = \; \tfrac{1}{2},$$

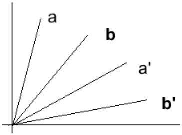
i.e.,

quantum entanglement, and observation disturbance

and also btw... Any *one* of P_{++}, P_{+-}, or $E(AB)$ imply the others for the QM distribution.

The physics:
Specific angles for experimental detection

Relative angle settings of detectors
yielding the most egregious
purported violation of Bell's inequality



(a,b) = -π/8

(a,b') = - 3π/8

(a',b) = π/8

(a',b') = - π/8

BTW ... Double these angles ... $-\pi/4, -3\pi/4, \pi/4$ and $-\pi/4$

Why ? Remember $E[A(a^*)B(b^*)] = cos\,2(a^*, b^*)$

The Metaphysics ... a Gedankenexperiment

$$s(\lambda, \mathsf{a}, \mathsf{b}, \mathsf{a}', \mathsf{b}') \equiv$$

$$A(\mathsf{a},\lambda)\,B(\mathsf{b},\lambda) - A(\mathsf{a},\lambda)\,B(\mathsf{b}',\lambda) + A(\mathsf{a}',\lambda)\,B(\mathsf{b},\lambda) + A(\mathsf{a}',\lambda)\,B(\mathsf{b}',\lambda)$$

(incompatible observations on single photon pair) for $\lambda \in \Lambda$

As per Aspect/Bell/CHSH, under **local realism** this quantity is limited in the gedankenexperiment to exhibit only one of the two possibilities $\in \{-2, +2\}$ ... as we shall see ...

Thus,
$$E[s(\lambda)] = E[A(\mathsf{a},\lambda)B(\mathsf{b},\lambda)] - E[A(\mathsf{a},\lambda)B(\mathsf{b}',\lambda)] + E[A(\mathsf{a}',\lambda)B(\mathsf{b},\lambda)] + E[A(\mathsf{a}',\lambda)B(\mathsf{b}',\lambda)]$$

$$= E[A(\mathsf{a})B(\mathsf{b})] - E[A(\mathsf{a})B(\mathsf{b}')] + E[A(\mathsf{a}')B(\mathsf{b})] + E[A(\mathsf{a}')B(\mathsf{b}')]$$

should surely lie in the interval $[-2, 2]$.

The Aspect/Bell Quandary

Applying the QM probs and expectations
to all four egregious angles yields

$$cos\, 2(\mathrm{a}, \mathrm{b}) = cos\, 2(\mathrm{a}', \mathrm{b}) = cos\, 2(\mathrm{a}', \mathrm{b}') = 1/\sqrt{2}$$

and $\quad cos\, 2(\mathrm{a}, \mathrm{b}') \;=\; -1/\sqrt{2}$

So it seems $\quad E[s(\lambda, \mathrm{a}, \mathrm{b}, \mathrm{a}', \mathrm{b}')] \;=\; 2\sqrt{2} \;>\; 2$!!!

Hmmmm ... Let's see !

Let's suppose we could do the gedankenexperiment, **think** about what might happen, and **think** about what quantum theory says about it.

OK let's think!

What are we talking about ? ... All this and more !

Let's consider the "realm matrix" of *all*

(im)possible observations ... smile ...

We'll look in *banks* of *columns* at possibilities for the observable quantities $A(\mathrm{a}), B(\mathrm{b}), A(\mathrm{a}'), B(\mathrm{b}')$;

their products

$A(\mathrm{a})B(\mathrm{b}),\ A(\mathrm{a})B(\mathrm{b}'),\ A(\mathrm{a}')B(\mathrm{b}),\ A(\mathrm{a}')B(\mathrm{b}')$;

and four symmetric function quantities

$\Sigma_{/(\mathrm{a},\mathrm{b})},\ \Sigma_{/(\mathrm{a},\mathrm{b}')},\ \Sigma_{/(\mathrm{a}',\mathrm{b})},\ \Sigma_{/(\mathrm{a}',\mathrm{b}')}$ . ... PLUS YACK

$$
\mathrm{R}\begin{pmatrix} A(\mathrm{a}) \\ B(\mathrm{b}) \\ A(\mathrm{a}') \\ B(\mathrm{b}') \\ *\,*\,*\,*\,* \\ A(\mathrm{a})B(\mathrm{b}) \\ A(\mathrm{a})B(\mathrm{b}') \\ A(\mathrm{a}')B(\mathrm{b}) \\ A(\mathrm{a}')B(\mathrm{b}') \\ *\,*\,*\,*\,* \\ \Sigma_{/(\mathrm{a},\mathrm{b})} \\ \Sigma_{/(\mathrm{a},\mathrm{b}')} \\ \Sigma_{/(\mathrm{a}',\mathrm{b})} \\ \Sigma_{/(\mathrm{a}',\mathrm{b}')} \\ *\,*\,*\,*\,* \\ s(\lambda) \\ 1 \end{pmatrix} =
$$

The columns of this matrix list

the ensemble of measurement possibilities

for the results of the gedankenexperiment

on a single pair of photons

emitted toward all four

of the tendered polarizer direction pairings

as restricted by the principle of local realism

$$
\begin{pmatrix}
A(\mathsf{a}) \\
B(\mathsf{b}) \\
A(\mathsf{a}') \\
B(\mathsf{b}') \\
\,\,*\,*\,* \\
A(\mathsf{a})B(\mathsf{b}) \\
A(\mathsf{a})B(\mathsf{b}') \\
A(\mathsf{a}')B(\mathsf{b}) \\
A(\mathsf{a}')B(\mathsf{b}') \\
\,\,*\,*\,* \\
\Sigma_{/(\mathsf{a},\mathsf{b})} \\
\Sigma_{/(\mathsf{a},\mathsf{b}')} \\
\Sigma_{/(\mathsf{a}',\mathsf{b})} \\
\Sigma_{/(\mathsf{a}',\mathsf{b}')} \\
\,\,*\,*\,* \\
s(\lambda) \\
1
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
 & & & & & & & & & & & & & & & \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
 & & & & & & & & & & & & & & & \\
3 & -1 & -1 & -1 & 1 & 1 & -3 & 1 & 1 & -3 & 1 & 1 & -1 & -1 & -1 & 3 \\
3 & 1 & -1 & 1 & 1 & -1 & -3 & -1 & -1 & -3 & -1 & 1 & 1 & -1 & 1 & 3 \\
3 & -1 & 1 & 1 & -1 & -1 & -3 & 1 & 1 & -3 & -1 & -1 & 1 & 1 & -1 & 3 \\
3 & 1 & 1 & -1 & -1 & 1 & -3 & -1 & -1 & -3 & 1 & -1 & -1 & 1 & 1 & 3 \\
 & & & & & & & & & & & & & & & \\
2 & 2 & -2 & 2 & 2 & -2 & -2 & -2 & -2 & -2 & -2 & 2 & 2 & -2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
$$

After the yack you know ...

$$\Sigma_{/(\mathsf{a}',\mathsf{b}')} \;=\; \Sigma\Big(A(\mathsf{a})B(\mathsf{b}),\; A(\mathsf{a})B(\mathsf{b}'),\; A(\mathsf{a}')B(\mathsf{b})\Big)$$

$$\equiv\; A(\mathsf{a})B(\mathsf{b}) \;+\; A(\mathsf{a})B(\mathsf{b}') \;+\; A(\mathsf{a}')B(\mathsf{b})$$

and similarly for other quantities named $\Sigma_{/(\mathsf{a}^*,\mathsf{b}^*)}$

and

$$A(\mathsf{a}')B(\mathsf{b}') \;=\; (\Sigma_{/(\mathsf{a}',\mathsf{b}')} \;=\; 3 \; or -1) - (\Sigma_{/(\mathsf{a}',\mathsf{b}')} \;=\; -3 \; or +1)$$

$$\equiv\; \mathsf{G}\,[A(\mathsf{a})B(\mathsf{b}), A(\mathsf{a})B(\mathsf{b}'), A(\mathsf{a}')B(\mathsf{b})]$$

and similarly for other quantities named $A(\mathsf{a}^*)B(\mathsf{b}^*)$

These are **completely symmetric functional relations.**

The neglected functional relations imply ...

Well $E[s(\lambda)] = E[A(\lambda, \mathrm{a})B(\lambda, \mathrm{b})] - E[A(\lambda, \mathrm{a})B(\lambda, \mathrm{b}')]$

$+ E[A(\lambda, \mathrm{a}')B(\lambda, \mathrm{b})] + E[A(\lambda, \mathrm{a}')B(\lambda, \mathrm{b}')]$

... sure enough, BUT ... this equals

$$= E[A(\mathrm{a})B(\mathrm{b})] - E[A(\mathrm{a})B(\mathrm{b}')] + E[A(\mathrm{a}')B(\mathrm{b})] + E\{\mathrm{G}[A(\mathrm{a})B(\mathrm{b}), A(\mathrm{a})B(\mathrm{b}'), A(\mathrm{a}')B(\mathrm{b})]\}$$

In fact there are FOUR such representations

... enter Bruno de Finetti and FTP

The fundamental theorem of probability says ...

Whatever probabilities or expectations are asserted for any vector of quantities whatsoever

then bounds on the range of cohering probability or expectation for any further quantity, are specified by a linear programming computation.

If there is no feasible solution to the LP problem then your array of asserted probabilities or expectations is incoherent.

... because an expectation vector must sit within the convex hull of the space of observation possibilities

What do coherent assertions of QM probs specify ?

$$\mathrm{E}\begin{pmatrix} 1 \\ A(\mathrm{a})B(\mathrm{b}) \\ A(\mathrm{a})B(\mathrm{b}') \\ A(\mathrm{a}')B(\mathrm{b}) \\ A(\mathrm{a}')B(\mathrm{b}') \\ s(\lambda) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 2 & 2 & -2 & 2 & 2 & -2 & -2 & -2 \end{pmatrix} \mathrm{q}_8$$

for some $\mathrm{q}_8 \in \mathcal{S}^7$ ... non-negative components summing to 1.

The FTP tells us that

*QM probs for a polarized pair are coherent *for any angle setting*

*QM probs for the *same photon pair* are coherent for any two

*QM probs for the *same photon pair* are coherent for any three

*QM probs for the *same photon pair* at **all four angle settings** are INCOHERENT ! , i.e. they do not cohere with one another.

What *do* assertions for any three angles imply for the fourth?

What does the FTP say about $E_{QM}(s)$?

...Results of 8 LP problems ... in a Table

Table 1: Bounding values of coherent QM expectation for $s(\lambda)$

LP problem	$E[s(\lambda)]$	$P_{++}(a^*, b^*)$	$P_{+-}(a^*, b^*)$	$E[A(a^*)\,B(b^*)]$
min $E[s(\lambda)](a, b')$	1.1213	.5	0	1.0
max $E[s(\lambda)](a, b')$	2.0	.2803	.2197	.1213
min $E[s(\lambda)](a', b')$	1.1213	0	.5	-1.0
max $E[s(\lambda)](a', b')$	2.0	.2197	.2803	$-.1213$
min $E[s(\lambda)](a, b)$	1.1213	0	.5	-1.0
max $E[s(\lambda)](a, b)$	2.0	.2197	.2803	$-.1213$
min $E[s(\lambda)](a', b)$	1.1213	0	.5	-1.0
max $E[s(\lambda)](a', b)$	2.0	.2197	.2803	$-.1213$

Their solution vectors are columns of extreme q_8

	$_{min}(a', b')$	$_{max}(a', b')$	$_{min}(a', b)$	$_{max}(a', b)$	$_{min}(a, b')$	$_{max}(a, b')$	$_{min}(a, b)$	$_{max}(a, b)$
q_1	0	0.1464	0	0.1464	0.5607	0.7803	0	0.1464
q_2	0.7803	0.5607	0	0.1464	0.1464	0	0	0.1464
q_3	0.0732	0	0.0732	0	0	0.0732	0	0
q_4	0	0.1464	0.7803	0.5607	0.1464	0	0	0.1464
q_5	0	0.1464	0	0.1464	0.1464	0	0.7803	0.5607
q_6	0.0732	0	0	0	0	0.0732	0.0732	0
q_7	0.0732	0	0.0732	0	0	0	0.0732	0
q_8	0	0	0.0732	0	0	0.0732	0.0732	0

This is a matrix with rank of only 4.

These column vectors represent the vertices of a 4-dimensional polytope.

Vertices of the Prevision Polytope ... in P_{++} space

Suppose we assess the $P_{++}(\mathrm{a}^*, \mathrm{b}^*)$ values at these vertices

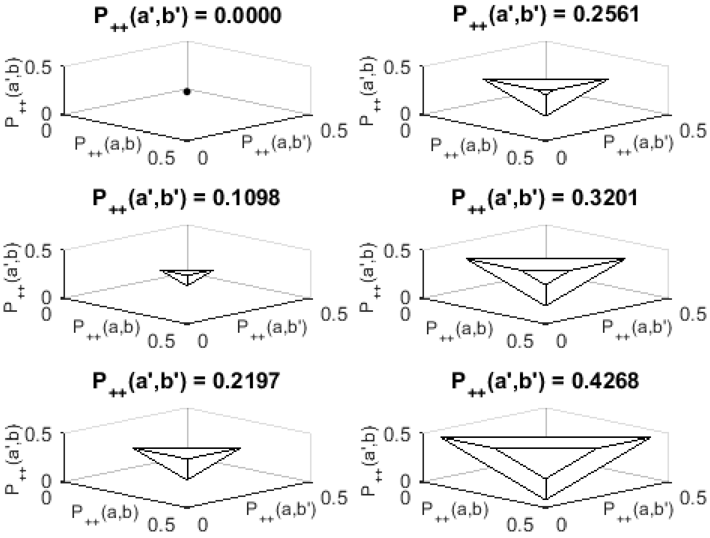
Table 2: Vertex vectors of coherent QM probability polytope

$P_{++}(\mathrm{a}, \mathrm{b})$	0.4268	0.4268	0.4268	0.4268	0.0000	0.2197	0.4268	0.4
$P_{++}(\mathrm{a}, \mathrm{b}')$	0.5000	0.2803	0.0732	0.0732	0.0732	0.0732	0.0732	0.0
$P_{++}(\mathrm{a}', \mathrm{b})$	0.4268	0.4268	0.4268	0.4268	0.4268	0.4268	0.0000	0.2
$P_{++}(\mathrm{a}', \mathrm{b}')$	0.4268	0.4268	0.0000	0.2197	0.4268	0.4268	0.4268	0.4
$E[s(\lambda)]$	1.1213	2.0000	1.1213	2.0000	1.1213	2.0000	1.1213	2.0

A movie of this 4-D polytope passing through 3-D space.

Rachael Tappenden, director

In SLO MO, Slices of the 4-D P_{++} polytope



What to make of Aspect's Empirical Estimations ?

Estimate each product moment by method of moments ...

$$\hat{E}[A(\mathrm{a})B(\mathrm{b})] = \frac{[N_{++}(\mathrm{a},\mathrm{b}) - N_{+-}(\mathrm{a},\mathrm{b}) - N_{-+}(\mathrm{a},\mathrm{b}) + N_{--}(\mathrm{a},\mathrm{b})]}{[N_{++}(\mathrm{a},\mathrm{b}) + N_{+-}(\mathrm{a},\mathrm{b}) + N_{-+}(\mathrm{a},\mathrm{b}) + N_{--}(\mathrm{a},\mathrm{b})]} \;,$$

using experiments on distinct photon pairs

and similarly for the other components of s, $\hat{E}[A(\mathrm{a}^*)B(\mathrm{b}^*)]$

Well OK, ... BUT

DON'T PRETEND THAT all four product pairs are free!

Let's check consequences of recognition using simulation data

Simulation Results ... requiring some Yack

"Bell's Theorem: the Naive View of an Experimentalist",
Alain Aspect, 2002

Table 3: Corrections to Aspect's estimate of $E[s(\lambda)]$

	(a, b)	(a, b′)	(a′, b)	(a′, b′)
$\hat{E}_{AA}[A(a^*)B(b^*)]$	0.707232	−0.706186	0.706840	0.707480
Aspect $\hat{E}[s]$	2.827738	2.827738	2.827738	2.827738
$\hat{E}_{G_{fctn}}[A(a^*)B(b^*)]$	−0.353078	0.354348	−0.354766	−0.353934
Corrected $\hat{E}[s]$	1.767180	1.767204	1.765740	1.766964

with an average value of 1.766772

Note the tantalizing tease of an "estimate"
near to $2.5/\sqrt{2} =$ 1.767766952966369

Hmmm ... , ... Well, who cares?

Conclusion and Available Extensive Discussions

*** Bell's inequality is **not** defied by QM probabilities with realism

*** Local realism is resurrected and the prospect of supplementary variables can be sensibly entertained !

Five extensive papers available on my Researchgate page:

*Quantum violation of Bell's inequality: a misunderstanding based on a mathematical error of neglect

*The GHSZ argument: a gedankenexperiment requiring more denken

*Resurrecting the principle of local realism and the prospect of supplementary variables

*More Hoojums than Boojums: quantum mysteries for no one

*Probability and Quantum Physics

*with a Preface to "Just Plain Wrong: the dalliance of quantum theory with the defiance of Bell's inequality"

Aspect's argument, in his own words (2002) p2

"Following Bell, I will first explain the motivations for considering supplementary parameters theories:

the argument is based on an analysis of the famous Einstein-Podolsky-Rosen (EPR) Gedankenexperiment.

Introducing a reasonable Locality Condition, we will then derive Bell's theorem, which states:

i. that Local Supplementary Parameters Theories are constrained by Bell's Inequalities; and

ii. that certain predictions of Quantum Mechanics violate Bell's Inequalities (if locality is presumed),

and therefore that Quantum Mechanics is incompatible with Local Supplementary Parameters Theories."