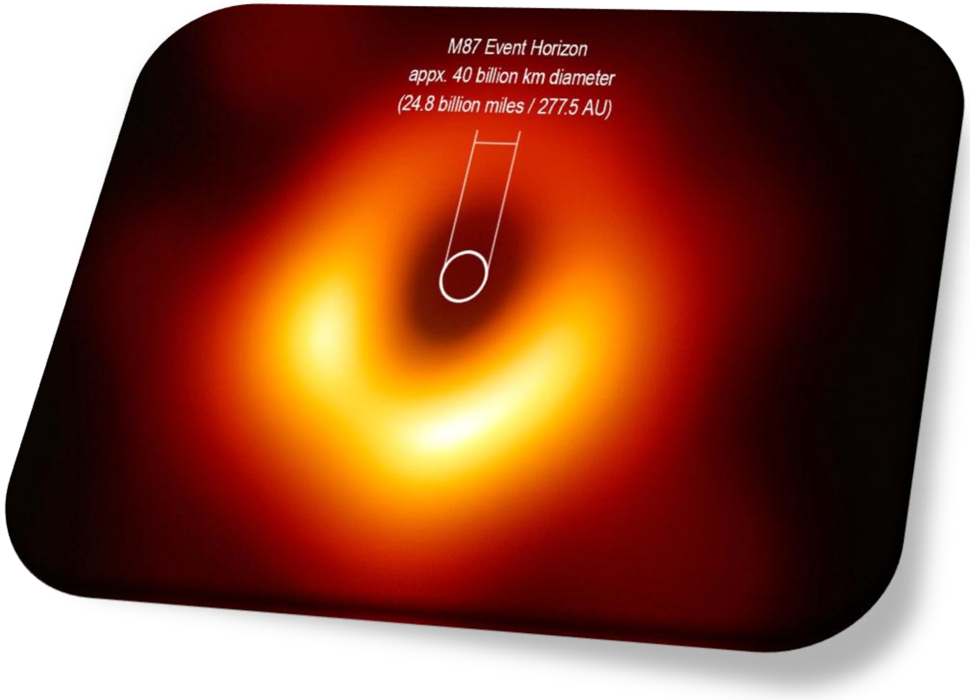
TECNOLÓGICO NACIONAL DE MÉXICO

The Entropy of Supermassive Black Holes during its Evaporation Time



Supermassive Black Holes Entropy Formula

A Dimensional Analysis

Leticia Corral Bustamante

National Technological of México/ Campus of Cuauhtémoc City

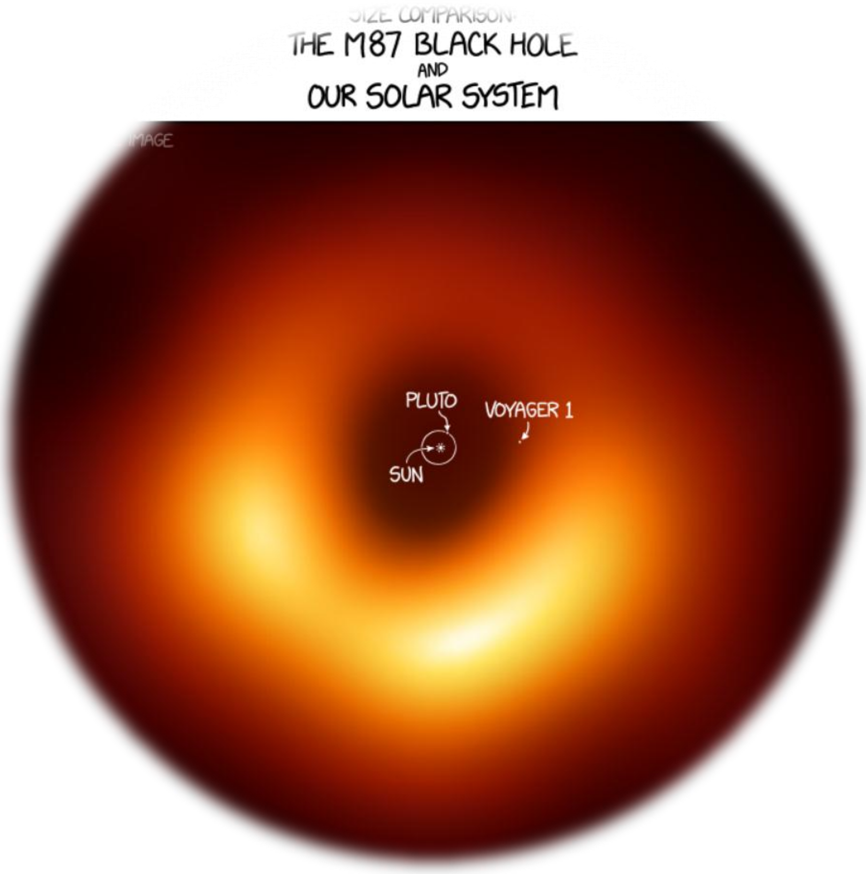
Av. Tecnológico 137, Cd. Cuauhtémoc, Chihuahua, México

Phone: +52 (625) 115 1575

E-mails: rcorral@itcdcuauhtemoc.edu.mx, leticia.corral@cimav.edu.mx

Agenda

- ✓Introduction
- ✓Modeling
- ✓Results and Discussion
- ✓Forthcoming Research
- ✓Conclusions
- ✓References



Introduction

✓S_{tevap} formula - Dimensional analysis - Constants of nature

✓Different types of BHs:

- Schwarzschild, Reissner-Nordström, Kerr , Kerr-Newman
- Remnant BH of BBH: GW150914, GW151226 and LTV151012 detected by LIGO

✓Quantify the "insignificant" quantum effects - GSL

✓Results: Formula could have some relationship with the detection of the shadow's image BHs (M87-BHs by EHT)

Modeling

$$S_{t_{evap}} = \frac{t_{evap} hc^6}{G^2 M^2 T}$$

Formula manages to join 3 different fields of physics
- ✓ Thermodynamics
- ✓ General Relativity
- ✓ Quantum Mechanics

$$t_{evap_{BH}} = \frac{G^2 M^3}{hc^4}$$

$$S_{t_{evap}} = S_{GSL} = S + S_{BH}$$

$$T = \frac{1}{4}\frac{h\kappa}{\pi^2 ck} = \frac{10^{23}}{M}$$

$$S_{BH} = \frac{1}{2}\frac{\pi c^3 kA}{hG}$$

$$\kappa = \frac{4\pi}{A}\sqrt{G^2M^2 - GQ^2 - c^2a^2}$$

$$A = \frac{4\pi G}{c^4}\left(\frac{2GM^2 - Q^2 +}{2M\sqrt{G^2M^2 - GQ^2 - c^2a^2}}\right)$$

a	Spin
A	Event horizon área
S_{BH}	Bekenstein-Hawking entropy
S_{GSL}	Bekenstein's generalized 2nd law of thermodynamics
c	Speed of light
G	Gravitational constant
h	Planck constant
k	Boltzmann constant
κ	Surface gravity
M	Mass
Q	Electric charge
t_{evap}	Evaporation time
T	Thermal Hawking radiation

Bekenstein, J. D. 1974, Phys. Rev. D, 9, 3292–3300.

Metric

Energy-stress tensor

✓ Schwarzschild

$$g_{11}=-e^{u(r)},\quad g_{22}=-r^2,\quad g_{33}=-r^2\sin^2(\theta),\quad g_{44}=-e^{v(r)}\ \longrightarrow$$

$$\begin{cases} T_{11}=-\dfrac{1}{8\pi G}\left(-\dfrac{1}{4r}\left(4\dfrac{du}{dr}\right)-2r\dfrac{d^2v}{dr^2}-r\left(\dfrac{dv}{dr}\right)^2+r\left(\dfrac{du}{dr}\right)\left(\dfrac{dv}{dr}\right)\right)-\dfrac{1}{2} \\ \cdot\left(\dfrac{1}{2r^2e^u}\left(\begin{array}{l}4r\left(\dfrac{du}{dr}\right)-2r^2\dfrac{d^2v}{dr^2}-r^2\left(\dfrac{dv}{dr}\right)^2 \\ +r^2\left(\dfrac{du}{dr}\right)\left(\dfrac{dv}{dr}\right)-4+4e^u-4r\left(\dfrac{dv}{dr}\right)\end{array}\right)\cdot e^u+\Lambda\cdot\left(-e^u\right)\right) \\ u(r)=-\ln\left(-\dfrac{\_C2-\_C3r}{r}\right)+\_C1,\quad v(r)=\ln\left(-\dfrac{\_C2-\_C3r}{r}\right) \\ \_C1=0,\quad \_C2=\dfrac{2GM}{\left(1+\dfrac{E}{c^2}\right)c^2}\quad \_C1=0 \end{cases}$$

✓ Reissner-Nordström

$$g_{11}=-c^2\left(1-\frac{2GM}{c^2r}+\frac{Ge^2}{c^4r^2}\right),\quad g_{22}=\frac{1}{1-\frac{2GM}{c^2r}+\frac{Ge^2}{c^4r^2}},\quad g_{33}=r^2,\quad g_{44}=r^2\sin^2(\theta)\ \longrightarrow\ T_{11}=-\frac{1}{8\pi G}\left(\frac{\left(-r^2+2Mr-e^2\right)e^2}{r^6}-\frac{1}{2}R\left(-1+\frac{2M}{r}-\frac{e^2}{r^2}\right)+\Lambda\left(-1+\frac{2M}{r}-\frac{e^2}{r^2}\right)\right)$$

✓ Kerr

$$g_{11}=-1+\frac{2Mr}{r^2+a^2\cos^2(\theta)},\quad g_{14}=-\frac{4aMr\sin^2(\theta)}{r^2+a^2\cos^2(\theta)},\quad g_{22}=\frac{r^2+a^2\cos^2(\theta)}{r^2-2Mr+a^2},$$

$$g_{33}=r^2+a^2\cos^2(\theta),\quad g_{41}=-\frac{4aMr\sin^2(\theta)}{r^2+a^2\cos^2(\theta)},\quad g_{44}=r^2+a^2+\frac{2a^2Mr\sin^2(\theta)}{r^2+a^2\cos^2(\theta)}$$

$$\longrightarrow\ T_{11}=-\frac{1}{8\pi G}\left(-\frac{2a^2M\left(18M^3r^6-12M^2r^7+3Mr^8+3Ma^2r^6\right)}{12r^{10}M^2a^2-2r^{13}M+r^{12}a^2+r^{14}}-\frac{1}{2}R\left(-1+\frac{2M}{r}\right)+\Lambda\left(-1+\frac{2M}{r}\right)\right)$$

✓ Kerr-Newman

$$g_{11}=1-\frac{2Mr-Q^2}{r^2},\quad g_{14}=\frac{4aM\sin^2(\theta)}{r},\quad g_{22}=-1+\frac{2Mr-Q^2}{r^2},$$

$$g_{33}=-r^2\left(-1+\frac{2Mr-Q^2}{r^2}\right),\quad g_{41}=\frac{4aM\sin^2(\theta)}{r},\quad g_{44}=-r^2\sin^2(\theta)\left(-1+\frac{2Mr-Q^2}{r^2}\right)$$

$$T_{11}=-\frac{1}{8\pi G}\left(R_{11}-\frac{1}{2}Rg_{11}+\Lambda g_{11}\right)$$

Analysis of the Entropy Formula

For BHs, at the limit, when T tends to absolute zero, the rate of change of the entropy generated during its evaporation time and eventual disappearance with respect to T is

This rate of change with respect to time is

$$\begin{cases} \lim_{T\to 0} \frac{d}{dT}\left(S_{t_{evap}}\right) = \lim_{T\to 0} \frac{d}{dT}\left(\frac{thc^6}{G^2M^2T}\right) = -Float(\infty) \\ \lim_{T\to 0} \frac{d}{dt}\left(S_{t_{evap}}\right) = \lim_{T\to 0} \frac{d}{dt}\left(\frac{thc^6}{G^2M^2T}\right) = Float(undefined) \end{cases}$$

BHs of Kerr, Reissner-Nordström, and the remnant BH from BBH merge have T that tend to absolute zero, especially the former ones → **BHs are not primordial black holes!!!**

$$\begin{cases} \lim_{t\to 0} S_{t_{evap}} = \lim_{t\to 0} \frac{thc^6}{G^2M^2T} = Float(undefined) \\ \lim_{t\to 0} \frac{d}{dt}\left(S_{t_{evap}}\right) = \lim_{t\to 0} \frac{d}{dt}\left(\frac{thc^6}{G^2M^2T}\right) = Float(undefined) \end{cases}$$

BHs tend to cancel, as it could have happened at the **Big Bang!!!**

$$\begin{cases} \lim_{T\to \infty} S_{t_{evap}} = \lim_{T\to \infty} \frac{thc^6}{G^2M^2T} = 0 \\ \lim_{T\to \infty} \frac{d}{dT}\left(S_{t_{evap}}\right) = \lim_{T\to \infty} \frac{d}{dT}\left(\frac{thc^6}{G^2M^2T}\right) = 0 \end{cases}$$

$Float(\infty)$ Represents a floating-point infinity. **This value is used to indicate a floating-point value that is too large to be otherwise represented.**

$Float(undefined)$ Represents a non-numeric object in the floating-point system. **This value can be returned by a function or operation if the input operands are not in the domain of the function or operand.**

LIGO Astronomical Databases of Remnant Black Hole of Binary Black Hole Mergers

M_{Sun}	$M\times10^{32}$ (kg)	a	a	A (m²)	κ (m s⁻²)	T (K)	S_{BH} (J K⁻¹)	t_{evap} (s)	S_{evap} (J K⁻¹)	$A_{S_{evap}}$ (m²)
GW150914										
$62.3^{+3.7+0.9}$	1.3306	0.73	$0.68^{+0.05+0.01}$	4.89E+11	2.28E+11	9.24E-10	6.48E+57	1.96E+75	1.3E+58	9.79E+11
$62^{+4.1+0.7}$	1.3287	0.61	$0.67^{+0.05+0.01}$	4.88E+11	2.28E+11	9.25E-10	6.46E+57	1.95E+75	1.29E+58	9.76E+11
$62.5^{+3.9}$	1.3207	0.74	$0.68^{+0.05}$	4.82E+11	2.3E+11	9.31E-10	6.38E+57	1.91E+75	1.28E+58	9.64E+11
$62^{+4.4}$	1.3207	0.63	$0.67^{+0.06}$	4.82E+11	2.3E+11	9.31E-10	6.38E+57	1.91E+75	1.28E+58	9.64E+11
62^{+4}	1.3127	0.74	$0.67^{-0.04}$	4.76E+11	2.31E+11	9.37E-10	6.31E+57	1.88E+75	1.26E+58	9.53E+11
$62.3^{+3.7}$	1.3127	0.72	$0.68^{+0.05}$	4.76E+11	2.31E+11	9.37E-10	6.31E+57	1.88E+75	1.26E+58	9.53E+11
$62^{+3.7}$	1.3068	0.64	$0.66^{+0.04}$	4.72E+11	2.32E+11	9.41E-10	6.25E+57	1.85E+75	1.25E+58	9.44E+11
$62.1^{+3.3}$	1.3008	0.6	$0.68^{+0.06}$	4.68E+11	2.33E+11	9.45E-10	6.19E+57	1.83E+75	1.24E+58	9.35E+11
$62_{+4.1-0.7}$	1.3008	0.63	$0.67_{+0.05-0.01}$	4.68E+11	2.33E+11	9.45E-10	6.19E+57	1.83E+75	1.24E+58	9.35E+11
$62.3_{+3.7-0.9}$	1.2948	0.62	$0.68_{+0.05-0.01}$	4.63E+11	2.34E+11	9.5E-10	6.14E+57	1.8E+75	1.23E+58	9.27E+11
$62.3^{-3.1+0.2}$	1.1815	0.73	$0.68^{-0.06+0.02}$	3.86E+11	2.57E+11	1.04E-09	5.11E+57	1.37E+75	1.02E+58	7.72E+11
$62.1_{-2.8}$	1.1795	0.59	$0.68_{-0.05}$	3.85E+11	2.57E+11	1.04E-09	5.09E+57	1.36E+75	1.02E+58	7.69E+11
$62.3_{-3.1}$	1.1775	0.7	$0.68_{-0.06}$	3.83E+11	2.58E+11	1.04E-09	5.07E+57	1.35E+75	1.01E+58	7.66E+11
$62.5_{-3.5}$	1.1735	0.6	$0.68_{-0.07}$	3.81E+11	2.59E+11	1.05E-09	5.04E+57	1.34E+75	1.01E+58	7.61E+11
62_{-3}	1.1735	0.73	$0.67_{-0.05}$	3.81E+11	2.59E+11	1.05E-09	5.04E+57	1.34E+75	1.01E+58	7.61E+11
$62.3_{-3.1-0.2}$	1.1735	0.71	$0.68_{-0.06-0.02}$	3.81E+11	2.59E+11	1.05E-09	5.04E+57	1.34E+75	1.01E+58	7.61E+11
$62^{-3.7+0.6}$	1.1715	0.62	$0.67^{-0.07+0.02}$	3.79E+11	2.59E+11	1.05E-09	5.02E+57	1.33E+75	1E+58	7.59E+11
$62_{-3.3}$	1.1675	0.58	$0.66_{-0.06}$	3.77E+11	2.6E+11	1.05E-09	4.99E+57	1.32E+75	9.98E+57	7.54E+11
62_{-4}	1.1536	0.73	$0.67_{-0.08}$	3.68E+11	2.63E+11	1.07E-09	4.87E+57	1.27E+75	9.74E+57	7.36E+11
$62_{-3.7-0.6}$	1.1477	0.62	$0.67_{-0.07-0.02}$	3.64E+11	2.64E+11	1.07E-09	4.82E+57	1.25E+75	9.64E+57	7.28E+11
GW151226										
35^{+14}	0.97461	0.78	$0.66^{+0.09}$	2.63E+11	3.11E+11	1.26E-09	3.48E+57	7.68E+74	6.95E+57	5.25E+11
35_{-4}	0.61659	0.67	$0.66_{-0.1}$	1.05E+11	4.92E+11	1.99E-09	1.39E+57	1.95E+74	2.78E+57	2.1E+11
$20.8^{+6.1+2}$	0.57482	0.82	$0.74^{+0.06+0.03}$	9.13E+10	5.28E+11	2.14E-09	1.21E+57	1.58E+74	2.42E+57	1.83E+11
$20.06^{+7.6}$	0.55016	0.7	$0.73^{+0.05}$	8.37E+10	5.52E+11	2.23E-09	1.11E+57	1.38E+74	2.22E+57	1.67E+11
$20.8^{+6.1}$	0.53504	0.83	$0.74^{+0.06}$	7.91E+10	5.67E+11	2.3E-09	1.05E+57	1.27E+74	2.1E+57	1.58E+11
$20.9^{+4.8}$	0.51117	0.1	$0.75^{+0.07}$	7.22E+10	5.94E+11	2.41E-09	9.56E+56	1.11E+74	1.91E+57	1.44E+11
$20.8_{+6.1-2}$	0.49526	0.71	$0.074_{+0.06-0.03}$	6.78E+10	6.13E+11	2.48E-09	8.98E+56	1.01E+74	1.8E+57	1.36E+11
$20.8_{-1.7-0.1}$	0.38189	0.65	$0.74_{-0.06-0.03}$	4.03E+10	7.95E+11	3.22E-09	5.34E+56	4.62E+73	1.07E+57	8.06E+10
$20.8_{-1.7}$	0.3799	0.8	$0.74_{-0.06}$	3.99E+10	7.99E+11	3.24E-09	5.28E+56	4.55E+73	1.06E+57	7.98E+10
$20.9_{-1.8}$	0.3799	0.68	$0.75_{-0.05}$	3.99E+10	7.99E+11	3.24E-09	5.28E+56	4.55E+73	1.06E+57	7.98E+10
$20.6^{-1.6}$	0.37791	0.75	$0.73_{-0.06}$	3.95E+10	8.03E+11	3.25E-09	5.23E+56	4.48E+73	1.05E+57	7.9E+10
$20.8^{-1.7+0.1}$	0.37791	0.56	$0.74^{-0.06+0.03}$	3.95E+10	8.03E+11	3.25E-09	5.23E+56	4.48E+73	1.05E+57	7.9E+10
LVT151012										
35^{+14+4}	1.0542	0.74	$0.66^{+0.09+0}$	3.07E+11	2.88E+11	1.17E-09	4.07E+57	9.72E+74	8.13E+57	6.14E+11
36^{+15}	1.0144	0.55	$0.65^{+0.09}$	2.84E+11	2.99E+11	1.21E-09	3.77E+57	8.66E+74	7.53E+57	5.69E+11
35^{+11}	0.91494	0.74	$0.66^{+0.08}$	2.31E+11	3.32E+11	1.34E-09	3.06E+57	6.36E+74	6.13E+57	4.63E+11
35_{+14-4}	0.89505	0.56	$0.66_{+0.09-0}$	2.21E+11	3.39E+11	1.37E-09	2.93E+57	5.95E+74	5.86E+57	4.43E+11
36_{-4}	0.63648	0.75	$0.65_{-0.1}$	1.12E+11	4.77E+11	1.93E-09	1.48E+57	2.14E+74	2.97E+57	2.24E+11
35_{-3}	0.63648	0.75	$0.66_{-0.1}$	1.12E+11	4.77E+11	1.93E-09	1.48E+57	2.14E+74	2.97E+57	2.24E+11
35^{-4+0}	0.61659	0.58	$0.66^{-0.10+0.02}$	1.05E+11	4.92E+11	1.99E-09	1.39E+57	1.95E+74	2.78E+57	2.1E+11
35_{-4-0}	0.61659	0.54	$0.66_{-0.10-0.02}$	1.05E+11	4.92E+11	1.99E-09	1.39E+57	1.95E+74	2.78E+57	2.1E+11

Results

- ✓ Calculations were made with different parameters of M, Q and a, according with lows of BH mechanics from Hawking's Physics.
- ✓ Kerr-Newman calculations with $T_{11} = 4.5\times10^{10}$ kg ($\theta = \pi/2$, $\Lambda = 10^{-51}$ m^{-2}, $a = 0.7$, $Q = 0.2$ & $Q = M$), $t_{evap} \approx$ 2400 years → primordial BH.
- ✓ K-N: $G^2M^2 < GQ^2 + a^2c^2$ → 2 EH desappear, but $\kappa \neq 0$ → they are not destroyed and therefore do not leave the naked singularity singularity → Penrose's Cosmic Censorship Conjecture✦ .
- ✓ $T\uparrow$ as the mass evaporates genetating an increase in ordinary S outside the BH★.
- ✓ Generalized S of the state of the region outside the BH should be increases.
- ✓ S & A decrease individually, but the S_{tot} always increases.

✦Penrose, R. 1999, J. Astrophys. Astr., 20, 233-248.

★

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Wald, R. M. 1994, Chicago Lectures in Physics (1st ed.,). Chicago: University of Chicago Press.
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Results

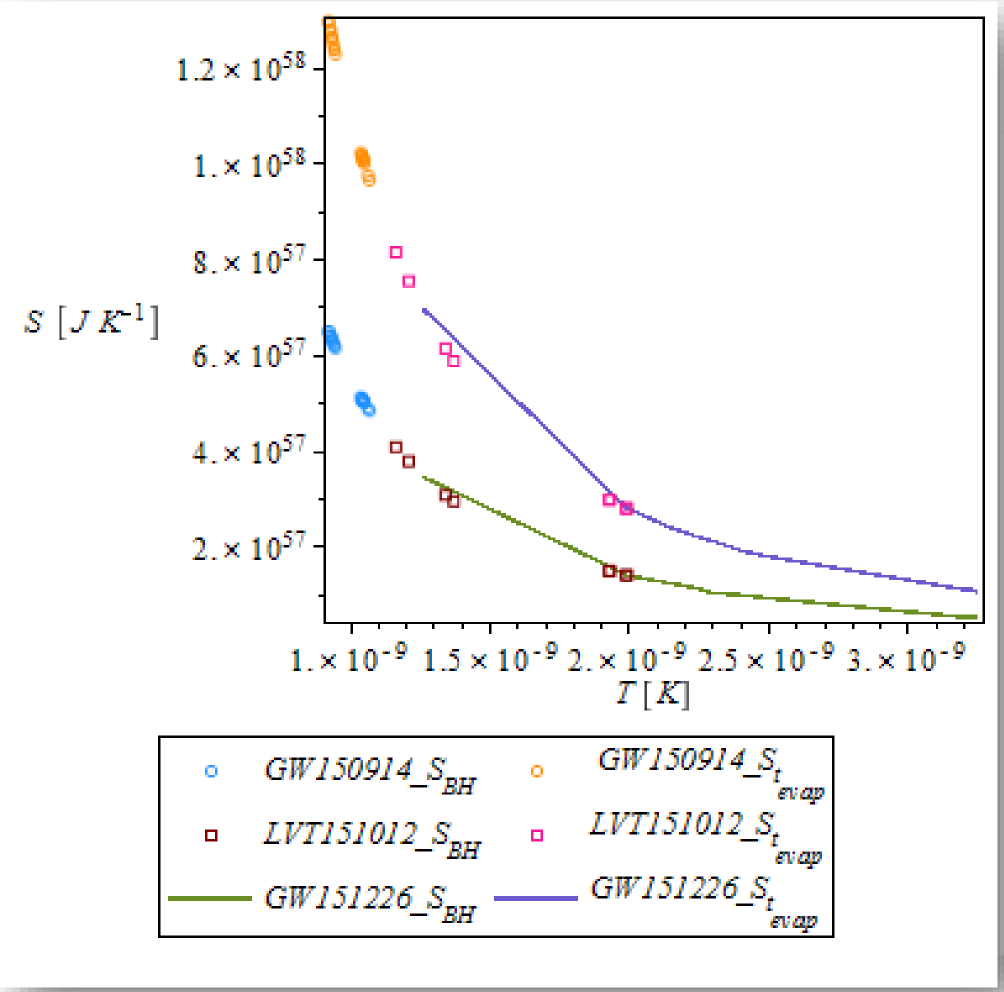

Figure 1 Graphical Abstract*

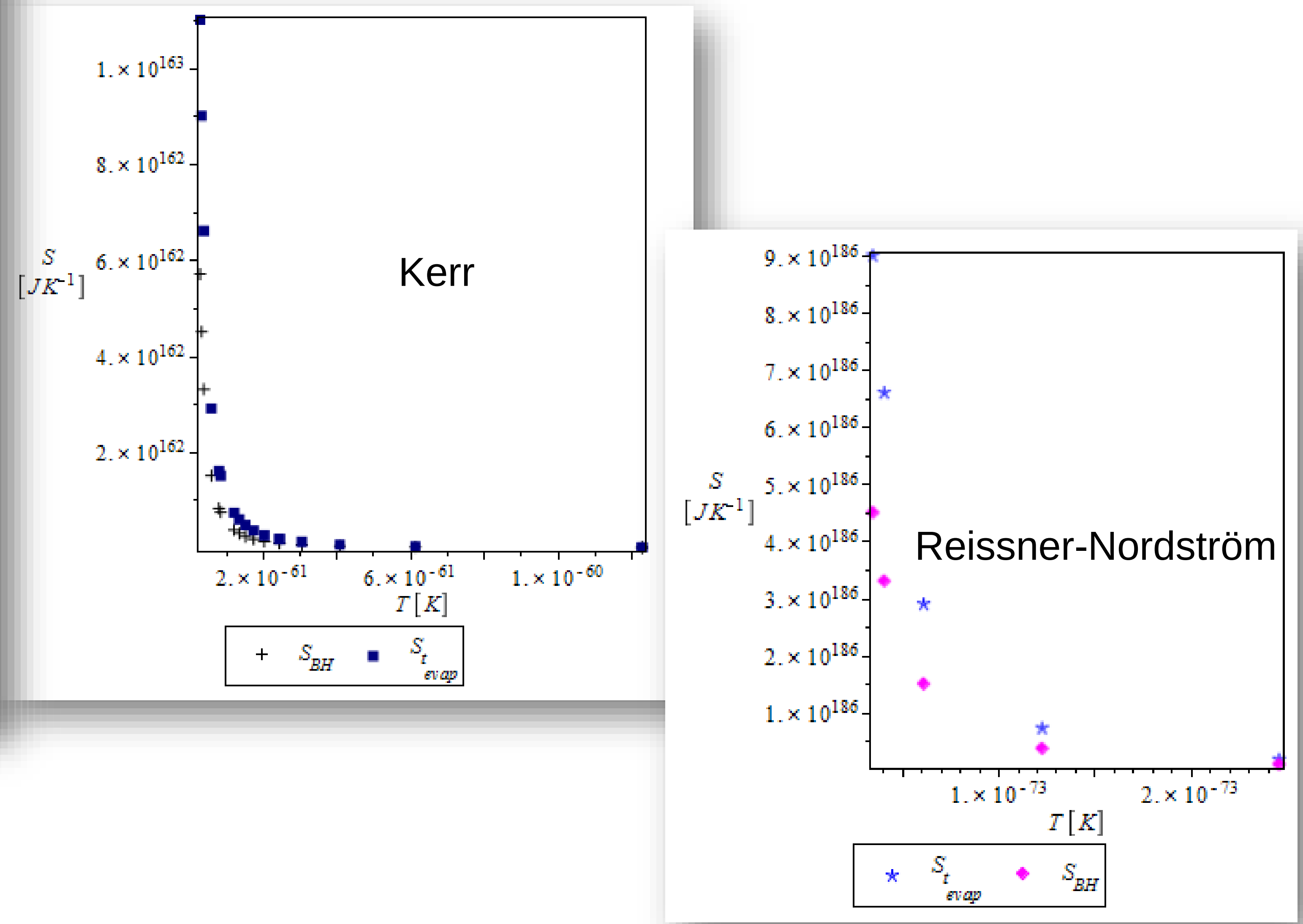

Figure 2 Entropy vs thermal Hawking radiation at the Hawking temperature of BHs

*
Abbott, B. P., Abbott, R., Abbott, T. D. et al. 2016a, Phys. Rev. Lett., 116, 1-19.
Abbott, B. P., Abbott, R., Abbott, T. D. et al. 2016b, Phys. Rev. X., 6, 041015-1-041015-36.

Results

TABLE 1 Thermophysical properties of black holes calculated from three intrinsic properties: mass, spin, and electric charge.

Black hole type	T_{11} $(M(kg))$	a	Q	A (m^2)	κ (ms^{-2})	$T(K)$	$S_{\mathbf{BH}}$ $(J\,K^{-1})$	$t_{\mathbf{evap}}(s)$	$S_{t_{evap}}$ $(J\,K^{-1})$	$A_{S_{t_{evap}}}$
GW151226	9.75×10^{31}	0.78	0	2.63×10^{11}	3.11×10^{11}	1.26×10^{-9}	3.48×10^{57}	7.68×10^{74}	6.95×10^{57}	5.25×10^{11}
	5.75×10^{31}	0.82	0	9.13×10^{10}	5.28×10^{11}	2.14×10^{-9}	1.21×10^{57}	1.58×10^{74}	2.42×10^{57}	1.83×10^{11}
	4.95×10^{31}	0.71	0	6.78×10^{10}	6.13×10^{11}	2.48×10^{-9}	8.98×10^{56}	1.01×10^{74}	1.8×10^{57}	1.36×10^{11}
	3.78×10^{31}	0.56	0	3.95×10^{10}	8.03×10^{11}	3.25×10^{-9}	5.23×10^{56}	4.48×10^{73}	1.05×10^{57}	7.9×10^{10}
Schwarzschild	9.89×10^{30}	0	0	2.7×10^{9}	3.07×10^{12}	1.24×10^{-8}	3.5×10^{55}	8.03×10^{71}	7.16×10^{55}	5.41×10^{9}
R-Nordström	3.5×10^{96}	0	0.2	3.4×10^{140}	8.67×10^{-54}	3.51×10^{-74}	4.5×10^{186}	3.6×10^{268}	9.0×10^{186}	6.8×10^{140}
	5.0×10^{95}	0	0.2	6.9×10^{138}	6.07×10^{-53}	2.46×10^{-73}	9.2×10^{184}	1.0×10^{266}	1.8×10^{185}	1.4×10^{139}
Kerr	7.07×10^{91}	0.7	0	1.4×10^{131}	4.29×10^{-49}	1.74×10^{-69}	1.8×10^{177}	2.9×10^{254}	3.7×10^{177}	2.8×10^{131}
	2.5×10^{86}	0.7	0	1.7×10^{120}	1.21×10^{-43}	4.92×10^{-64}	2.3×10^{166}	1.3×10^{238}	4.6×10^{166}	3.5×10^{120}
	3.94×10^{84}	0.7	0	4.3×10^{116}	7.7×10^{-42}	3.12×10^{-62}	5.7×10^{162}	5.1×10^{232}	1.1×10^{163}	8.6×10^{116}
	1.42×10^{84}	0.7	0	5.6×10^{115}	2.1×10^{-41}	8.66×10^{-62}	7.4×10^{161}	2.4×10^{231}	1.5×10^{162}	1.1×10^{116}
	1.31×10^{32}	0.7	0	4.76×10^{11}	2.31×10^{11}	9.37×10^{-10}	6.31×10^{57}	1.88×10^{75}	1.26×10^{58}	9.53×10^{11}
Messier 87	1.23×10^{40}	0.95	0	4.2×10^{27}	2.46×10^{3}	9.97×10^{-18}	5.57×10^{73}	1.6×10^{99}	1.11×10^{74}	8.41×10^{27}

- ✓ M87 BH 6,200,000,000 M⊙, $a = 0.90\pm0.05$♦, EHT the first shadow's image.
- ✓ Schwarzschild and Kerr-Newman BHs - T "mostly significant" than M87 BH.
- ✓ T is so low that it makes the BHs "undetectable": BHs's mass & quantum effects by entropy formula.
- ✓ EHT was able to get the first M87 BH photograph despite the "insignificant" quantum effects♠.
- ✓ T just above absolute zero: 3rd law of black hole mechanics and 3rd law of thermodynamics satisfied - the extreme black hole state cannot be reached.
- ✓Mass decreases during the evaporation process, S_{tevap}, A_{Stevap} t_{evap}, S_{BH} and $A\downarrow$ while κ & $T\uparrow$

M87 Supermassive Black Hole image

♦ Tamburini, F., Thidé, B., & Della Valle, M. 2019, MNRAS Lett., 000, 1-14.
♠
Akiyama, K., Alberdi, A., Alef, W. et al. 2019a, Astrophys. J. Lett., 875: L1, 17.
Akiyama, K., Alberdi, A., Alef, W. et al. 2019b, Astrophys. J. Lett., 875: L4, 52.
Akiyama, K., Alberdi, A., Alef, W. et al. 2019c, Astrophys. J. Lett., 875: L6, 44.

Forthcoming Research

The **time of life** of the material content of the BHs would have to be analyzed from the **quantum point of view of the particles** that make it up, to verify that these are particles that can be detected by a highly sensitive detector such as EHT, since the calculations showed that

$$t_{evap} >> \frac{h}{Mc^2}$$

Conclusions

✓The entropy formula equated with Bekenstein's generalized 2nd law of thermodynamics, fulfill the objective of elucidating that the intrinsic quantum effects, occupy a central role in the compliance with physical laws.

✓BHs can be detected despite that the Hawking effect, which reveals their presence, is not significant in this type of holes.

✓The formula can contribute to the detection of the image of these BHs, since the detection of this type of black holes is directly linked to the "insignificance" of the intrinsic Hawking effect in obtaining the shadow image of the event horizon of M87 BH achieved by EHT.

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Thank you for attending this talk!