

# Complexity as Causal Information Integration <sup>†</sup>

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Complexity measures in the context of the Integrated Information Theory of consciousness, developed mainly by Tononi [1], try to assess the strength of the causal connections between different neurons. This is done by minimizing the Kullback-Leibler-Divergence between a full system and one without causal connections. Various measures have been proposed in this setting and compared in, for example, [2–6]. Oizumi et al. develop in [7] a unified framework for these measures and postulate properties that they should fulfill. Furthermore, they introduce an important candidate of these measures, denoted by  $\Phi$ , based on conditional independence statements. Unfortunately it cannot be computed analytically in general and the KL-Divergence has to be optimized numerically.

We propose an alternative approach using a latent variable which models a common exterior influence. This leads to a measure, causal information integration, that satisfies all of the required conditions provided the state space of the latent variable is large enough and it can serve as an upper bound for  $\Phi$ . Our measure can be calculated using an iterative information geometric algorithm, the em-algorithm. Therefore we are able to compare its behavior to existing integrated information measures.

## References

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