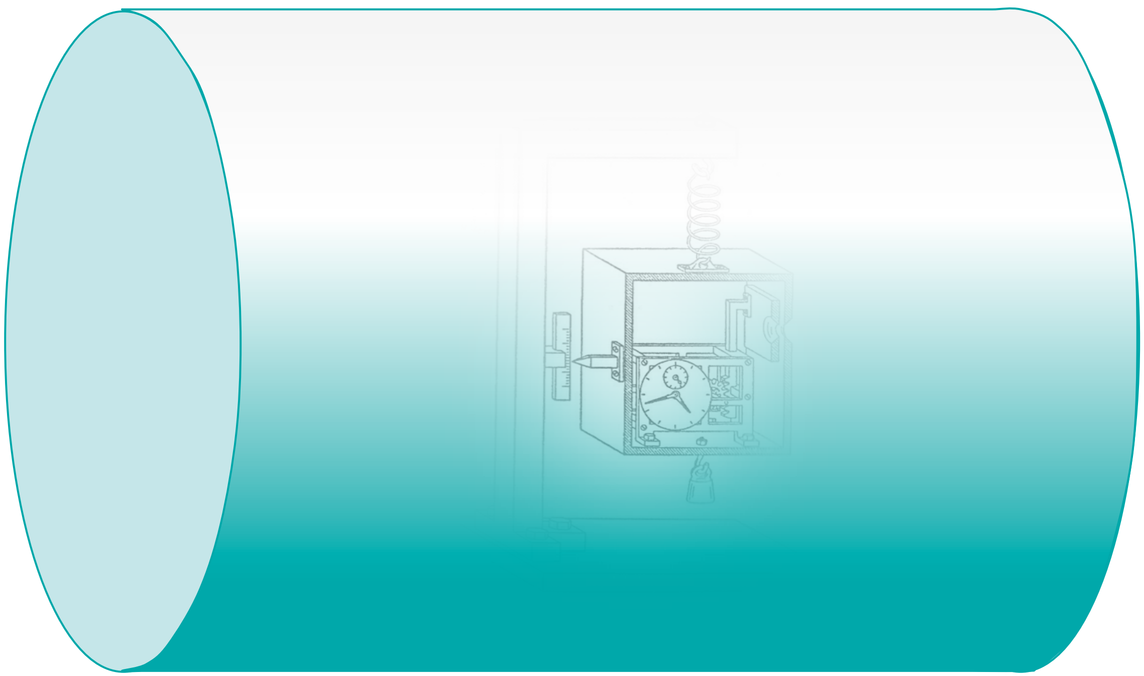
Quantum chaos and quantum measurement: paradigms of quantum entropy production



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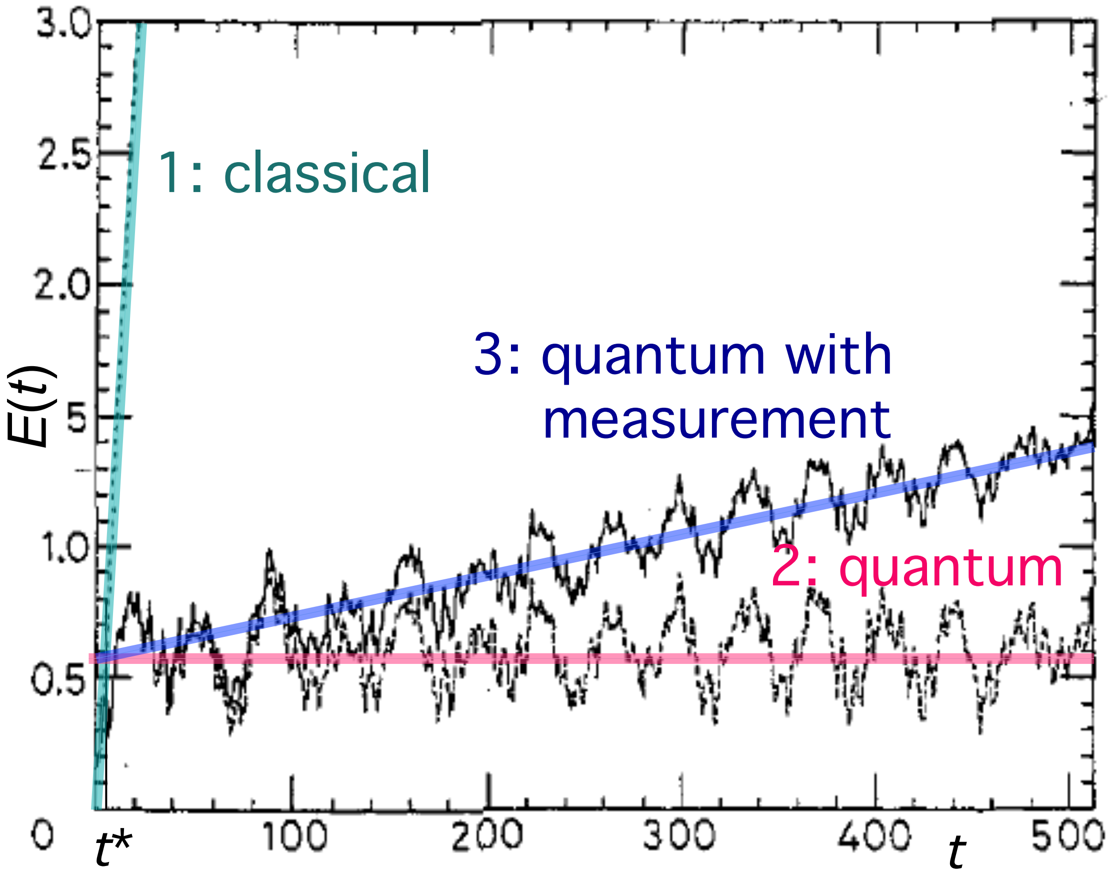
1 Classical bloom and quantum death of deterministic chaos

1.1 Quantum chaos and decoherence (a 1990 result)

- Revival of angular-momentum diffusion in the quantum kicked rotor with continuous momentum measurement



Trace 1:

Momentum-space diffusion in the classical kicked rotor

Trace 2:

Suppression of classical chaos after a finite time t^* in the quantum kicked rotor

Trace 3:

Recovery of chaotic energy growth in the observed quantum kicked rotor

1.2 There is no entropy production in closed quantum systems

Divergences of information content / production in deterministic chaos ...

Static structures:

- Self-similar phase-space patterns imply unbounded information density.

Dynamical scenario:

- Chaotic time evolution permanently "produces" entropy, lifting it from invisibly small to large scales.

... are regularized in unitary quantum mechanics

Static structures:

- Quantum uncertainty limits the resolution of phase-space structures.

Dynamical scenario:

- In systems with a finite number of freedoms, quantum time-evolution eventually becomes (quasi-)periodic.

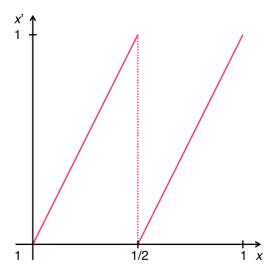
- Fundamental bounds on the information content of isolated quantum systems prevent permanent entropy production, characteristic of deterministic classical chaos

1.3 Breaking the splendid isolation

macro

entropy flow

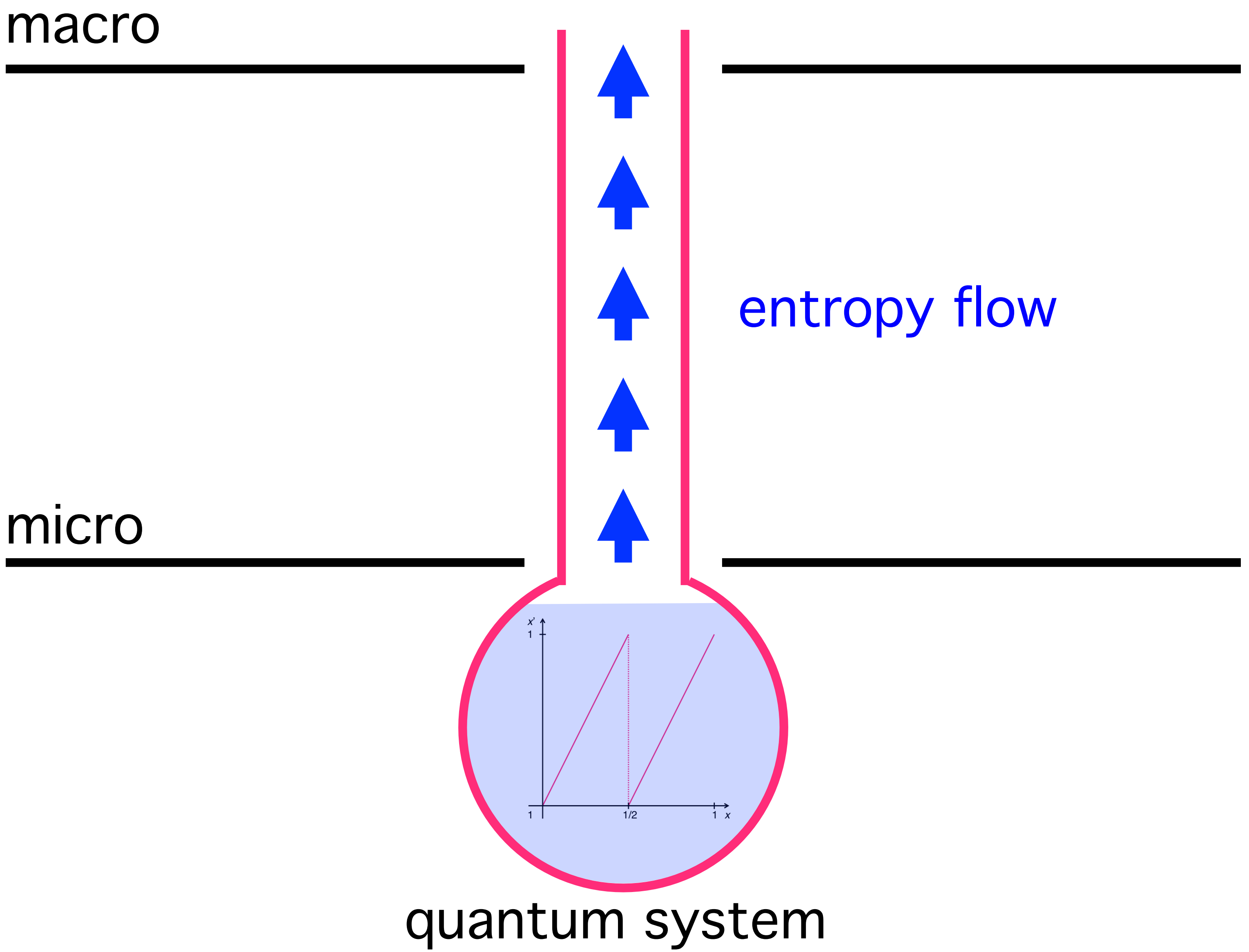
micro



classical system

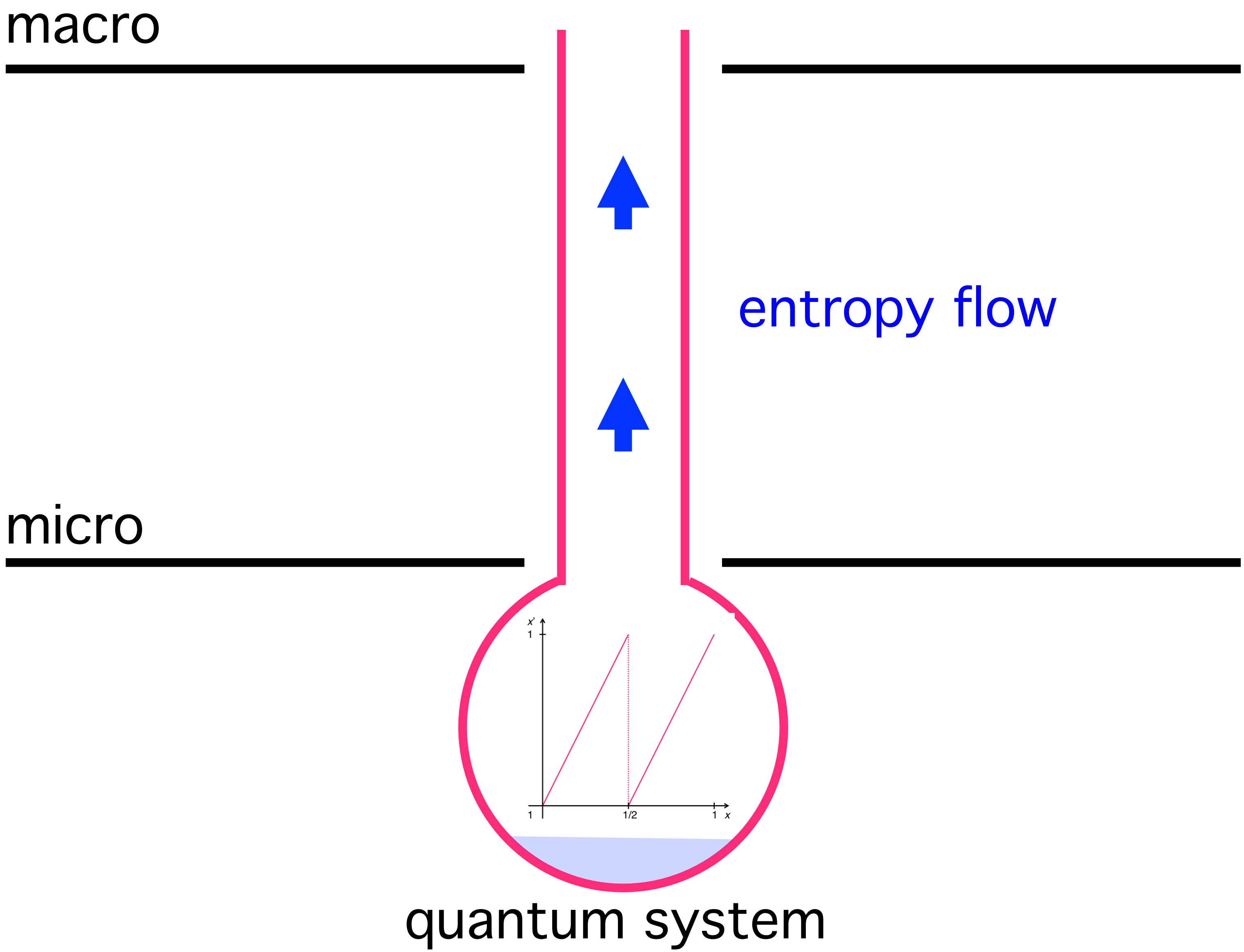
1 Classical bloom and quantum suppression of chaos

1.3 Breaking the splendid isolation

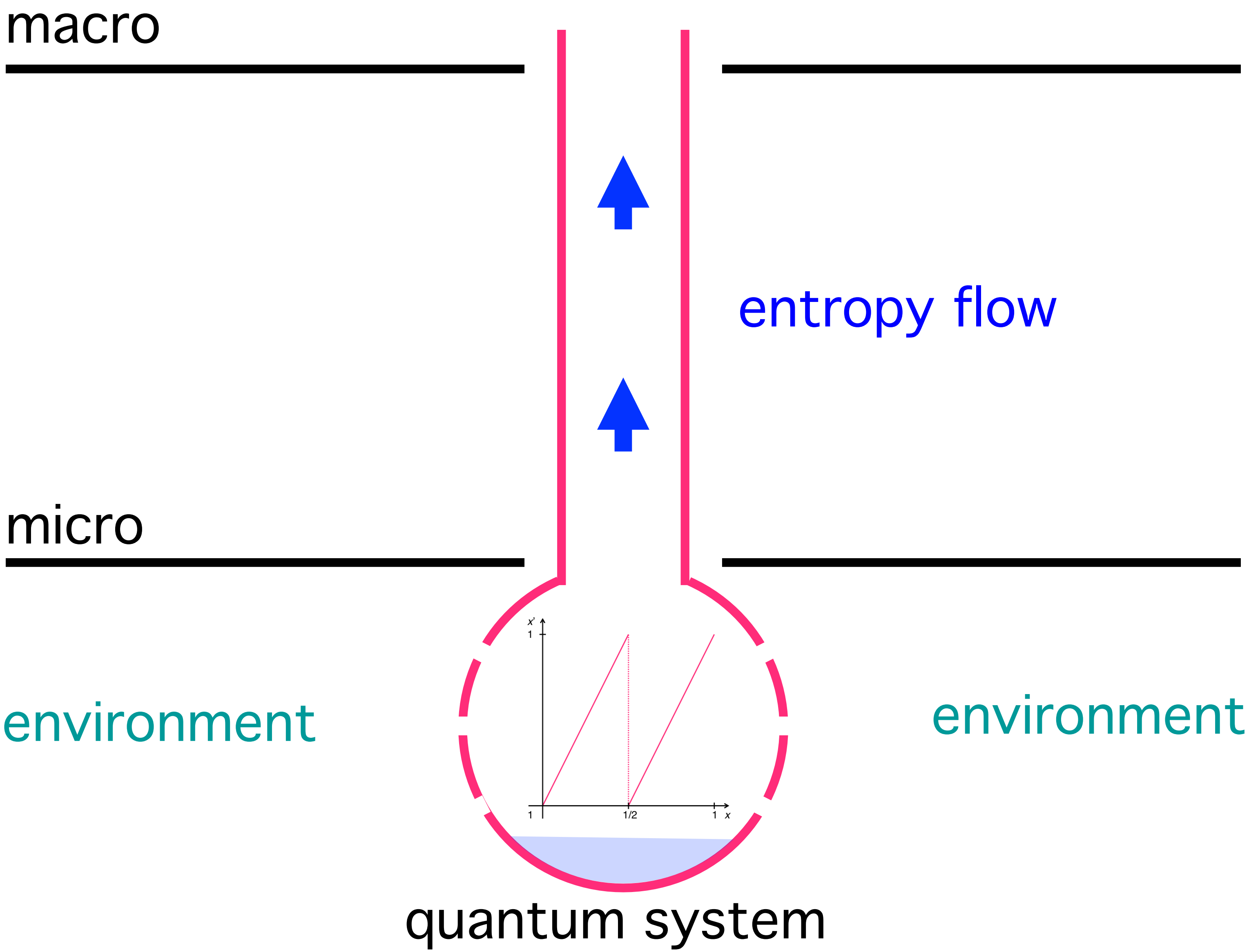


1 Classical bloom and quantum suppression of chaos

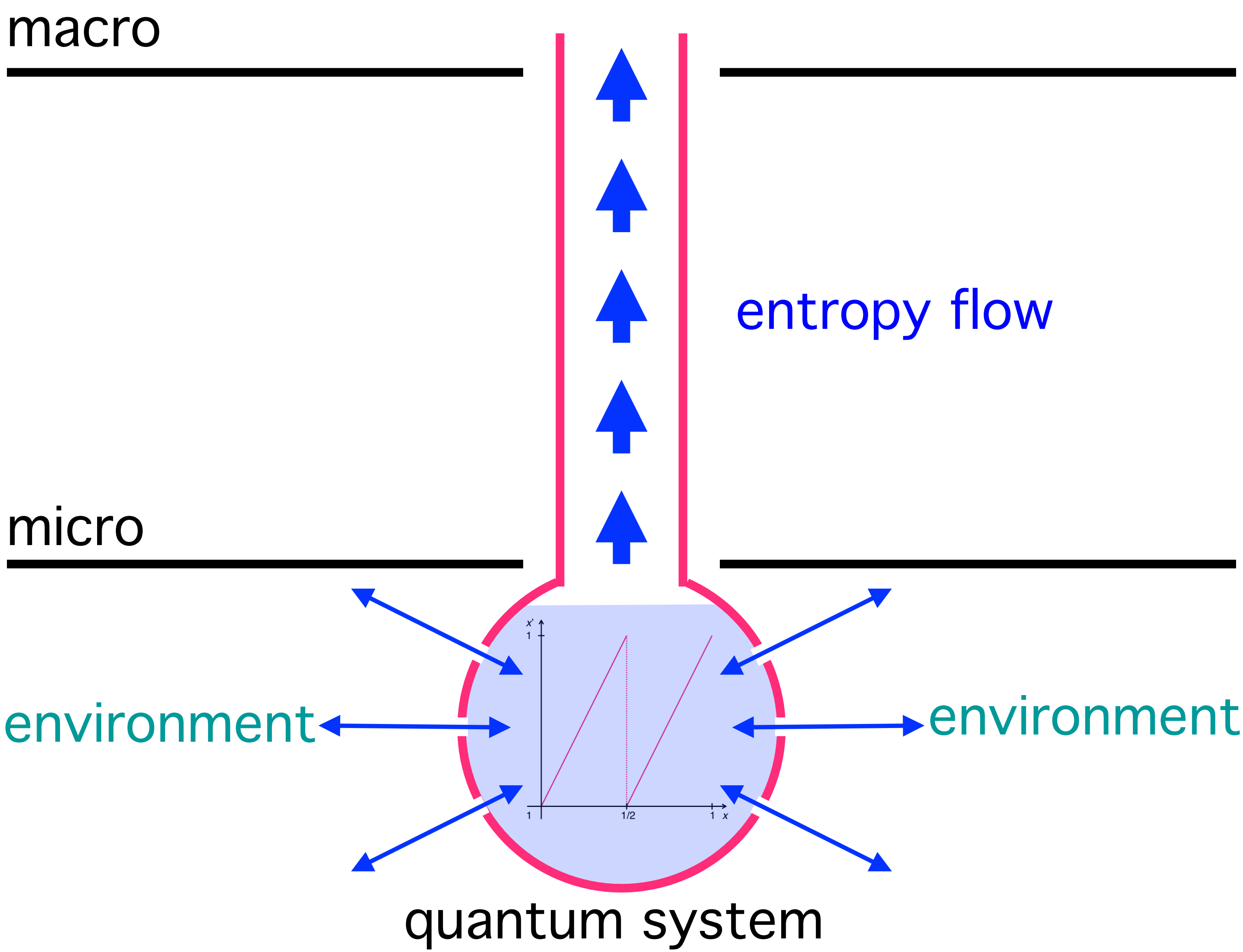
1.3 Breaking the splendid isolation



1.3 Breaking the splendid isolation



1.3 Breaking the splendid isolation



1.3 Breaking the splendid isolation

- The limits quantum mechanics imposes with respect to information measures apply only in strictly closed systems.

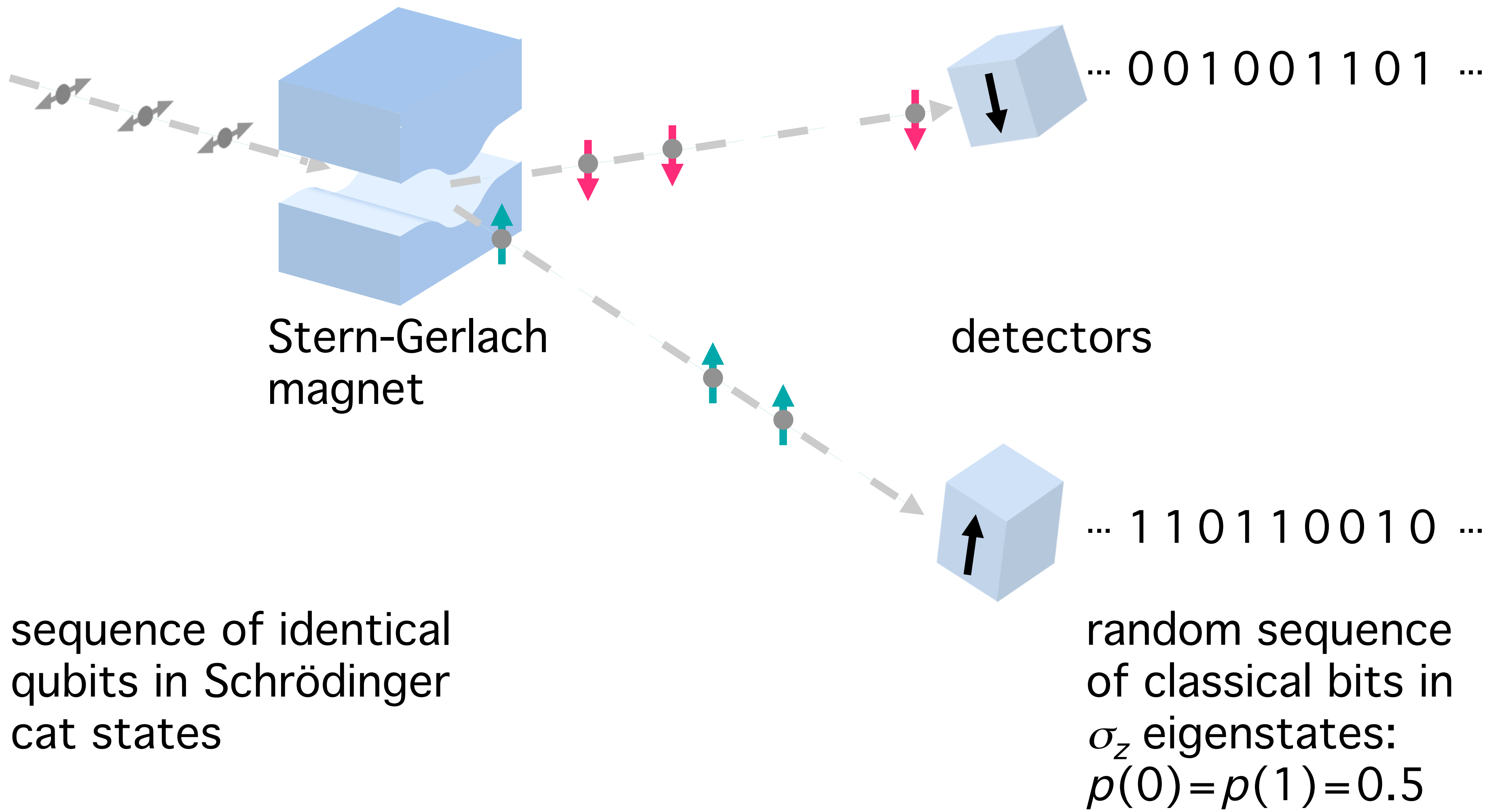
- No real system is perfectly closed (or else we could not observe it).

- Any coupling, however weak, to an environment with a (quasi-) continuous spectrum opens access to an external source of entropy.

- Paradigmatic cases are
  - decoherence (e.g., measurement) and
  - dissipation (e.g., friction).

2 Quantum measurement and quantum randomness

2.1 Spin measurements and randomness



2.2 Hypothesis

- In spin measurement, one classical bit of information per detected particle is produced “from nothing”.
- Such entropy production is incompatible with deterministic unitary time evolution of the spin alone.
- Consider spin, meter, and environment as a closed system evolving unitarily as a whole: The total information content is conserved!
- If a bit is “created” in the measurement (“quantum randomness”), it can only come from the macroscopic ensemble meter + environment.
- Can it be the result of a unitary time evolution, depending on the initial state of the full system, spin + meter + environment?

2.3 Three steps of a quantum measurement (von Neumann)

1 Generate correlations between object and meter states

through a suitable coupling, typically local in time

2 Turn quantum superpositions into classical alternatives

“first collapse of the wave packet”

creates entanglement due to the coupling of the meter
to a macroscopic environment,
leaves the measured object in a mixed state,
which only specifies probabilities for the outcomes of the measurement

3 Turn classical alternatives into sequences of random events

“second collapse of the wave packet”

upon leaving the apparatus, the measurement object returns to
a pure state, one of the eigenstate of the measured observable

2.3 Three steps of a quantum measurement

Entanglement is reciprocal!

As the measured system gets entangled with the apparatus and its environment, information on the state of the measured system is shared with the environment and recorded in it:

The environment as a witness.

By reciprocity of entanglement, information on the state of the apparatus is also shared with the measured system.

In this way, the time evolution of the measured system must also depend on the initial state of the environment.

2.4 Information balance of quantum measurement

1 Before the measurement: object, meter and environment in pure states

$$S_{\mathrm{o}} = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{o}}\,\mathrm{lb}\left(\hat{\rho}_{\mathrm{o}}\right)\right] = 0$$

2 During the measurement: object and meter entangled with the environment

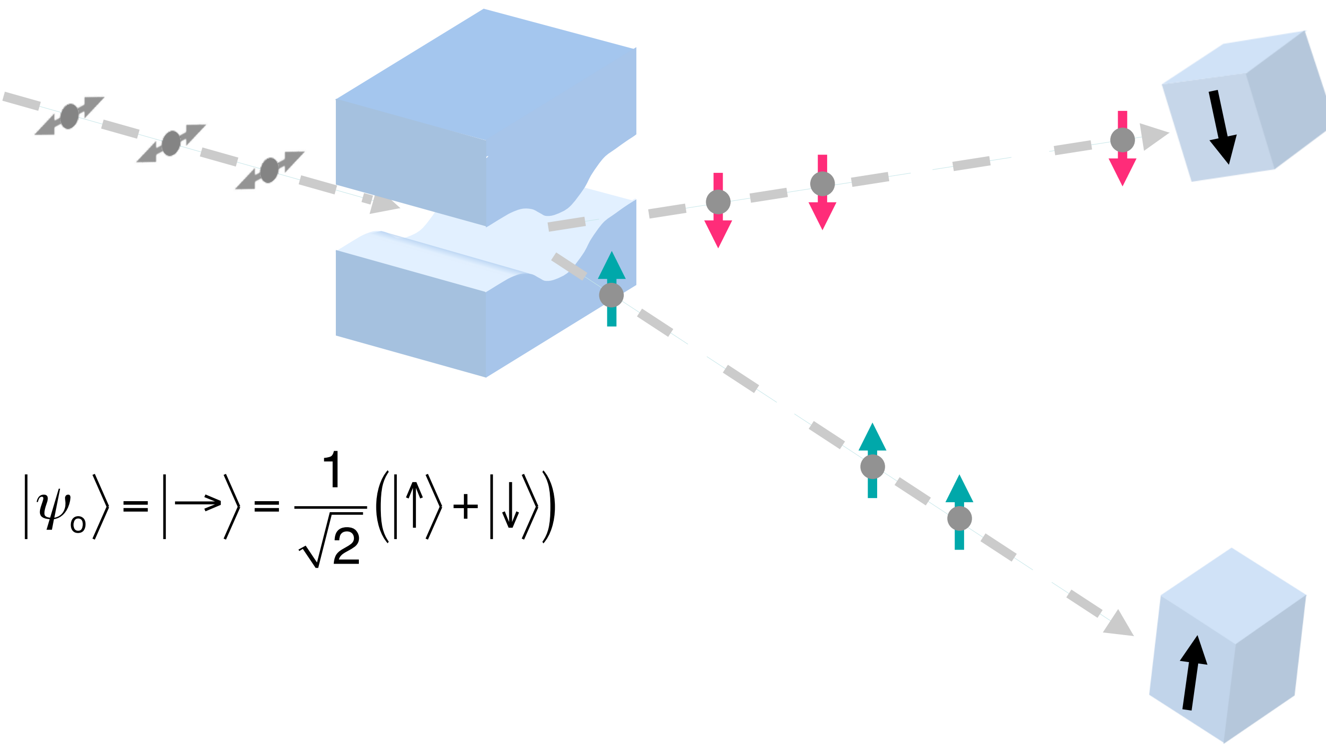
$$S_{\mathrm{o}} > 0$$

Note that the initial environment state $\left|\psi_{\mathrm{e}}\right\rangle$ is featureless so far.

3 After the measurement: object leaves in a pure state

$$S_{\mathrm{o}} = 0$$

2.4 Information balance of quantum measurement



… 0 0 1 0 0 1 1 0 1 …

$$|\psi_o\rangle = |\downarrow\rangle$$

$$\hat{\rho}_o = |\psi_o\rangle\langle\psi_o| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|\psi_o\rangle = |\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

… 1 1 0 1 1 0 0 1 0 …

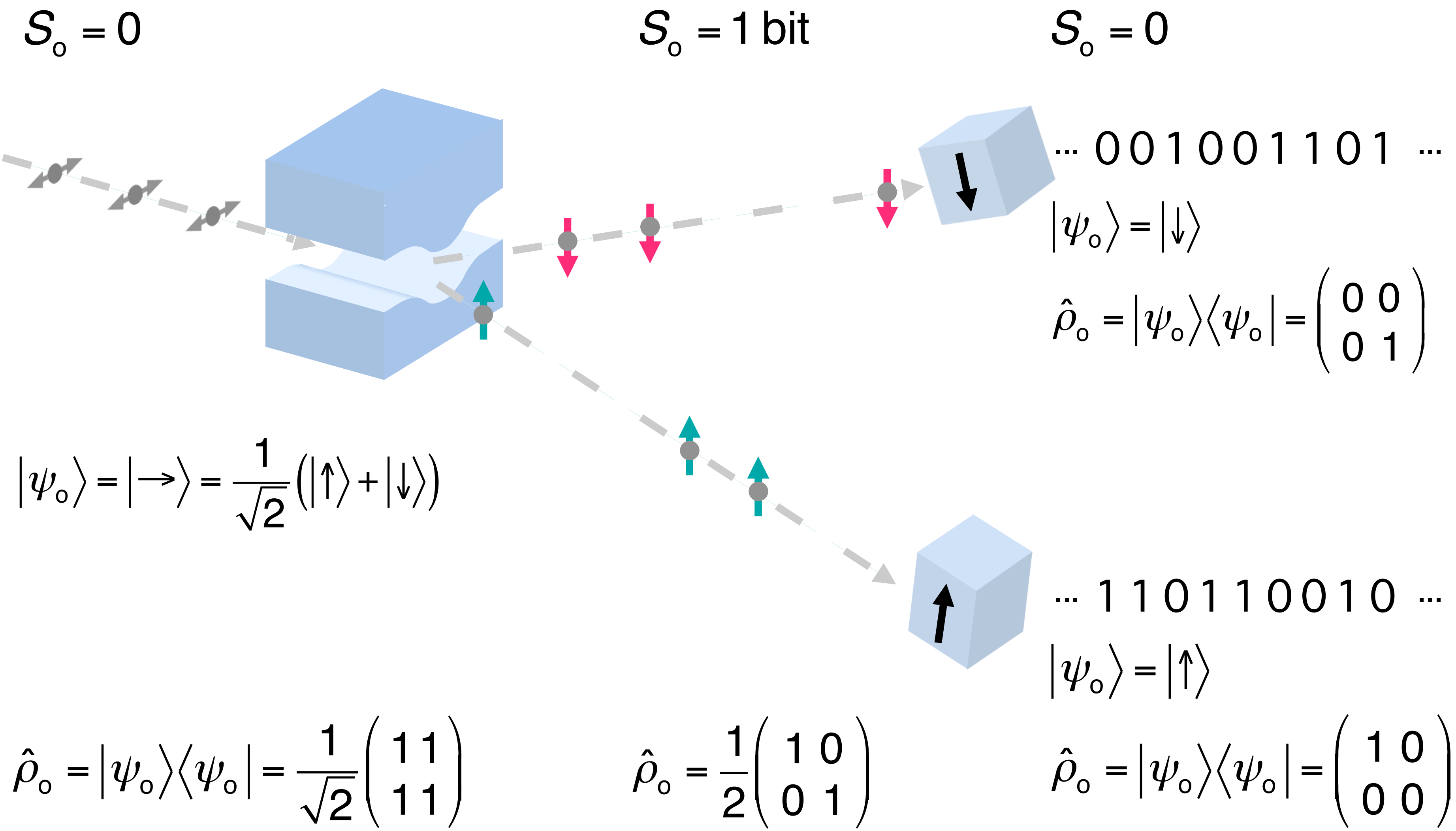
$$|\psi_o\rangle = |\uparrow\rangle$$

$$\hat{\rho}_o = |\psi_o\rangle\langle\psi_o| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\hat{\rho}_o = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

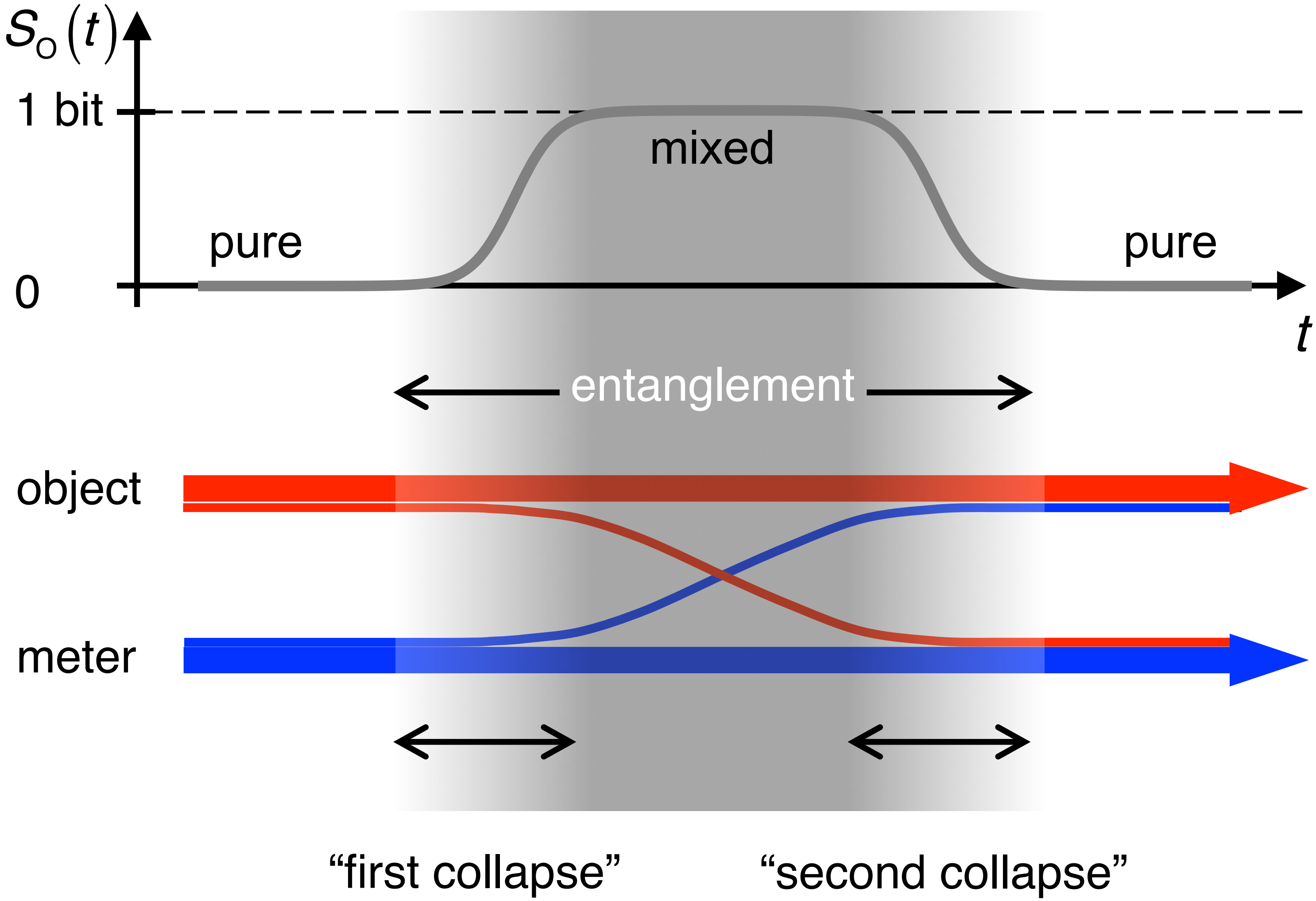
$$\hat{\rho}_o = |\psi_o\rangle\langle\psi_o| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

2.4 Information balance of quantum measurement



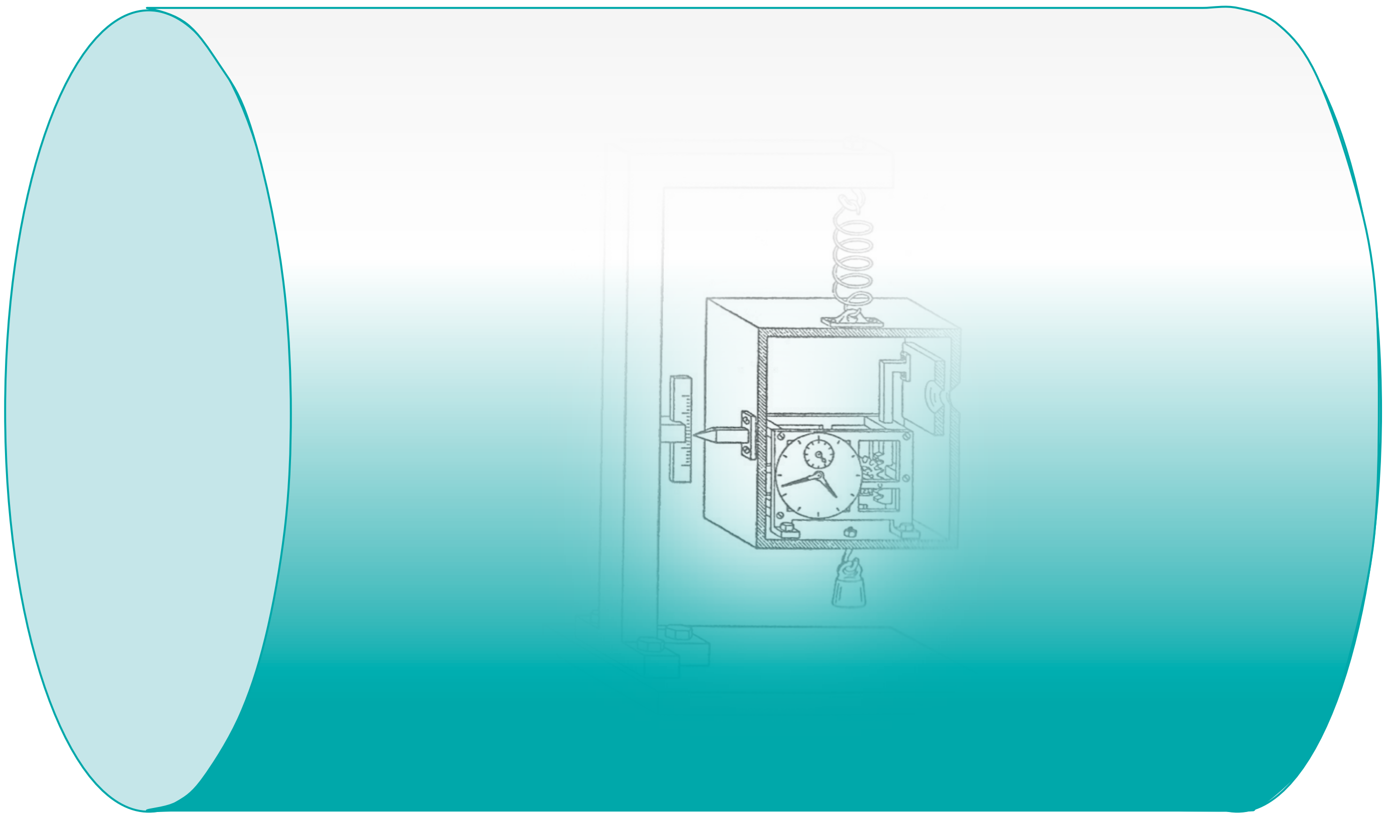
2.4 Information balance of quantum measurement

Partial entropy of the object



3 Numerical / experimental test

Measurement inside a closed cavity



3 Numerical / experimental test

3.1 Strategy

- The finite heat bath approach, developed for decoherence and dissipation in quantum optics and chemistry, treats system and environment as one closed system with unitary time evolution.

- Combine a model of decoherence in quantum measurement à la Zurek et al. with the finite heat bath approach for meter + environment.

- Apply powerful algorithms for numerical simulation of the unitary time evolution of object + (meter + environment).

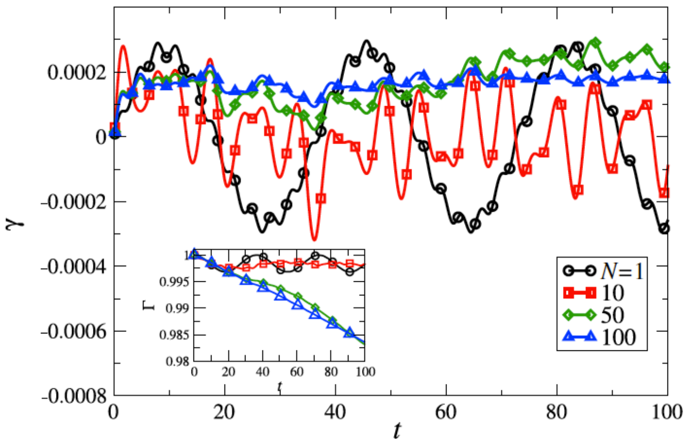
- Increase the number N of heat bath modes as far as possible.

- Register the dependence of the measurement results on the initial state of meter + environment.

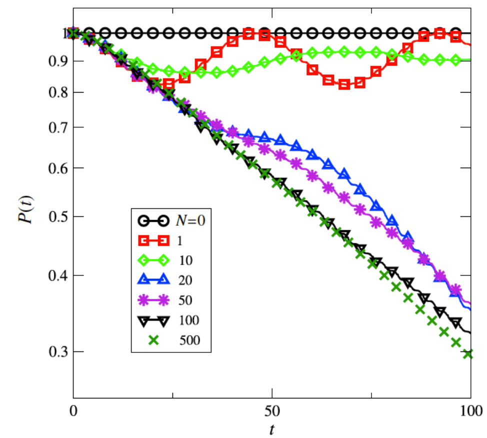
3 Numerical / experimental test

3.2 Finite heat bath approach

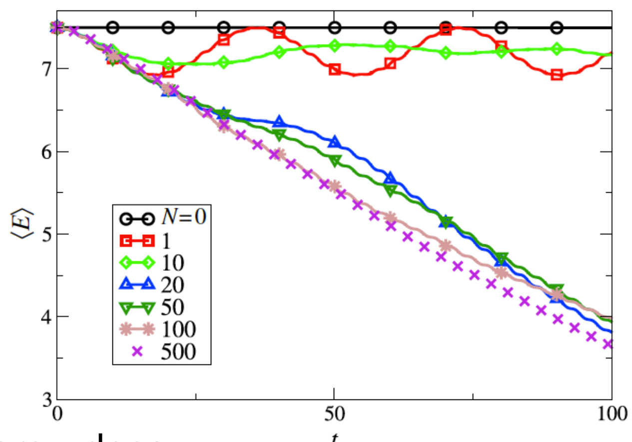
A parallel case: decoherence in a harmonic oscillator + finite bath*



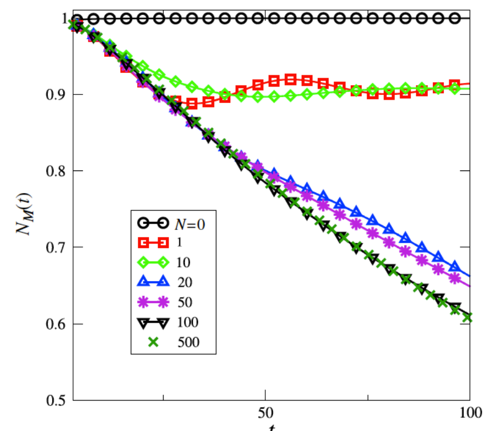
time-dependent dissipation rate



purity



energy decay



non-Markovianity

*M. Galiceanu, M. W. Beims, W. T. Strunz: “Quantum energy and coherence exchange with discrete baths”, Physica A 415, 294 (2014).

3 Numerical / experimental test

3.3 Protocol

1 Prepare the object system in a neutral (Schrödinger cat) state with respect to the observable to be measured (for example, in an eigenstate of $\hat{\sigma}_x$ or $\hat{\sigma}_y$, if $\hat{\sigma}_z$ will be measured).

2 Perform measurements with a meter that comprises a large but finite number N of degrees of freedom (bath modes)

3 Register the evolution and the outcome of the measurements as they vary with growing N,

observe how, as $N \to \infty$, a typical few-body quantum dynamics (e.g., collapses and revivals), crosses over to that of a measurement with a definite result that depends on the initial state of the meter.

3.4 Quantum model

Hamiltonians (spin-boson model with N boson modes)

$$\hat{H} = \hat{H}_{\mathrm{o}} + \hat{H}_{\mathrm{om}} + \hat{H}_{\mathrm{m}}$$

$$\hat{H}_{\mathrm{o}} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_x$$ two-level system

$$\hat{H}_{\mathrm{om}} = \sum_{n=1}^{N}\hat{\sigma}_z\left(g_n\hat{a}_n^\dagger + g_n^*\hat{a}_n\right)$$ coupling

$$\hat{H}_{\mathrm{m}} = \sum_{n=1}^{N}\hbar\omega_n\left(\hat{a}_n^\dagger\hat{a}_n + \frac{1}{2}\right)$$ meter (cavity with N modes)

Initial state

$$\left|\Psi(0)\right\rangle = \left|\psi_{\mathrm{o}}(0)\right\rangle\left|\Psi_{\mathrm{m}}(0)\right\rangle$$

two-level system $\left|\psi_{\mathrm{o}}(0)\right\rangle$:

pure state with $\left\langle\hat{\sigma}_z\right\rangle = 0$, e.g.,

$$\left|\psi_{\mathrm{o}}(0)\right\rangle = \left|\pm_x\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow_z\right\rangle \pm \left|\downarrow_z\right\rangle\right)$$

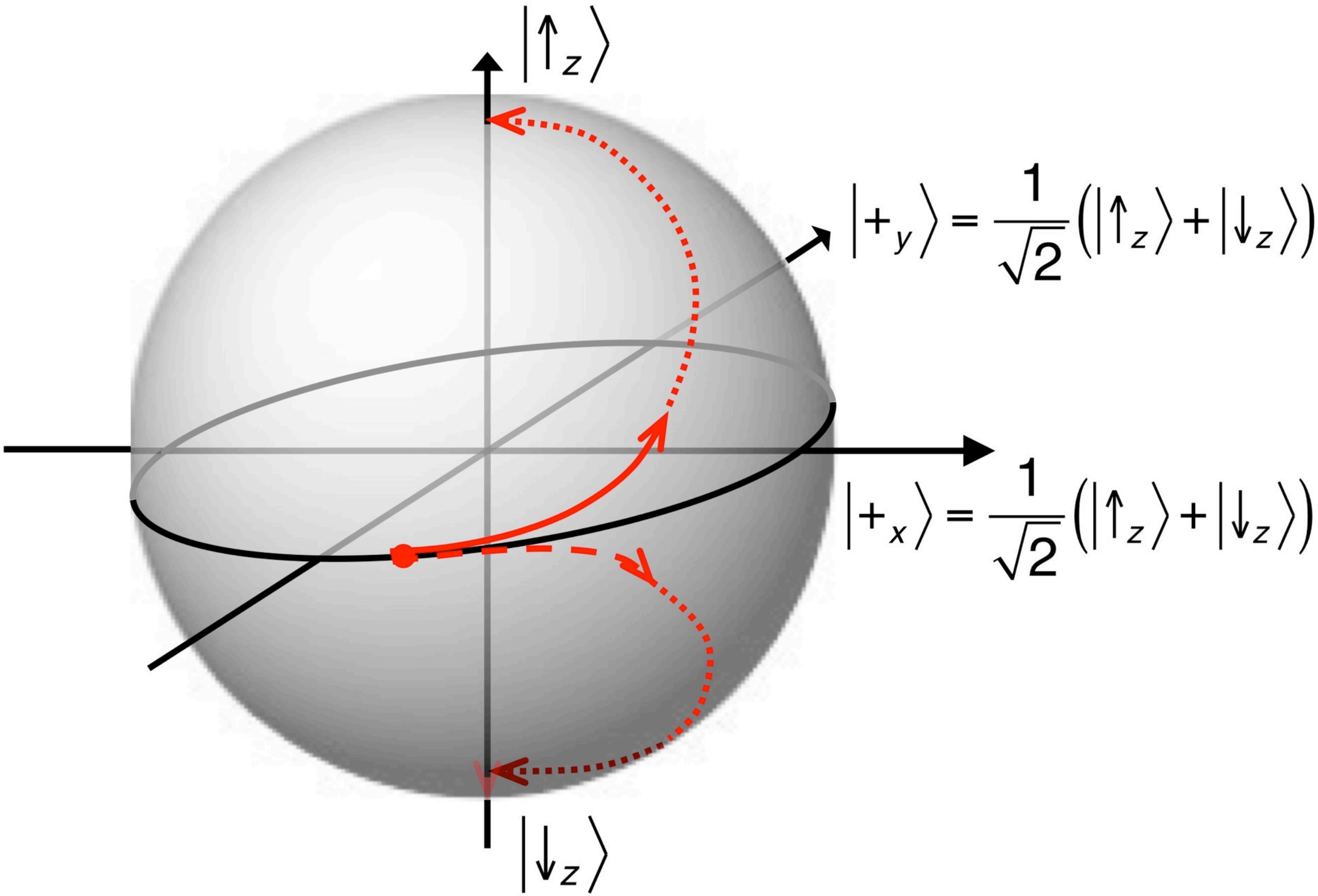
meter $\left|\Psi_{\mathrm{m}}(0)\right\rangle$:

pure state with

$$\left|\Psi_{\mathrm{m}}(0)\right\rangle = \bigotimes_{n=1}^{N}\left|\psi_{\mathrm{m,n}}(0)\right\rangle, \quad \left\langle\hat{\mathrm{P}}\right\rangle = \left\langle\hat{\mathrm{X}}\right\rangle = 0$$

3 Numerical / experimental test
3.4 Quantum model

Expected initial time evolution of the state vector on the Bloch sphere



3 Numerical / experimental test

3.5 Numerical results for $N = 1$

Time evolution of the reduced density matrix of the spin, represented on the Bloch sphere, numerical simulations for $\omega_0 = 1.0$, $N = 150$

"God does not throw dice."

Yes He does.
But they fall deterministically.