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Network Analysis of Multivariate Transfer Entropy of Cryptocurrencies in Times of Turbulence

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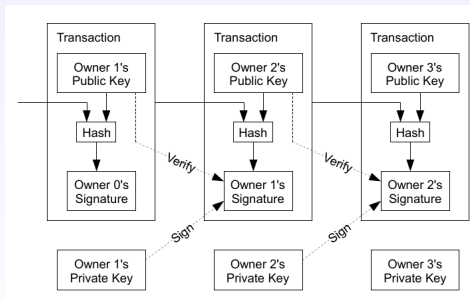
Entropy 2021: The Scientific Tool of the 21st Century

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Cryptocurrencies

- Cryptocurrencies are new financial instruments based on the technology of blockchains ¹.
- A coin is defined as a chain of digital signatures.



- In Bitcoin, each owner transfers the coin by digitally signing a hash of the previous transaction and the public key of the next owner, adding them to the end of the coin.

¹S. Nakamoto, Bitcoin: A Peer-to-Peer Electronic Cash System (2009)

Some facts about cryptos ²

- Cryptos: 9,418
- Exchanges: 369
- Market Cap: \$2,072,106,076,497.00usd (top ten compared to GDP)
- Diem is a permissioned blockchain-based payment system proposed by Facebook (experimental code has been released).
- American users of Paypal can now make payments through: Bitcoin, Ethereum, Litecoin y Bitcoin Cash.
- Federal Reserve Chairman Jerome Powell referred to the digital dollar as a monetary alternative to be evaluated at the IMF's annual meeting (October 2020)
- At the same event, Kristalina Georgieva, the director of the IMF, mentioned that *Today we face a new Bretton Woods moment.*

²At April 21 2021

Objectives

- We analyze the cryptocurrency network induced by the estimation of the multivariate transfer entropy as proposed by ³
- We are especially interested to understand the effects of the financial turbulence of 2020 on the market of cryptocurrencies taking into account the price and volume of transactions as a variable of interest.
- We study the systematic risk and contagion between the currencies through the transfer entropy when the cryptocurrency market is in a turbulent situation.
- Further, to obtain deeper insights about the structure of the induced network, the March 2020 turmoil is explored by quantifying the clustering coefficient and estimating the degree distributions of nodes.

³Novelli, L.; Wollstadt, P.; Mediano, P.; Wibral, M.; Lizier, J.T. Large-scale directed network inference with multivariate transfer entropy and hierarchical statistical testing. *Netw. Neurosci.* 2019, 3, 827–847

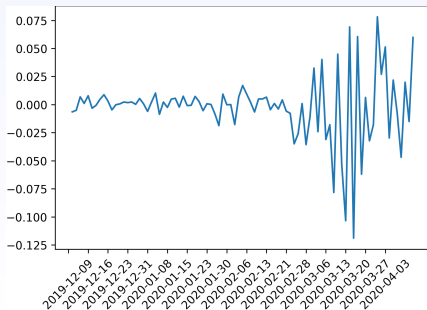
Research questions

- Do the directed networks associated to the multivariate transfer entropy of volume and price of cryptocurrencies present complex properties?
- Is it possible to characterize and in some sense to anticipate the turmoils on the cryptocurrency markets through the properties of the network induced by the multivariate transfer entropy?
- Can the clustering coefficients of these networks play the role of an early indicator of turbulence in these markets?
- Is there self-similarity in the induced networks, and if so, how do we interpret this characteristic in turbulent times?

Motivation

All Country World Index (ACWI):

- It is an index published by Morgan Stanley Capital International or MSCI inc.
- It is a *market capitalization-weighted index* designed to measure the behavior of equity markets around the world.
- It is weighted with the markets of 23 developed and 24 emerging countries.
- It is observed a 10% drop in a single day, the largest since 2008.



Transfer Entropy

- The Transfer Entropy (TE) from a process X to a process Y measures the amount of uncertainty reduced in future values of X by knowing the past states of Y and X itself.
- The multivariate Transfer Entropy (mTE) from X_i to Y can be defined as the information that the past of X_i provides about $Y = y_t$, in the context of both Y 's past and all the other relevant sources in X
- In principle, the mTE from X_i to Y is computed by conditioning on all the other sources in the network, i.e., $X \setminus X_i$. However, in practice, the sizes of the conditioning set must be reduced in order to avoid the curse of dimensionality.
- The idea is to restrict the conditioning set by finding and including the sources that participate with X_i in reducing the uncertainty about $Y = y_t$, in the context of Y 's own past. The set of relevant sources will be denoted as Z .
- To infer Z from X it is followed the greedy algorithm approach suggested by Lizier et.al.(2012) and Novelli et.al. (2019), where Z is built iteratively by maximizing the conditional mutual information (CMI) criterion.

Transfer Entropy

- As a first step, a set of candidate variables $c \in C$ is defined from the past values of X .
- At each iteration, the algorithm selects the past variable $c \in C$ that maximizes the conditional mutual information $I(c, y_t | Z)$ at significance level α of a maximum information test. The set of selected variables forms a multivariate, *non-uniform embedding* of X (Faes, 2011).
- Then, the mTE between a single source X_i and target Y can be computed from the inferred non-uniform embedding Z .
- To this end, it is collected from Z all of X_i 's selected past variables, X_i , and calculated the mTE as $I(X_i; y_t | Z \setminus X_i)$.
- The delay can be estimated as the lag of the past variable which provides the maximum individual information contribution, where the maximum contribution is indicated either by the maximum raw TE estimate or by the minimum *p-value* over all variables from the source.
- Finally, the algorithm must be repeated for every node (or variable) in the network.

Data

- We consider the price and volume in dollars of $p = 146$ cryptocurrencies, for the period from 00:00 2019/12/01 to 00:00 2020/04/05, at an hourly frequency, resulting in a total of $n = 3025$ observations.
- The cryptocurrencies are selected under the condition of having each less of 1% . In case of no record, a spline interpolation of order three was used.
- The $2p$ time series were transformed into price-returns $r_k^{(price)}(t)$, and volume-returns $r_k^{(volume)}(t)$ by

$$r_k^{(price)}(t) = \log(s_k(t)) - \log(s_k(t-1)) \quad (1)$$

$$r_k^{(volume)}(t) = \log(v_k(t)) - \log(v_k(t-1)) \quad (2)$$

where $k = 1, \dots, p$; $t = 1, \dots, n$; and $s_k(t)$, $v_k(t)$ stands for the price and volume of cryptocurrency k at time t , respectively.

- we have verified stationarity using the augmented Dickey-Fuller test and the Phillips-Perron test. In both tests it has been obtained a p -value less than 0.001 for all the $2p$ time series considered.

Implementation

- We consider each time series of price-returns and volume-returns of cryptocurrencies to be a stochastic process in order to detect the causal relations between the variables.
- The procedure is to fix the target variable $Y = X_i$, the source set as $X \setminus X_i$, and apply the mTE algorithm described above for each time series $i = 1, \dots, 2p$.
- we design a temporal analysis of time windows of 21 days, sliding by seven days, and using an overlapping of 14 days.
- Under this procedure, it is obtained $k = 16$ time windows.
- the first from the 01:00 of 12/01/2019 to 00:00 of 12/22/2019
- the last from 01:00 of 03/15/2020 to 00:00 of 04/05/2020.
- Thus, each data frame contain $q = 504$ observations of hourly price-returns and volume-returns, i.e., having dimensions $q \times 2p$.

Settings

- the number of permutations for the surrogate distribution used in the statistical tests (*maximum, mininum, omnibus*): 500
- significance level: 0.05
- Significant links were tested for one to three lags.
- The adjacency matrix has elements $A_{ij} = 1$ if there exit a statistically significant causal relation in this range of lags, and $A_{ij} = 0$ on the contrary, where $i, j = 1, \dots, 2p$.
- The graph visualization is made using the Kamada-Kawai algorithm.
- We have discriminated the price and volume variables separating their corresponding nodes by a fixed distance to the upper right if it corresponds to a price-return node, and to the lower left if it corresponds to a volume-return node.
- The magnitude of the nodes is according to their clustering coefficient.

mTE network of cryptocurrencies

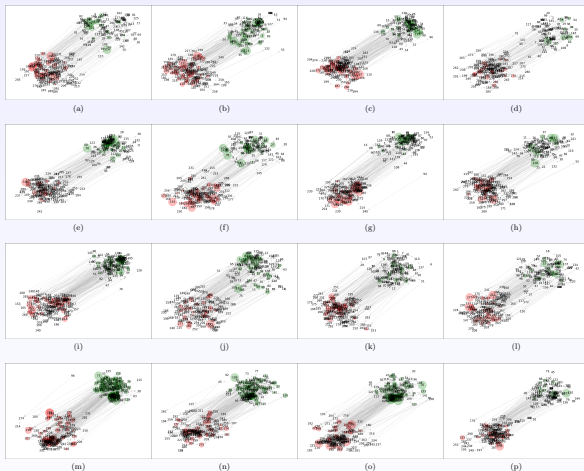
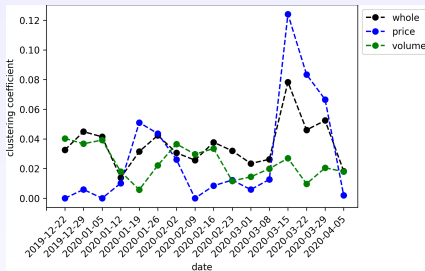


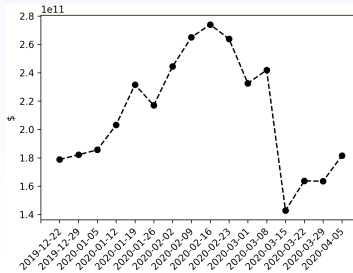
Figura: Network representation of mTE results for cryptocurrency variables. The green nodes represent price-return variables, while the red nodes represent volume-return variables.

Clustering coefficient

(a) Overall clustering coefficient as a function of time. Black dotted lines show the results for the whole networks, blue and green dotted lines show results for the price-return and volume-return nodes, respectively. (b) Market capitalization averaged over the cryptocurrencies under study at the end of the time window. The units in the y-axis are measured in hundred million dollars.

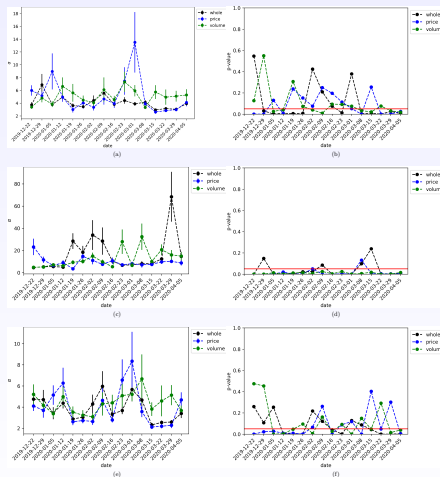


(a)



Power law

(a-b) show the dynamic of the estimated power α and corresponding p -value for the degree distribution. (c-d) show the dynamic of the estimated power α and corresponding p -value for the in-degree distribution. (e-f) show the dynamic of the estimated power α and corresponding p -value for the out-degree distribution. In all cases, the black line represents the results for the whole network, the blue line for the price-returns subgraph, and the green line for the volume-returns subgraph. The red straight line is the significance threshold of 0.05 and the vertical lines centered at each point represents the standard deviations.



Conclusions

- The graph visualization of the interactions of price-returns and volume-returns distinguish three time windows where the strength of the attractive forces between nodes stand out to be stronger than the other scenarios. Interestingly, these periods coincide with the most severe fall of the recent worldwide stock markets crash on March 2020.
- Our greatest contribution arose finding that the clustering coefficient of the whole network, as well as the price-returns subnetwork, increases dramatically during the same periods of major financial contraction, where we use as an indicator of turbulence the market capitalization of the cryptocurrencies under study.
- The log-likelihood in all cases bent over a power law distribution, giving evidence of the complex nature of the network. Most importantly, it was found that the power of the distribution has higher estimated values during March 2020, which provides further support to our hypothesis: the structure of the induced cryptocurrency network by mTE changes during times of turbulence in the sense of higher clustering coefficient and complexity.

Thank you!

